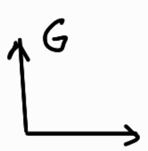
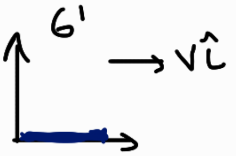
## Lorentz Transformations

25.09.24





$$ct' = \gamma(ct - \beta x)$$
$$x' = \gamma(x - \beta ct)$$
$$\beta = v/c$$

$y' = y$
$z' = z$

an event
$(ct, x)$

$(ct', x)$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \Rightarrow |\beta| < 1$$

① if $|\beta| \ll 1$ they go to Gallileau transformations
$x' = x - vt$ , $t' = t$

② Simultaneity is NOT universal!
if $\Delta t = 0 \not\Rightarrow \Delta t' = 0$

③ $\Delta t > 0 \not\Rightarrow \Delta t' > 0$ either!

④ Lengths? Durations? $L = \sqrt{1-\beta^2}\, L_0$ $\quad T = \frac{T_0}{\sqrt{1-\beta^2}}$

- Measuring A Rod...

event left event right

G $\begin{pmatrix} ct = 0 \\ x = 0 \end{pmatrix}$ $\begin{pmatrix} ct = 0 \\ x = L \end{pmatrix}$

G' $\begin{pmatrix} ct' = 0 \\ x' = 0 \end{pmatrix}$ $\begin{pmatrix} ct' = \gamma(ct - \beta L) = -\beta \overset{L_0}{\boxed{\gamma L}} \\ x' = L \\ x' = \gamma(x - \beta ct) \end{pmatrix}$ $L_0$ $\quad L = 0$

Length Contraction : $\boxed{L_0 = \gamma L \rightarrow L = \sqrt{1-\beta^2}\, L_0}$

G' measures the rod shorter! Sees event right earlier

## Is there a shape change perpendicular to motion?

No! It'd be a paradox.

Think of railroad & wheels,
barrel & bomb examples!

## Measuring Duration (Dilation)

$$\begin{array}{ccc}
 & \text{tic} & \text{tac} \\
G & \begin{pmatrix} ct=0 \\ x=0 \end{pmatrix} & \begin{pmatrix} ct = cT_0 \\ x = 0 \end{pmatrix} \\
G' & \begin{pmatrix} ct'=0 \\ x'=0 \end{pmatrix} & \begin{pmatrix} ct' = cT = \gamma(cT_0 - \beta \overset{0}{\cancel{x}}) \\ x' = \gamma(\underset{0}{\cancel{x}} - \beta c T_0) = -vT \end{pmatrix}
\end{array}$$

$$\boxed{T = \gamma T_0 = \frac{T_0}{\sqrt{1-\beta^2}}}$$

$$cTL = cT_0 L_0$$

Observers agree on the "area" of the universe.

## Why Twin Problem doesn't work.

- Two inertial observers meet only once! (Relative velocities const.)
  ↳ No acceleration.

# Invariant Interval

Consider two events seperated

G : $(c\Delta t, \Delta x, \Delta y, \Delta z)$

G' : $(c\Delta t', \Delta x', \Delta y', \Delta z')$

under Lorentz transformations.

$$\Delta s^2 \equiv c^2\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2 = c^2\Delta t'^2 - \Delta x'^2 - \Delta y'^2 - \Delta z'^2$$

$\Delta s^2 > 0$ time-like interval

$\Delta s^2 < 0$ space-like interval (NO causal relation possible!)

$\Delta s^2 = 0$ light-like (null) "

Proposition 1: If two events are spacelike seperated, There is a "physical" inertial obs. s.t. these events occur simultaneously.

Say for $G'$ $\Delta t' = 0$ $\beta = \frac{c\Delta t}{\Delta x}$ is $|\beta| < 1$?

$$c^2\Delta t^2 - \Delta x^2 < 0$$

$$\Delta x^2 \left( \underbrace{\frac{c^2\Delta t^2}{\Delta x^2}}_{\beta^2} - 1 \right) < 0$$

if $\Delta t' = 0$ in one frame, it is different in other frames.