

# Nested Models and Simulated Out-of-Sample Data

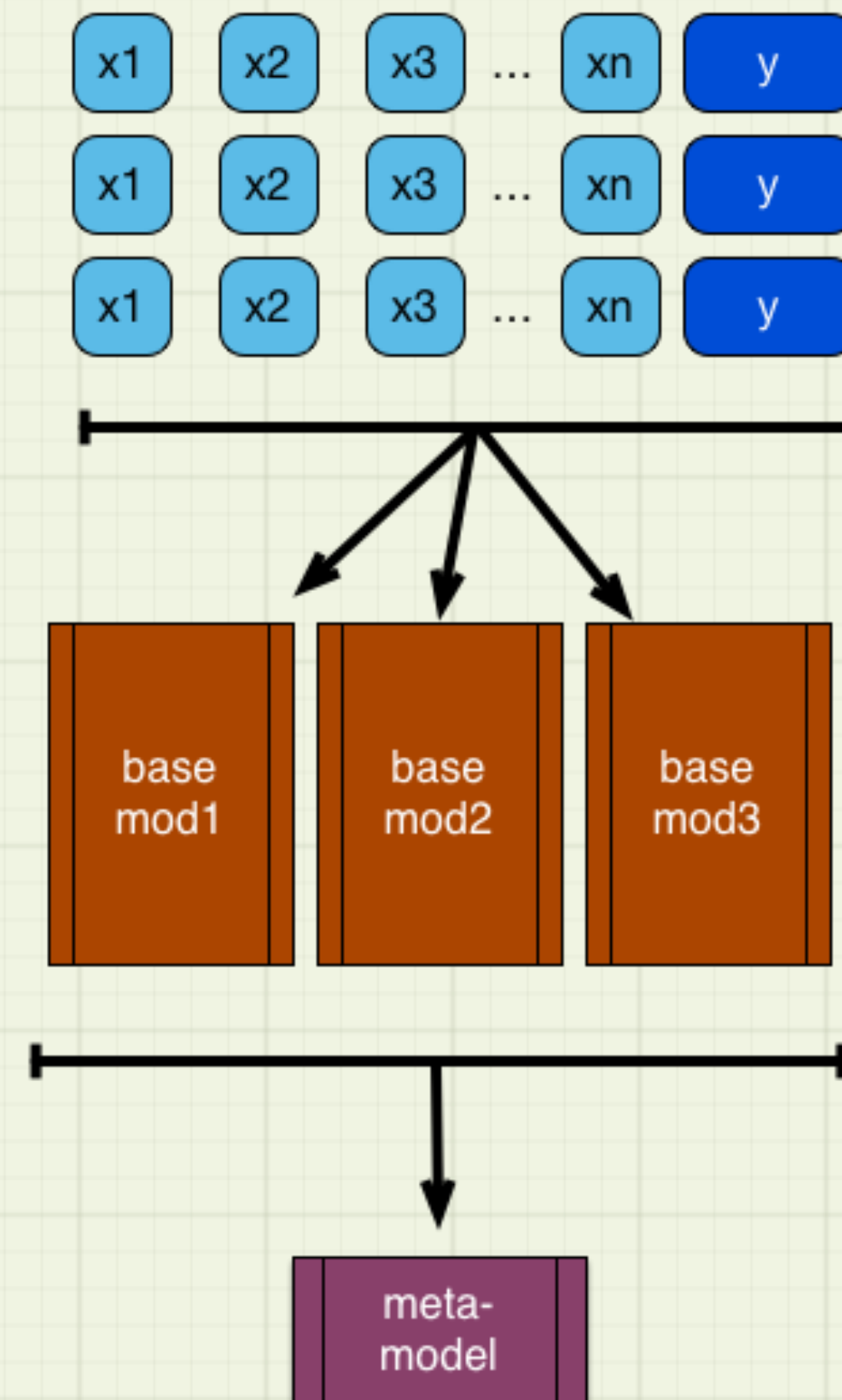
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# Outline

- What are Nested Models
  - Why do we care?
  - Types of Nested Models
- Examples of What Can Go Wrong
- Solutions
- Conclusion

# What do We Mean by *Nested Models*?

- Use data to train several base models to predict outcome
- Use a "meta-model" to combine the base models' predictions into an overall prediction



