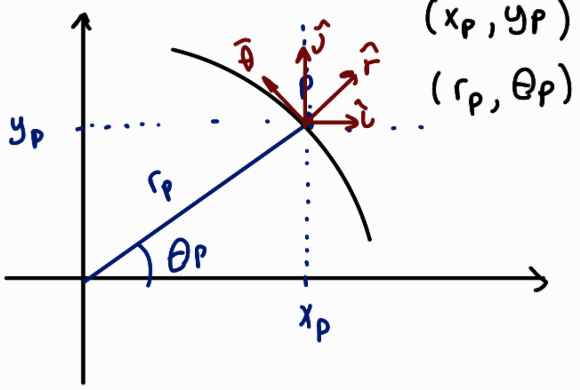
Coordinate Systems

25.09.24



$$x_p = r_p \cos\theta_p$$
$$y_p = r_p \sin\theta_p$$
$$r_p = \sqrt{x_p^2 + y_p^2}$$
$$\theta_p = \tan^{-1}\left(\frac{x_p}{y_p}\right)$$

Unit Vectors:

$\hat{i}_p, \hat{j}_p$
$\hat{r}_p, \hat{\theta}_p$

Transforming units algebraically:

$\hat{e}_1, \hat{e}_2$

$\vec{A} = A_1\hat{e}_1 + A_2\hat{e}_2 \longrightarrow A_i = \vec{A}\cdot\hat{e}_i$

$\hat{r}_p = a\hat{i}_p + b\hat{j}_p \qquad a = \hat{r}_p \hat{i}_p = \cos\theta_p$
$b = \sin\theta_p$

$$\hat{r}_p = \cos\theta_p\,\hat{i}_p + \sin\theta_p\,\hat{j}_p$$
$$\hat{\theta}_p = -\sin\theta_p\,\hat{i}_p + \cos\theta_p\,\hat{j}_p$$

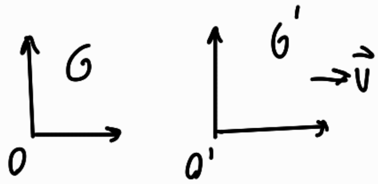
"A position vector is "unambiguously" defined"

How is a scalar different than a vector?

- What you can't rotate, is a scalar wrt rotations.
  Can you rotate red?

- Time is scalar
  ↳ But not under Lorentz transformation.

→ "1st law says there are inertial observers"



Galileo Transformations

$\vec{r}\,'_p = \vec{r} - \vec{v}t \qquad t' = t$

$\vec{u}\,'_p = \vec{u}_p - \vec{v}$

$\boxed{\vec{a}\,'_p = \vec{a}_p}$

observers agreed on acceleration.

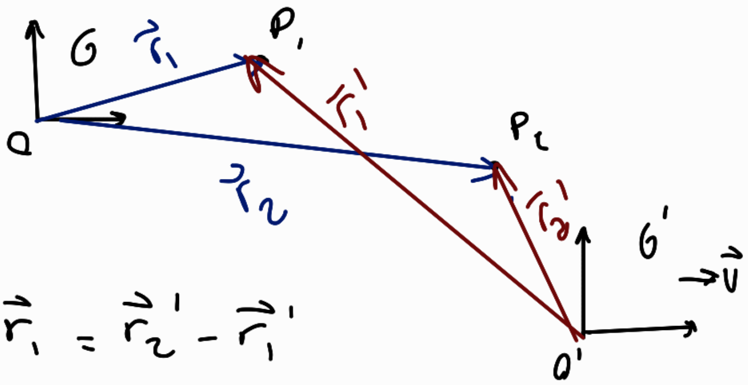
Because this is agreed on, anything leading to can be a candidate for a natural law

$$\vec F_p = \underbrace{m_p}_{\text{invariant between inertial observers.}} \vec a_p + \cdots$$

Thus, $\boxed{\vec F_p' = \vec F_p}$

Inertial observers exist!! → Proof:

We need a vector that's the same for this two reference frames:



Difference!

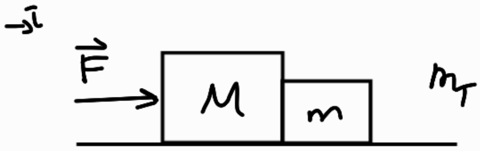
This leads us to $\vec F_p' = \vec F_p$

$$\vec r_2 - \vec r_1 = \vec r_2' - \vec r_1'$$

$$\boxed{\vec F_{12} = (\widehat{\vec r_2 - \vec r_1})\, f(|\vec r_2 - \vec r_1|)}$$

So by measuring motions you find forces like Keplers.

