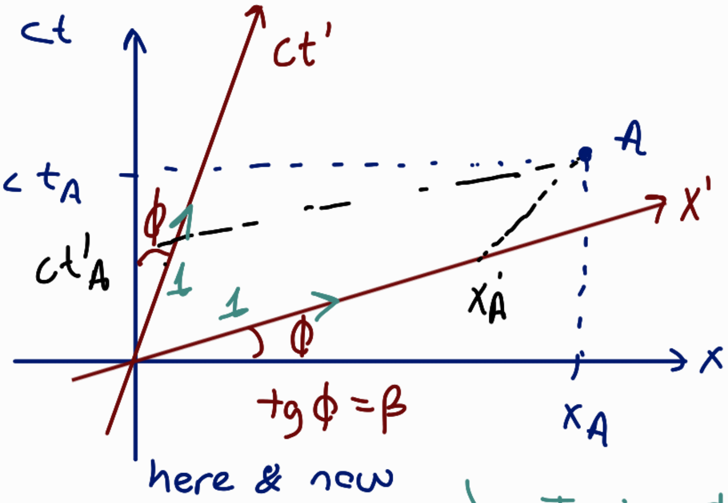
PHYS 311 Lecture 5 07.10.24

Minkowski Diagrams: A special case of Penrose diagrams



$$ct' = \gamma(ct - \beta x)$$
$$x' = \gamma(x - \beta ct)$$

$ct' = 0 \Rightarrow ct = \beta x$ · x' axis

$x' = 0 \Rightarrow ct = \frac{1}{\beta} x$ ct' axis

Trying to put a non-euclidian space into an euclidian space.

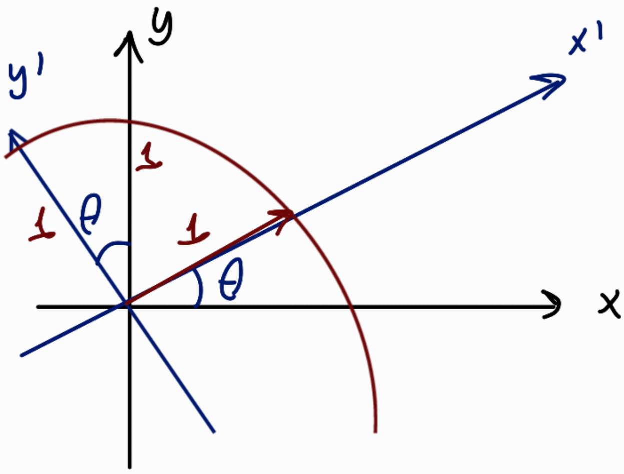
- What are the coordinates of this event according to the observer? (x'_A, ct'_A)

Ex:

$x' = \cos\theta\, x + \sin\theta\, y$
$y' = -\sin\theta\, x + \cos\theta\, y$

$x' = 0 \longrightarrow y'$ axis
$y' = 0 \longrightarrow x'$ axis

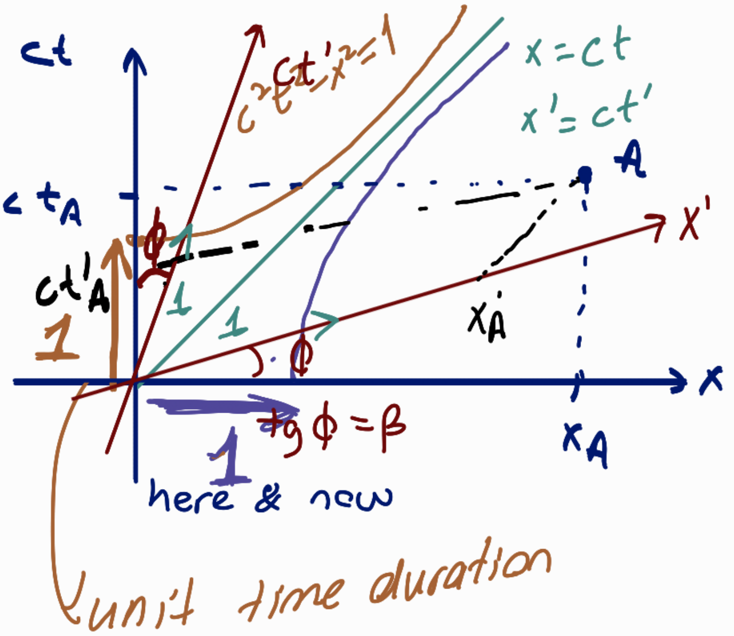
$y' = 0 \quad y = tg\,\theta\, x$
$x' = 0 \quad y = -\cot\theta\, x$



