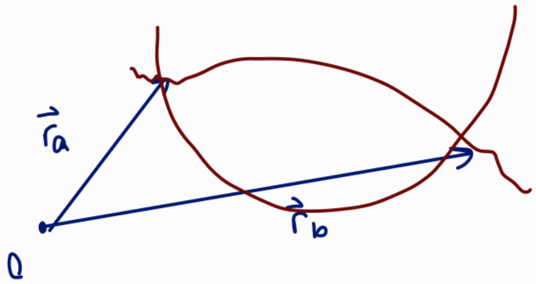
14.10.2024

Conservative Forces and Potential (Equilibrium)

$$\frac{d}{dt}(K+U) = 0 \iff \vec{F} = -\vec{\nabla}U$$

$$\underbrace{\oint_a^b}_{\text{path from } \vec{r}_a \text{ to } \vec{r}_b} \underbrace{\vec{F}\cdot d\vec{r}}_{\text{line integral from } \vec{r}_a \text{ to } \vec{r}_b}$$



(many paths possible)

If $\vec{F} = -\vec{\nabla}U \Rightarrow \oint_a^b \vec{F}\,d\vec{r} = -\oint_a^b \underbrace{(\vec{\nabla}U)\cdot d\vec{r}}_{dU}$

$$(*) \qquad \Rightarrow \oint_a^b \vec{F}\,d\vec{r} = -(U_b - U_a)$$

What (*) Tells Us? :

- If the work done by $\vec{F}$ between $\vec{r}_a$ & $\vec{r}_b$ does not depend on the path for all $\vec{r}_a$ & $\vec{r}_b$ $\Rightarrow$ $\boxed{\vec{F} = -\vec{\nabla}U}$ (potential force)

Equivilantly:

- If $\vec{r}_b = \vec{r}_a$, $\boxed{\oint_{loop} \vec{F}\cdot d\vec{r} = 0} \Rightarrow \vec{F} = -\vec{\nabla}U$

$\vec{\nabla}\times\vec{F} = 0$ (local)

This two don't always imply each other (space should be simply connected [no holes])

↳ PHYS325 notes

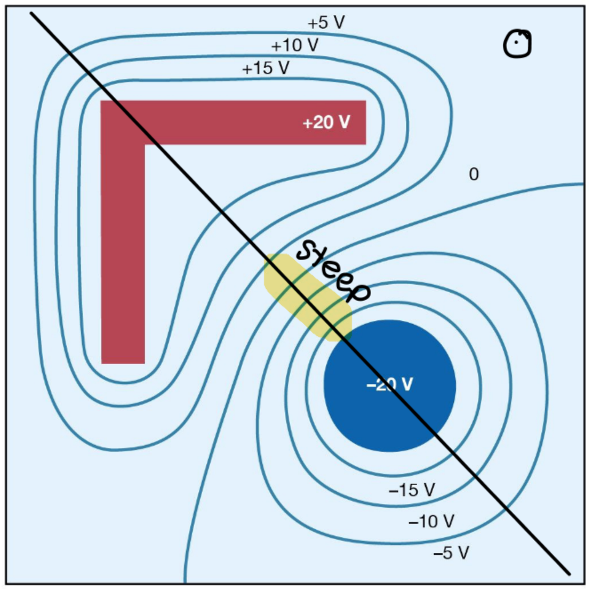
Equipotential Surfaces

$U(x,y,z) = U_0 \longrightarrow$ usually a surface

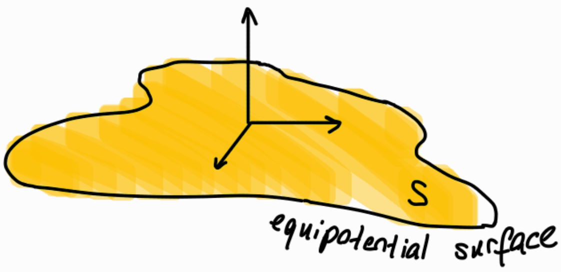
- Remember $\vec{F} = -\vec{\nabla} U$.

