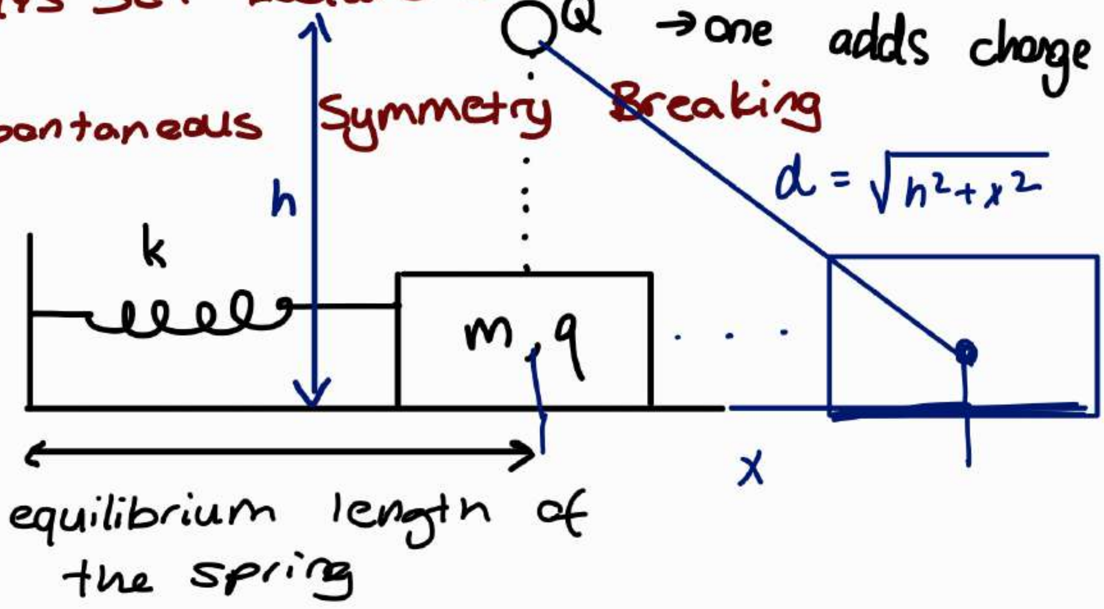
PHYS 301 Lecture 8 16.10.2024

Spontaneous Symmetry Breaking



$$V = \frac{1}{4\pi\epsilon_0}\frac{Qq}{\sqrt{h^2+x^2}} + \frac{1}{2}kx^2$$

$$\frac{a}{\sqrt{1+x^2}} + x^2 \qquad a \geq 0$$

- There is a critical value of a, where the minima change.

- Now we Taylor Expand the function around $x=0$

$$a(1+x^2)^{-1/2} + x^2$$

Remember Taylor Expansion:

$$f(x) \approx f(0) + f'(0)x + \frac{1}{2!}f''(0)x^2 + \cdots$$

Binomial Expansion

$$(1+\epsilon)^{\alpha} = 1 + \alpha\epsilon + \frac{1}{2}\alpha(\alpha-1)\epsilon^2 + \cdots$$

$$a(1+x^2)^{-1/2}+x^2$$

$$\simeq a\left(1-\frac{1}{2}x^2+\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)x^4+\cdots\right.$$

$$= a-\frac{a}{2}x^2+\frac{3a}{8}x^4$$

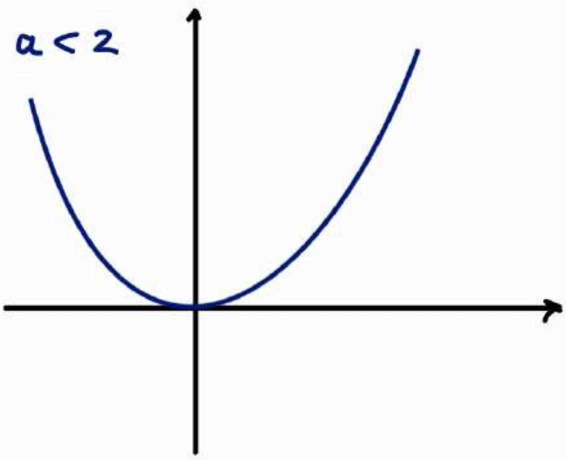
(*) $$\simeq a+\boxed{\left(1-\frac{a}{2}\right)}x^2+\boxed{\frac{3a}{8}}x^4$$

