№ 2582

$$\frac{(n!)^2}{2n^2}$$

$$\frac{\cancel{((n+1)!)^2}\,\cancel{2}n^2}{\cancel{2}(n+1)^2\,\cancel{(n!)^2}} = \frac{\cancel{(n+1)^2}\,n^2}{\cancel{(n+1)^2}} = n^2$$

№ 2583

$$\frac{1000}{1} + \frac{1000\ 1001}{1\ \ 3} + \frac{1000\ 1001\ 1002}{1\ \ 3\ \ 5}$$

$$\frac{a(n+1)}{a(n)} = \frac{1000+n+1}{2(n+1)-1} = \frac{1001+n}{2n+1} = \lim \frac{1+\frac{1001}{n}}{2+\frac{1}{n}} = 1/2 \quad \text{сх-ся}$$

№ 2584

$$\frac{4}{2} + \frac{4\ 7}{2\ 6} + \frac{4\ 7\ 10}{2\ 6\ 10} \quad \frac{\cdot(1+3n)}{\cdot(4n-2)}$$

$$\frac{a(n+1)}{a(n)} = \frac{1+3(n+1)}{4(n+1)-2} = \frac{2+3n}{4n+2} = \frac{3}{4} \quad \text{сх-ся}$$

2585

$$\sum_{n=1}^{\infty} (\sqrt{2}-\sqrt[3]{2})(\sqrt{2}-\sqrt[5]{2}) \quad (\sqrt{2}-\sqrt[2n+1]{2})$$

$$\lim \left(\sqrt{2}-\sqrt[2(n+1)+1]{2}\right)=\sqrt{2}-\lim 2^{\frac{1}{2n+3}}=\sqrt{2}-1<1 \quad \text{сх-ся}$$

2585 1

$$a_n = \begin{cases} 1/n & n=m^2 \\ 1/n^2 & n\neq m^2 \end{cases}$$

№2812

$$\sum \frac{x^n}{n^p} \qquad \frac{1}{R}=\lim \sqrt[n]{a_n}$$

$$R=\lim \frac{a_n}{a_{n+1}}$$

$$\lim \frac{n^p}{(n+1)^p}=\left(\frac{n}{n+1}\right)^p=\left(\frac{1}{1+\frac{1}{n}}\right)^p=1^p$$

$R=1$

$(-1,1)$

$x=-1$

$$\sum_{1}^{\infty}\left|\frac{(-1)^n}{n^p}\right| \quad p>1 \text{ сх-ся абс} \qquad x=1 \quad \frac{1}{n^p} \quad \begin{matrix} p>1 \text{ сх-ся} \\ p\leq 1 \text{ рас-ся} \end{matrix}$$

№2813

$$\cancel{\sum_{n=1}^{\infty} \frac{3^n+(-2)^n}{n}(x+1)^n}$$

2814

$$\sum \frac{(n!)^2}{(2n)!} x^n$$

$$R = \lim \frac{a_n}{a_{n+1}} = \frac{(n!)^2}{(2n)!}\frac{(2n+2)!}{((n+1)!)^2} = \frac{2(2n+1)(2n+2)}{(n+1)^2} = 2\lim \frac{2+\frac{1}{n}}{1+\frac{1}{n}} = 4$$

$(-4, 4)$

$x = -4$ $\quad \frac{(n!)^2(-4)^n}{(2n)!}$

$$\lim \frac{((n+1)!)^2(-4)^{n+1}}{(2n+2)!}\frac{(2n)!}{(n!)^2(-4)^n} = \left|\frac{-4(n+1)^2}{(2n+1)(2n+2)}\right| = \frac{+4}{2}\lim\frac{n+1}{2n+1} = 1$$

$$\frac{1\cdot(-4)}{2} < \frac{\cancel{4}\cdot 16\,8}{\cancel{4}\,3\,\cancel{2}} < \frac{\cancel{36}\cdot\cancel{64}\,16}{\cancel{6}\,5\,\cancel{4}\,\cancel{3}\,\cancel{2}} \Rightarrow$$ расх-ся т.к. 1п пр Лейбница не вып

$2 < 2\frac{2}{3} < 3\frac{1}{5}$

2813

$$\sum_{n=1}^{\infty} \frac{3^n+(-2)^n}{n}(x+1)^n = \sum \frac{3^n}{n}(x+1)^n + \sum \frac{(-2)^n}{n}(x+1)^n$$

1) $\frac{1}{3}\lim \frac{\cancel{3^n}}{n}\frac{(n+1)}{\cancel{3^n}} = \frac{1}{3}$

2) $\lim \left|\frac{\cancel{(-2)^n}(n+1)}{n \cdot (-2)\cancel{(-2)^n}}\right| = \frac{1}{2}$

$\Rightarrow R = \frac{1}{3}$

№2815

$$\sum_{n=1}^{\infty} \alpha^{n^2} x^n \quad 0<\alpha<1$$

$$R = \lim \left|\frac{1}{\sqrt[n]{\alpha^{n^2}}}\right| = \lim_{n\to\infty} \frac{1}{\alpha^n} = \infty$$