3rd Law



$m_T = m+M$

$$\vec a_{whole} = \frac{\vec F}{m+M} \to \vec a_m = \vec a_M$$

$$\vec v_m = \vec v_M$$

$$\vec F_m = \vec a_{whole}\, m \qquad \vec F_M = \vec a_{whole}\, M \neq \vec F$$

$$\vec F_M = \frac{M}{m+M}\vec F$$

$$\vec F_M = \frac{M}{m+M}\vec F = \vec F + \vec F_{mM} \qquad \vec F_m = \frac{m}{m+M}\vec F = \vec F_{Mm}$$

$$F_{mM} = -\frac{m}{m+M}\vec F \qquad F_{Mm} = -\frac{m}{m+M}\vec F$$

3rd Law Says:

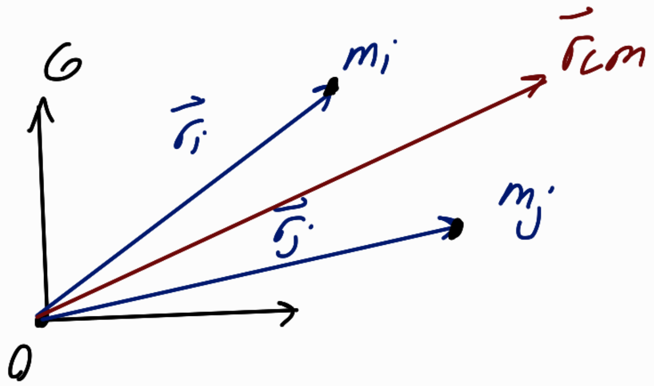
- You can apply 2nd law to the any subsystem of the system.

$$-\vec{F}_{12} = \vec{F}_{21} = (\vec{r}_1 - \vec{r}_2)\, f(|\vec{r}_1 - \vec{r}_2|)$$

Closed Systems : Defined by simplest properties (Conservation)

- Be careful using Newton's laws for the whole system.

Particle Systems



$i = 1, \dots, N$

m_i

r_i $\qquad \vec{r}_{cm} = \frac{1}{M_T} \sum_{i=1}^{N} m_i \vec{r}_i$

v_i $\qquad \vec{v}_{cm} = \frac{1}{M_T} \sum_{i=1}^{N} m_i \vec{v}_i$

a_i $\qquad \vec{a}_{cm} = \frac{1}{M_T} \sum_{i=1}^{N} m_i \vec{a}_i$

- $\vec{F}_i = m_i \vec{a}_i = \vec{F}_i^{(ext)} + \sum_{\substack{j=1 \\ j \neq i}}^{N} F_{ji}^{(int)}$

do:

$$\sum_{i=1}^{N} F_i = \sum_i m_i \vec{a}_i = \vec{F}_{Net}^{(ext)} + \underbrace{\sum_{i=1}^{N} \sum_{\substack{j=1 \\ i \neq j}}^{N} F_{ij}^{(int)}}_{0 \text{ (3rd law)}}$$

if $\vec{F}_{NET}^{ext} = 0 \Rightarrow a_{cm} = 0 \Rightarrow \vec{u}_{cm}$: constant.

→ internal observer.

Define momentum as $p_i = m_i v_i$

$$\sum_{i=1}^{N} \vec{p}_i = \vec{p}_{Total} \qquad \frac{d\vec{p}_i}{dt} = m_i \overset{a_i}{\frac{d\vec{u}_i}{dt}} = \vec{F}_i$$

$$\sum_{i=1}^{N} \vec{F}_i = \vec{F}_{NET}^{(ext)} = \sum \frac{d\vec{p}_i}{dt} = \frac{d}{dt} \sum_{i=1}^{N} \vec{p}_i$$

$$\frac{d\vec{p}_{TOT}}{dt} = \vec{F}^{ext}_{NET} \qquad \text{if} \quad \vec{F}^{ext}_{NET} = 0 \Rightarrow \vec{p}_{TOT} = \text{constant}$$

In special relativity, no 2nd law in sese of action - reaction

but $\quad p_{Tot} = \text{const}$

↳ different!