depends on a. >0

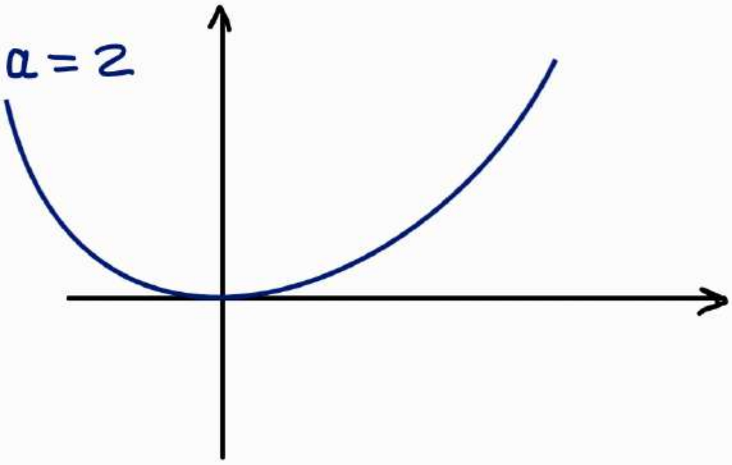
(*) $$\left(1-\frac{a}{2}\right)x+\frac{12}{8}ax^3=0$$

$$x_c\left(\left(1-\frac{a}{2}\right)+\frac{12}{8}ax_c^2\right)=0$$

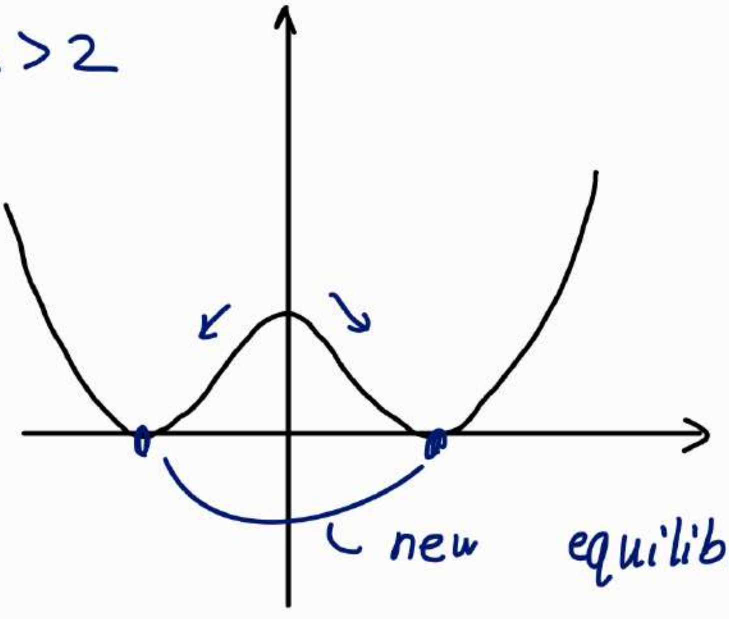
$a<2$



$a=2$



$a>2$



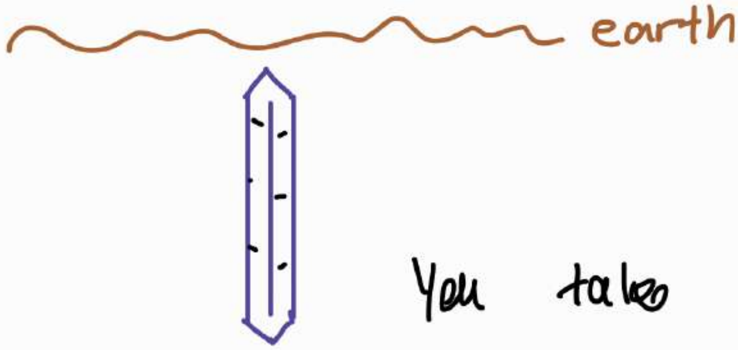
(The particle will choose one of them)

new equilibrium points...

Spontaneous symmetry breaking is a spontaneous process of symmetry breaking, by which a physical system in a symmetric state spontaneously ends up in an asymmetric state.[1][2][3] In particular, it can describe systems where the equations of motion or the Lagrangian obey symmetries, but the lowest-energy vacuum solutions do not exhibit that same symmetry. When the system goes to one of those vacuum solutions, the symmetry is broken for perturbations around that vacuum even though the entire Lagrangian retains that symmetry.

Example

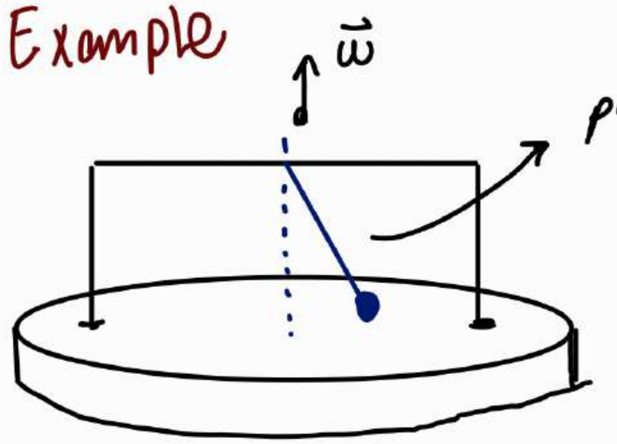
You want to load the obelisk to the truck.