№2816

$$\sum \left(1+\frac{1}{n}\right)^{n^2} x^n$$

$$R = \lim \frac{1}{\sqrt[n]{a_n}} = \lim\left(\left(1+\frac{1}{n}\right)^n\right)^{-1} = \frac{1}{e}$$

$\left(-\frac{1}{e}, \frac{1}{e}\right)$

$x = -\frac{1}{e}$ $\left(1+\frac{1}{n}\right)^{n^2}\left(-\frac{1}{e}\right)^n = \left(1+\frac{1}{n}\right)^{n^2}\left(\frac{1}{e}\right)^n(-1)^n$

по пр Лейбница

$\lim\left(1+\frac{1}{n}\right)^{n^2}\left(\frac{1}{e}\right)^n = 1 \neq 0 \Rightarrow$ ряд расход

№ 2817

$$\sum \frac{n!}{a^{n^2}} x^n \quad a>1$$

$$R = \lim_{n\to\infty} \left| \frac{n!\, a^{(n+1)^2}}{a^{n^2}(n+1)!} \right| = \frac{a^{2n+1}}{(n+1)} = \infty$$

$$(-\infty, +\infty)$$

№ 2818

$$\sum \left(\frac{1\,3 \ldots (2n-1)}{2\,4 \ldots 2n} \right)^p \left(\frac{x-1}{2} \right)^n$$

$$\lim \left| \frac{a_n}{a_{n+1}} \right| = \frac{(1\,3 \ldots (2n-1))(\cancel{2\,4 \ldots 2n})\,2^{n+2}}{(\cancel{2\,4 \ldots 2n})(1\,3 \ldots \cancel{(2n-1)})(2n+1)} = 2 \lim \left(\frac{2n+2}{2n+1} \right)^p = 2$$

$$-2 < x-1 < 2$$

$$-1 < x < 3 \quad (-1, 3)$$

$x = -1$

$\sim (-1)^n$ сх-ся абс $\quad \lim = \frac{1}{2}$

$x = 3 \qquad$ сх-ся

№ 2819

$$\sum \underbrace{(-1)^n \left[\frac{2^n (n!)^2}{(2n+1)!}\right]^p}\, x^n$$

$$R = \left|\frac{\cancel{2^n}(n!)^2\,(2(n+1)+1)!}{(2n+1)!\cdot 2\,\cancel{2^n}((n+1)!)^2}\right|^p = \left|\frac{\cancel{(2n+2)}(2n+3)}{\cancel{2}(n+1)^{\cancel{2}}}\right|^p = (2)^p$$

$$(-2^p,\ 2^p)$$

$$x = -2^p$$

$$\sum (-1)^n \left[\frac{2^n (n!)^2}{(2n+1)!}\right]^p (-2)^{pn} = (-1)^p (+2)^{np}$$

$$\lim \left(\frac{(n+1)}{(2n+3)}\right)^p \frac{\cancel{2}^{p(n+1)}}{\cancel{2^{pn}}}\quad 2^p$$

сх-ся абс

№ 2820

$$\sum \frac{m(m-1)\ \ (m-n+1)}{n!} x^n$$

$$R = \lim \frac{m(m-1)\ .(m-n+1)\,(n+1)!}{n!\ m(m-1)\ \ (m-n)} = \left|\frac{(n+1)}{(m-n)}\right| = 1$$

$(-1, 1)$

$x = -1$

$$\lim \left| \frac{m(m-1) \quad (m-n+1)}{n!} \right|$$

2821

$$\sum \left(\frac{a^n}{n} + \frac{b^n}{n^2} \right) x^n$$

$$\sum \frac{a^n}{n} x^n + \sum \frac{b^n}{n^2} x^n$$

$$R_1 = \lim \frac{1}{\sqrt[n]{a_n}} = \frac{1}{a}$$

$$R_2 = \lim \frac{1}{\sqrt[n]{a_n}} = \frac{1}{b}$$

$$R = \min \left(\frac{1}{a}, \frac{1}{b} \right)$$

$a < b$

$$\left(-\frac{1}{b}, \frac{1}{b} \right)$$

$x = -\frac{1}{b}$

$$(-1)^n \left(\frac{a^n}{n} + \frac{b^n}{n^2} \right) \frac{1}{b^n} = \lim_{n \to \infty} \left| \frac{a^n}{b^n n} \right| = \left(\frac{a}{b} \right)^n \frac{1}{n} = 0$$

