A case study for expanding the complex plane to a more attractive mathematical set

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Abstract

The aim of this document is to challenge the common fact within the mathematical world claiming that $\mathbb{C}$ is as complete as we would think about it. This paper does not deny any commonly known properties of the set such that "$\mathbb{C}$ is an algebraic closure of $\mathbb{R}$" but rather blame the mathematic community for not trying to extend this set and account for solving problems like the following equation,

$$z^* z = -1$$

and their interest in widening the range of possibilities in modern daily life mathematics.

1 Introduction

While maintaining the consistency of the complex plane, we wish to introduce an innovative approach to construct a set that solves for equations of the type,

$$z^* z = -1 \tag{1}$$

where * denotes the conjugation sign known in $\mathbb{C}$ *as* $(a + ib)^* = (a - ib)$. We will first briefly discuss the assumptions that denote the set that contains solutions to the equation (1), assumptions which are regarded as axiomatic definitions.

Definition 1 *Let there be* $\mathbb{L}$ *such that* (1) *admits at least one solution,* $l \in \mathbb{L}$
Following are a list of assumptions that hopefully characterize the set $\mathbb{L}$

1. $\mathbb{C} \subset \mathbb{L}$
2. $(\mathbb{L}, +, .)$ *is a vector space over the field* $\mathbb{C}$

[*] Bashar
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[‡] and GOD above all.

3. $(\mathbb{L}, +, \cdot)$ *is a field*

4. *The conjugation operator* * *keeps the same known qualities:*
   - *distributivity over* $+$: $(a+b)^* = a^* + b^*$
   - *distributivity over* $\cdot$: $(a \cdot b)^* = a^* \cdot b^*$
   - *involutivity:* $a^{**} = a$

Proposition 1 *From the above* 1.3 *it follows that equation* (1) *does not admit a unique solution. There are in fact infinitely many:*

$$\forall \phi \in \mathbb{R}, (e^{i\phi} \cdot l)^* \cdot (e^{i\phi} \cdot l) = -1$$

$$\forall n \in \mathbb{Z}, (l^{2n+1})^* \cdot (l^{2n+1}) = -1$$

A justification for the second clause follows from the following result of 1.1 *that is, "l is not a zero of any element of* $\mathbb{C}[X]$*". Otherwise, l would be in* $\mathbb{C}$ *For similar reasons, the following property holds as well:*

$$l^* \neq l$$

2 Announcing the traits of the set

Following our introduction, we want to discuss some traits of the set $\mathbb{L}$ in the following section and show some properties that do not need formal proofs in that they are easy to understand.
First, let us witness from *Proposition* 1 and earlier results that l is not distinguishable from any of its odd powers, nor of its phases with respect to equation (1). We will further see to it that we solve (1) in $\mathbb{L}$ and derive the solutions for equations of the type $y^*y = x$ with generic values of x, yet being open to complete $\mathbb{L}$ in that aspect if needed. Regarding the dimensionality of $\mathbb{L}$ being a field as well as a $\mathbb{C}$ vector space, we can derive from the above results that $(l^n)_{n\in\mathbb{Z}}$ is a linearly independent family. Thus $\mathbb{L}$ is infinite-dimensional. $\mathbb{C}$ being algebraically closed, together with the assumed field structure of $\mathbb{L}$ guarantees that property.

From $l^* = -l^{-1}$, we can consider that $(l^n)_{n\in\mathbb{Z}} \sim (\cdots, {l^*}^2, l^*, 1, l, l^2, \cdots)$. This family does not necessarily constitute a base for the vector space $\mathbb{L}$ as we will guide ourselves through solving the following set of equations

$$y^*y = x \mid x \in \mathbb{C} \tag{2}$$

considering the solutions to be elements of $\mathbb{L}$ and we will see what implications there are regarding the structure of this set.

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About the author:

Suffices to know that my testament holds enough information about me. All thanks are due to God, my Father and your Father. My discoveries concerning this set go far beyond what has been laid down in this document and both Marut and Bashar were of great help during this process. I thank them both for what they have done.