You want to roll it over.

You take the ground around it...

Example



pendulum forced to oscillate only $\perp$ to the axel about

merry go round

$\omega > \vec{\omega}_{critical} \rightsquigarrow$ oscillation angle?

$$V = a + bx^2 + cx^4 \qquad c > 0$$

If $b > 0$ no SSB
$b < 0$ SSB

"Study the potential function around its critical points"

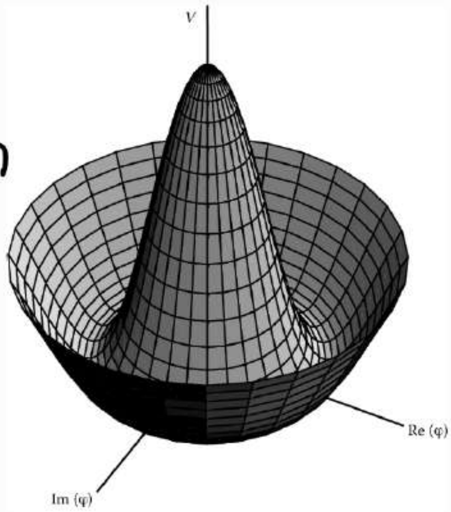
Structure of the minimum energy state in SSB

$$V(x,y) = \cancel{a} + b(x^2+y^2)^2 + (x^2+y^2)^4$$

"mexican hat potential"

Conf. with minimum energy are not discrete.
They are on the same circle.

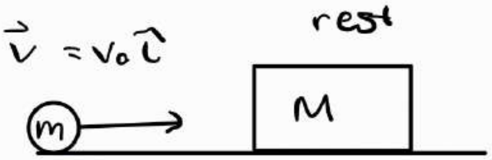
- If the particle is moving on that line, No energy needs to be provided to do that.



"gold stone mode": That zero cost motion

Momentum Conservation:

- In the absence of external forces, the total momentum of a system is conserved.



(here momentum cons. won't be enough)

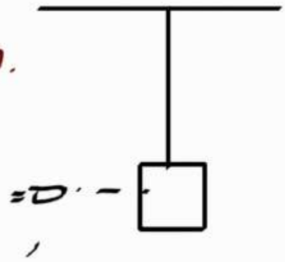
① m sticks to M. $\vec{P}_i^{Tot} = \vec{P}_f^{Tot} \Rightarrow m v_0 \hat{i} = (M+m) v_f \hat{i}$

$$v_f = \frac{m v_0}{(M+m)}$$ → No Kinetic Energy conservation!

How do you interpret that? ?

"Essentially, for sticking, one object should change shape."
(Inelastic Collisions)

Ex: Ballistic Pendulum.



② Elastic Collisions (Conserve Kinetic Energy)

$$\cancel{m} v_0 = \cancel{m} v_m + M v_M \Rightarrow v_0 = v_m + \frac{M}{m} v_M$$

$$\cancel{\tfrac{1}{2}} m v_0^2 = \cancel{\tfrac{1}{2}} m v_m^2 + \cancel{\tfrac{1}{2}} M v_M^2 \Rightarrow V_0^2 = V_m^2 + \frac{M^2}{m^2} V_M^2 + 2\frac{M}{m} V_m V_M$$

$$= V_m^2 + \frac{M}{m} V_M^2$$

$$\frac{M}{m}\left(1 - \frac{M}{m}\right) V_M^2 = 2\frac{M}{m} V_m V_M$$

$$V_m = \frac{1}{2}\left(\frac{m-M}{m}\right) V_M$$

$$\frac{1}{2}\left(1 - \frac{M}{m}\right) V_M + \frac{M}{m} V_M = V_0$$

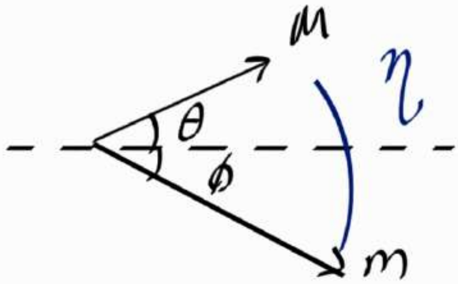
$$\Rightarrow \boxed{V_M = \frac{2V_0}{1 + \frac{M}{m}} \qquad V_m = \left(\frac{1 - \frac{M}{m}}{1 + \frac{M}{m}}\right) V_0}$$

Reason why stuff bounces back.

→ See what happens when a heavy object hits a small one

(Sudden acceleration of car accidents)

2D Elastic Collisions



$$\vec{p}_i^{\,TOT} = \vec{p}_f^{\,TOT}$$

x dir: $m v_0 = m V_m \cos\phi + M V_M \cos\theta$

y dir: $0 = M V_M \sin\theta - m V_m \sin\phi$

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m V_m^2 + \frac{1}{2} M V_M^2$$

Exercise:

Assume $M = m$ and figure out η