$$\frac{a}{b} + 1 > \frac{a^2}{2b^2} + \frac{\cancel{b^2}}{4\cancel{b^2}}$$

$$\frac{1}{2} \left(\frac{a}{b} \right)^2 + \frac{1}{4}$$

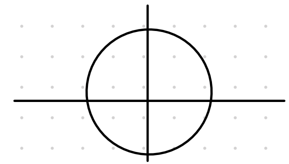
№ 2822

$$\sum_{n=1}^{\infty} \frac{x^n}{a^n+b^n}$$

№ 2936

$$f(x)=\sin^4 x=\left(\frac{1-\cos 2x}{2}\right)^2=\frac{1-2\cos 2x+\cos^2 2x}{4}=$$

$$\frac{1-2\cos 2x+\frac{1+\cos 4x}{2}}{4}=\frac{3-4\cos 2x+\cos 4x}{8}$$



$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx=\frac{1}{8\pi}\left(3x\Big|_{-\pi}^{\pi}-2\sin 2x\Big|_{-\pi}^{\pi}+\frac{1}{4}\sin 4x\Big|_{-\pi}^{\pi}\right)=\frac{3}{4}$$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}\left(\frac{3-4\cos 2x+\cos 4x}{8}\right)\cos nx=$$

$$\frac{1}{\pi}\int_{-\pi}^{\pi}\frac{3\cos nx}{8}dx=\frac{3}{8\pi n}\sin nx\Big|_{-\pi}^{\pi}=0$$

$$-\frac{1}{\pi 2}\int_{-\pi}^{\pi}\cos 2x\cos nx\,dx=-\frac{1}{2\pi}\int\frac{1}{2}\left(\cos(x(2-n))+\cos(x(2+n))\right)=\frac{1}{4\pi}\left(\frac{\sin x(2-n)}{(2-n)}\Big|_{-\pi}^{\pi}+\frac{\sin(2+n)x}{(2+n)}\Big|_{-\pi}^{\pi}\right)=0$$

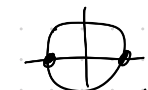
$$f(x)=\frac{3}{8}$$

2961

$f(x) = x^2$

а) $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 dx = \frac{1}{\pi}\frac{x^3}{3}\Big|_{-\pi}^{\pi} = \frac{2\pi^3}{3\pi} = \frac{2}{3}\pi^2$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \cos nx\, dx =$



$x^2\cos nx = \begin{bmatrix} v = x^2 & v' = 2x\,dx \\ u' = \cos nx & u = \frac{\sin nx}{n} \end{bmatrix} = \frac{x^2 \sin nx}{n} - \int \frac{2x \sin nx}{n} dx = \underbrace{\frac{x^2 \sin nx}{n}}_{0} - \left(\frac{-2x\cos nx}{n^2} + \underbrace{\int \frac{\cos nx\, dx}{n^2}}_{0}\right) = \frac{4\pi(-1)^{n+1}}{n^2}$

$f(x) = \frac{2}{3}\pi^2 + \sum (-1)^{n+1}\frac{4\pi}{n^2}\cos nx$

б) $[0, \pi)$

$b_n = \frac{2}{\pi}\int_0^{\pi} x^2 \sin nx\, dx = \frac{2}{\pi}\left(\frac{-x^2\cos nx}{n} + \int \frac{2x\cos nx\, dx}{n}\right) = \frac{2}{\pi}\left(\frac{-x^2\cos nx}{n} + \underbrace{\frac{2x\sin nx}{n^2}}_{0} + \frac{2\cdot\cos nx\, dx}{n^3}\right)$

$b_n = -\frac{2\pi(-1)^{n+1}}{n} + \frac{2(-1)^{n+1}}{\pi n^3}$

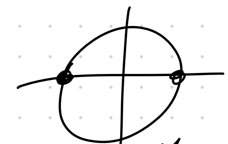
в) $[0, 2\pi)$

$a_0 = \frac{1}{\pi}\int_0^{2\pi} x^2 = \frac{x^3}{3\pi}\Big|_0^{2\pi} = \frac{8\pi^{\cancel{3}2}}{3\cancel{\pi}}$

$$a_n = \frac{1}{\pi}\int_0^{\pi} x^2 \cos nx\,dx = \frac{2x\cos nx}{n^2}\Big|_0^{2\pi} = \frac{2x}{n^2}$$

№2940

$f(x) = x$



$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x\,dx = \frac{1}{\pi}\,\frac{x^2}{2}\Big|_{-\pi}^{\pi} = 0$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x\cos nx\,dx = \frac{1}{\pi}\left(\frac{x\sin nx}{n}\Big|_{-\pi}^{\pi} - \int_{-\pi}^{\pi}\frac{\sin nx}{n}\,dx\right) = \frac{1}{\pi}\,\frac{\cos nx}{n^2}\Big|_{-\pi}^{\pi} = \frac{(-1)^{n+1}}{n^2\pi}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x\sin nx\,dx = \frac{1}{\pi}\left(\frac{-x\cos nx}{n}\Big|_{-\pi}^{\pi} + \underbrace{\int_{-\pi}^{\pi}\frac{\cos nx}{n}\,dx}_{=0}\right) = \frac{-2(-1)^{n+1}}{n} = \frac{2(-1)^n}{n}$$

№2951