Since length is conserved, You can show that:

$x^2 + y^2 = x'^2 + y'^2 \equiv 1$ is the standard ruler (standard curve)

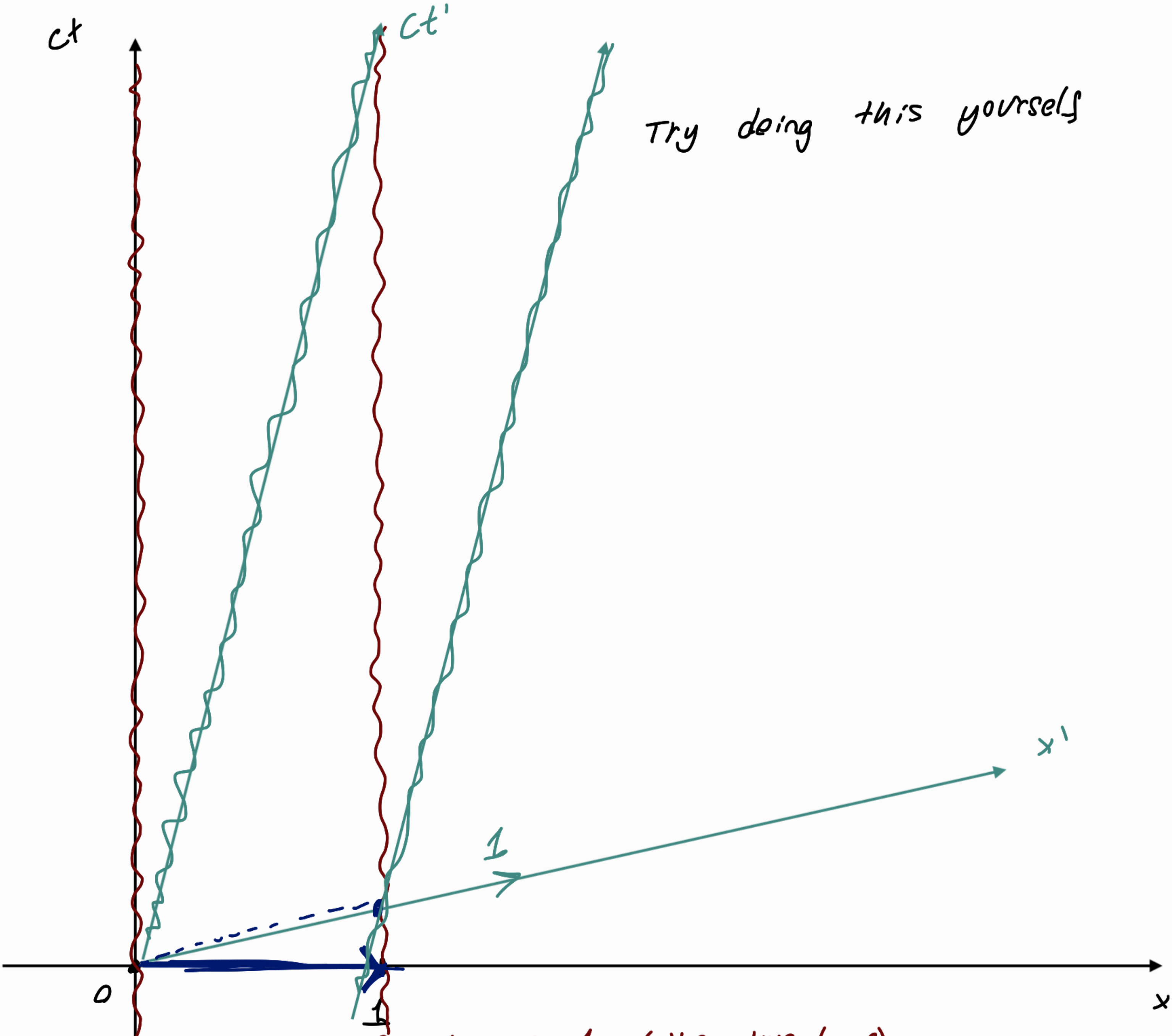
- Something similar exists for lorentz transfer motions as well.



