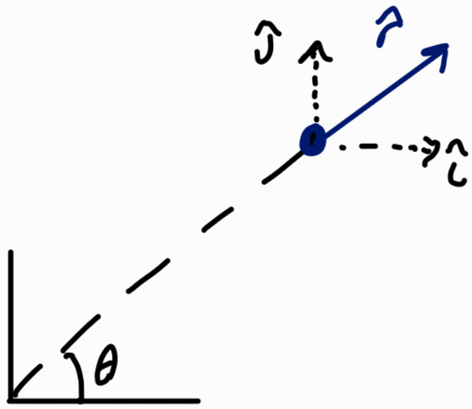
PHYS 301 LECTURE 4 02.10.2024

$\vec{r} = r\hat{r}$ • $\vec{v} = \frac{d\vec{r}}{dt} = \dot{r}\vec{r} + \vec{r}\dot{r} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$



• $\hat{\theta} \to \dot{\hat{\theta}} = -\dot{\theta}\cos\theta\,\hat{\imath} - \dot{\theta}\sin\theta\,\hat{\jmath} = -\dot{\theta}\hat{r}$

• $\hat{r} = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$

$\dot{\hat{r}} = -\dot{\theta}\sin\theta\,\hat{\imath} + \dot{\theta}\cos\theta\,\hat{\jmath}$

$= \dot{\theta}(-\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath})$

• $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} = \frac{d}{dt}\left(\dot{r}\hat{r} + r\dot{\theta}\hat{\theta}\right)$

$= \ddot{r}\hat{r} + 2\dot{r}\dot{\theta}\hat{\theta} + r\ddot{\theta}\hat{\theta} + r\dot{\theta}\dot{\hat{\theta}} - r\dot{\theta}^2\hat{r}$

$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}$ $\vec{a} = \ddot{x}\,\hat{\imath} + \ddot{y}\,\hat{\jmath}$

$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$ $\vec{v} = \dot{x}\,\hat{\imath} + \dot{y}\,\hat{\jmath}$

$\vec{r} = r\hat{r}$ $\vec{r} = x\hat{\imath} + y\hat{\jmath}$

PS. Holonimic vs. non holonimic constraints.

https://physics.stackexchange.com/questions/409951/what-are-holonomic-and-non-holonomic-constraints#409953

General Circular Motion

Constraint: Equality on coordinates (holonomic)

$$r = R \qquad \dot{r} = 0 \qquad \ddot{r} = 0$$

- $\vec{r} = R\hat{r}$ ⇒ remember this is not a constant vector.
- $\vec{r} \cdot \vec{r} = R^2$ ⇒ magnitude is constant
- $\vec{v} = R\dot{\theta}\hat{\theta}$
- $\vec{a} = -R\dot{\theta}^2\hat{r} + R\ddot{\theta}\hat{\theta}$

- Relationship between cartesian - polar is non-linear.

$$x = r\cos\theta \qquad \text{or} \qquad r = \sqrt{x^2+y^2}$$
$$y = r\sin\theta \qquad\qquad \theta = \arctan(y/x)$$

A further constraint: In uniform circular motion

$\ddot{\theta} = 0$. Say, define $\dot{\theta} \equiv \omega$.

$$\vec{v} = R\omega_0\hat{\theta}$$
$$\underbrace{\vec{a} = -R\omega_0^2\hat{r}}_{\text{centrifugal force}}$$

- You can't be an inertial frame when you are rotating with such an object.

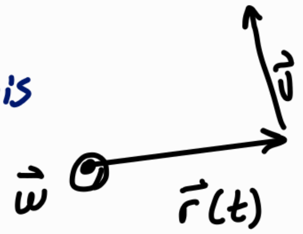
- Remember: obvious stuff like strings can't push, they can only pull.