$\vec{F}$ is dependent on the gradient. So the change of potential and the direction is important

The distance between two equipotential surfaces, represented by the lines, indicates how rapidly the potential changes.
The smallest distances correspond to the location of the greatest rate of change and therefore to the largest values of the electric field.



- $\vec{F} = -\vec{\nabla} U$ is $\perp$ to equipotential surfaces.



1D Potential U

Minima & Maxima

$$\left.\frac{dU}{dx}\right|_{x=x_i} = 0 \qquad i = 1,2,3,\dots \Big] \text{ equilibrium point}$$

$\Downarrow$ check if I'm just a local minima / maxima

$$\left.\frac{d^2U}{dx^2}\right|_{x=x_i} \begin{cases} >0 & \text{minima} \\ <0 & \text{maxima} \\ =0 & \text{inconclusive} \end{cases}$$

take another derivative!

Why is this enough? Generally double derivative isn't 0.

- Can there be a measure of probability, that a random function's double derivative isn't zero, opposed to being 0? "morse theory"

Multivariate Form

$$\vec{\nabla} u \Big|_{\vec{x}=x_0} = 0 \Rightarrow \underbrace{\frac{\partial u}{\partial x}}_{x_1} = \underbrace{\frac{\partial u}{\partial y}}_{x_2} = \underbrace{\frac{\partial u}{\partial z}}_{x_3} = 0$$

$$\frac{\partial^2 u}{\partial x_i \partial y_i} \qquad i,j = 1,2,3$$

$$\begin{pmatrix} \frac{\partial^2 u}{\partial x^2} & \frac{\partial^2 u}{\partial x \partial y} & \frac{\partial^2 u}{\partial x \partial z} \\ \vdots & \vdots & \vdots \end{pmatrix}\Bigg|_{x_0, y_0, z_0}$$

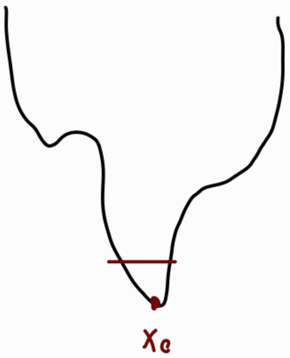
eigenvalues of this matrix

Any function $f(x)$ can be expanded about any point.

$$f(x) = f(x_0) + \cancelto{0}{f'(x_0)}(x-x_0) + \boxed{\frac{1}{2} f''(x_0)(x-x_0)^2} + \frac{1}{3!} f'''(x_0)(x-x_0)^3 \ldots$$

⊛ If $f'(x_0)=0$ this is an equilibrium point (Definition)

Idea of Small Motions about an equilibrium point:



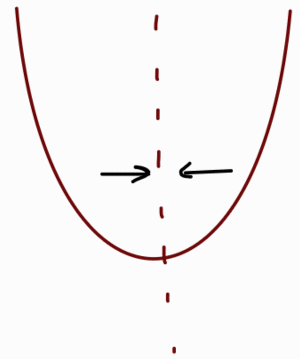
Whenever Justifiable:

(If $\boxed{}$ is the dominant term)

$$\boxed{f(x) = f(x_0) + \frac{1}{2} f''(x_0)(x-x_0)^2}$$

This is why Hooke's Law is everywhere.

$$-\frac{df}{dx} = -\underbrace{f''(x_0)}_{\text{spring constant.}}(x - x_0) = F(x)$$



"Hooke's kind of Law" → restoring force

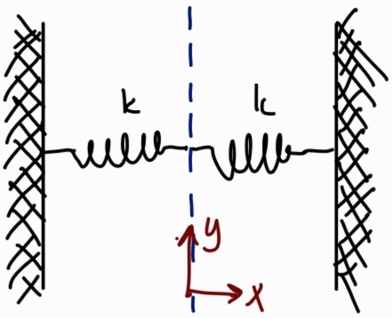
Resonance

- Imagine swinging a swing. You push it with $\tilde{F}$ in sync with its motion, thus increasing the amplitude.

(other examples were given)

will be

HW :



Any slight motion in x direction makes it harmonic