$$c^2t^2 - x^2 = c^2t'^2 - x'^2$$

$$= \begin{cases} +1 \\ -1 \\ 0 \end{cases}$$

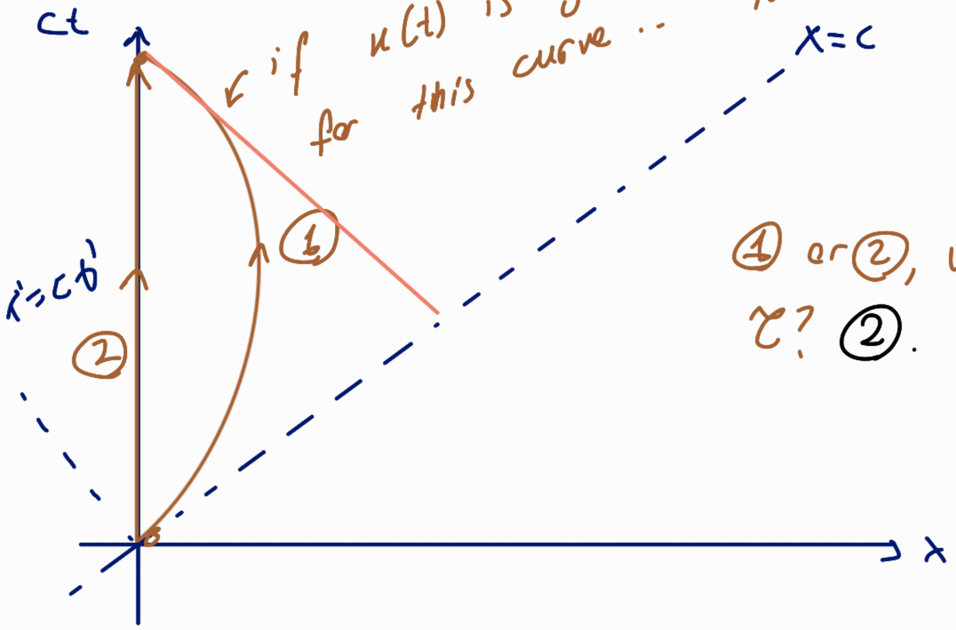
Understanding Length Contraction with This



Try doing this yourself

life lines of observer 1 (they live here)

Proper Time



you can calculate anything (you can measure ac. locally)

It will remain in the "forward light cone"

1 or 2, which has a bigger τ? 2.

Definition: Proper time (same for all observers)

$$\tau \equiv \int_{\text{origin}}^{\text{end}} dt \sqrt{1-\frac{u^2}{c^2}}$$

Clock Hypothesis: Observer can "recalibrate" their clock to match the proper time (?)

* Free fall is the closest thing to inertial observer.

It's actually I, who is accelerating.

Relativistic Decays (total) & Collisions (closed) $P_i^{TOT} = P_f^{TOT}$

P: four momentum of a system is conserved.

center of momentum (the frame in which $\vec{P}_{TOT}=0$)

Decay: One particle suddenly becoming two particles. (Not by internal chemical energy - explosion) but by internal mass energy.

① $M \longrightarrow m + m$? • is $\vec{v}_M$ important for decay?

• is it a relevant factor?

$E_i^{TOT} = Mc^2 = E_1 + E_2$

$\vec{p}_i^{\,TOT} = 0 = \vec{p}_1 + \vec{p}_2 = 0$

$$\left.\begin{aligned} \vec{p}_1 &= -\vec{p}_2 \\ p_1 &= \frac{m u_1}{\sqrt{1-\frac{u_1^2}{c^2}}} \end{aligned}\right\} \Rightarrow \underline{u_2 = -u_1} \equiv \underline{u}$$

→ No! $\vec{v}$'s change depending on the observer. And the frame is physically irrelevant. So we can calculate from wherever. We choose to calculate from where M is at rest.

$$\Rightarrow E_i^{TOT} = Mc^2 = E_1 + E_2 = \frac{mc^2}{\sqrt{1-u_1^2/c^2}} + \frac{mc^2}{\sqrt{1-u_2^2/c^2}}$$

$$\cdot\ Mc^2 = \frac{2mc^2}{\sqrt{1-u^2/c^2}}$$

$$\cdot\ M^2c^4 = \frac{4m^2c^2}{1 - u^2/c^2}$$

• m can also be 0 (You can create a massless particle)

$$\cdot\ \frac{u^2}{c^2} = \frac{M^2c^4 - 4m^2c^4}{M^2c^4}$$

$$u = \sqrt{1 - \left(\frac{2m}{M}\right)^2}\ c$$

$$\boxed{M \geq 2m}$$

↳ Any extra mass energy goes to motion of created particles.