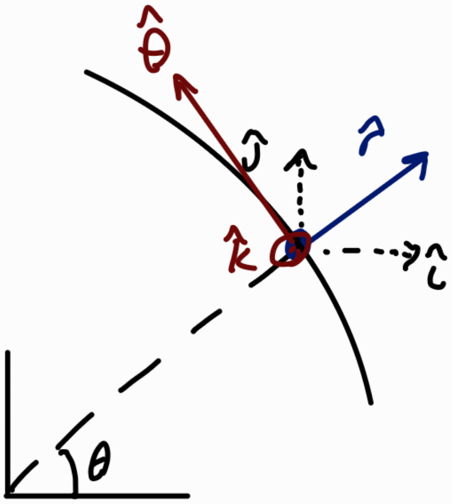
$\dot{\theta}$: Angular speed or Velocity?

- Should angular velocity even be a vector?

↳ Every rotational motion can be thought of performing a small circular motion instantaniously

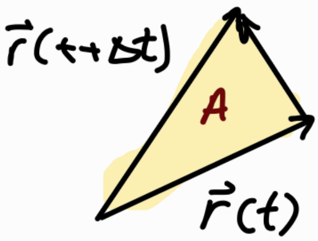
I want ω to point like this





- $w = (\tilde{w}\hat{k})$ (i want it in this direction)
- $\vec{V} = \vec{\omega} \times r$
- $R w_0 \hat{\theta} = R\tilde{w}(\underbrace{\hat{k} \times \hat{r}}_{\hat{\theta}})$
- $\tilde{\omega} = w_0 = \dot{\theta}$

- Now Kepler's law for areals start to make sense (2nd law)



$$\vec{r}(t) \times (\vec{r}(t) + \Delta t\, \vec{V}(t))$$

$$\Delta t\, \vec{r} \times \vec{v} = 2A\hat{k}$$

$$\vec{r}(t+\Delta t) \approx \vec{r}(t) + \Delta t\, \vec{v}(t)$$

Types of Forces

Contact Forces. (constraints) normal to the surface of contact. $\vec{N}$

Friction Forces:
- static friction forces. $|\vec{f}_s|_{max} = \mu_s |\vec{N}|$ (oppose the would-be motion)
- Kinetic Friction // to $-\hat{v}$
  ↳ Speed dependent or not $\vec{f}_k = \mu_k |\vec{N}| \hat{v}$

Drag

$\vec{f}_{n.t} = -\alpha \vec{v}$

$\vec{f}_t = -\frac{1}{2} C_D \rho A v^2 \hat{v}$

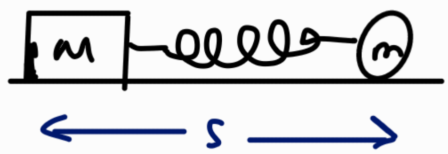
A – projected area

C_D – (Drag coefficient) dimless # depending on the shape

! Remember friction forces are generally not conservative.

Restoring Forces $(\vec{r}_2 - \vec{r}_1) f(|\vec{r}_2 - \vec{r}_1|)$

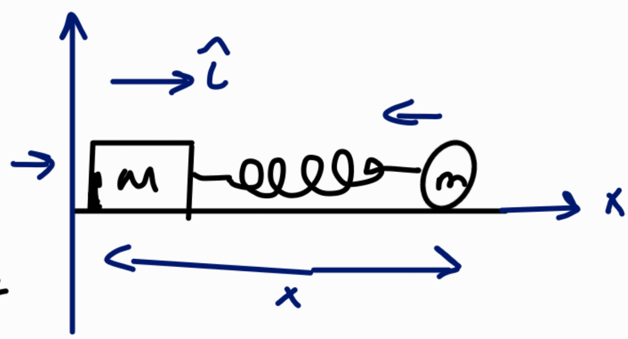
Hooke's Law: $\vec{f} = -k\vec{s}$



if $M >> m$:

$\vec{f}_m = -k x \hat{\imath}$

$\vec{f}_m = -\hat{\imath} \frac{dV}{dx}$ with $V = \frac{1}{2} k x^2$



(★) Unit vectors can't carry dimensions! They depend on the observers.

Newton's Law $\vec{f} = -\frac{GMm}{s^2}\hat{s}$

Coulomb's Law $\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{s^2}\hat{s}$

They look similar BUT!

inertial mass' cancel out in newtons law (everything falls at the same rate)

But not for Coulombs Law.