# Pro and Con

- **Pro:** Ensembles and Stacked models can improve performance over single models
  - Diversity of learning biases
- **Con:** They can also produce inferior overall models
  - We'll cover ways to avoid this in this talk

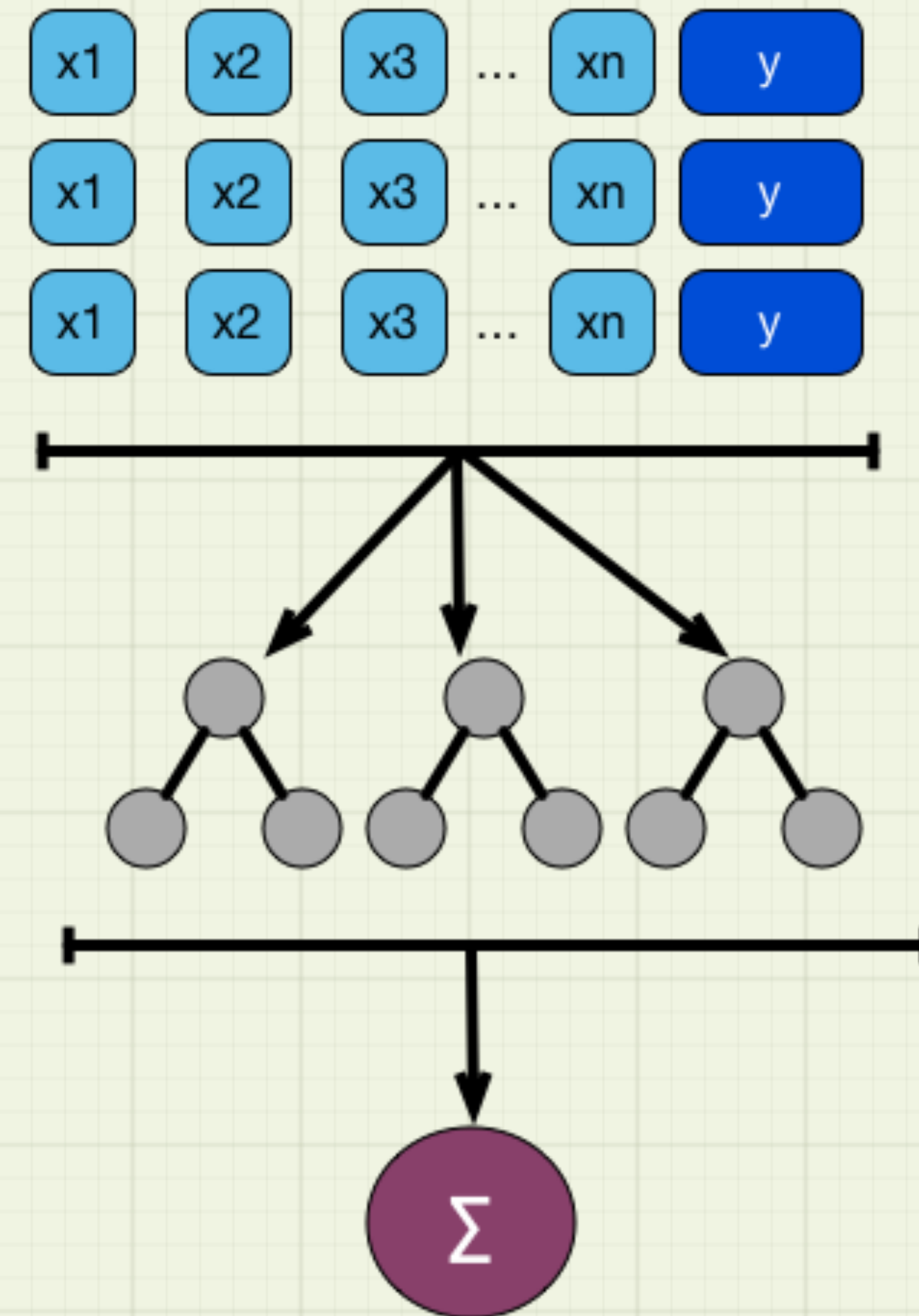
- Any nested model can introduce undesirable bias, to a greater or lesser degree
- But nested models are useful — and they are already everywhere
- This is basic statistical material, but it takes some care to properly apply it to modern data science workflows

# Types of Nested Models



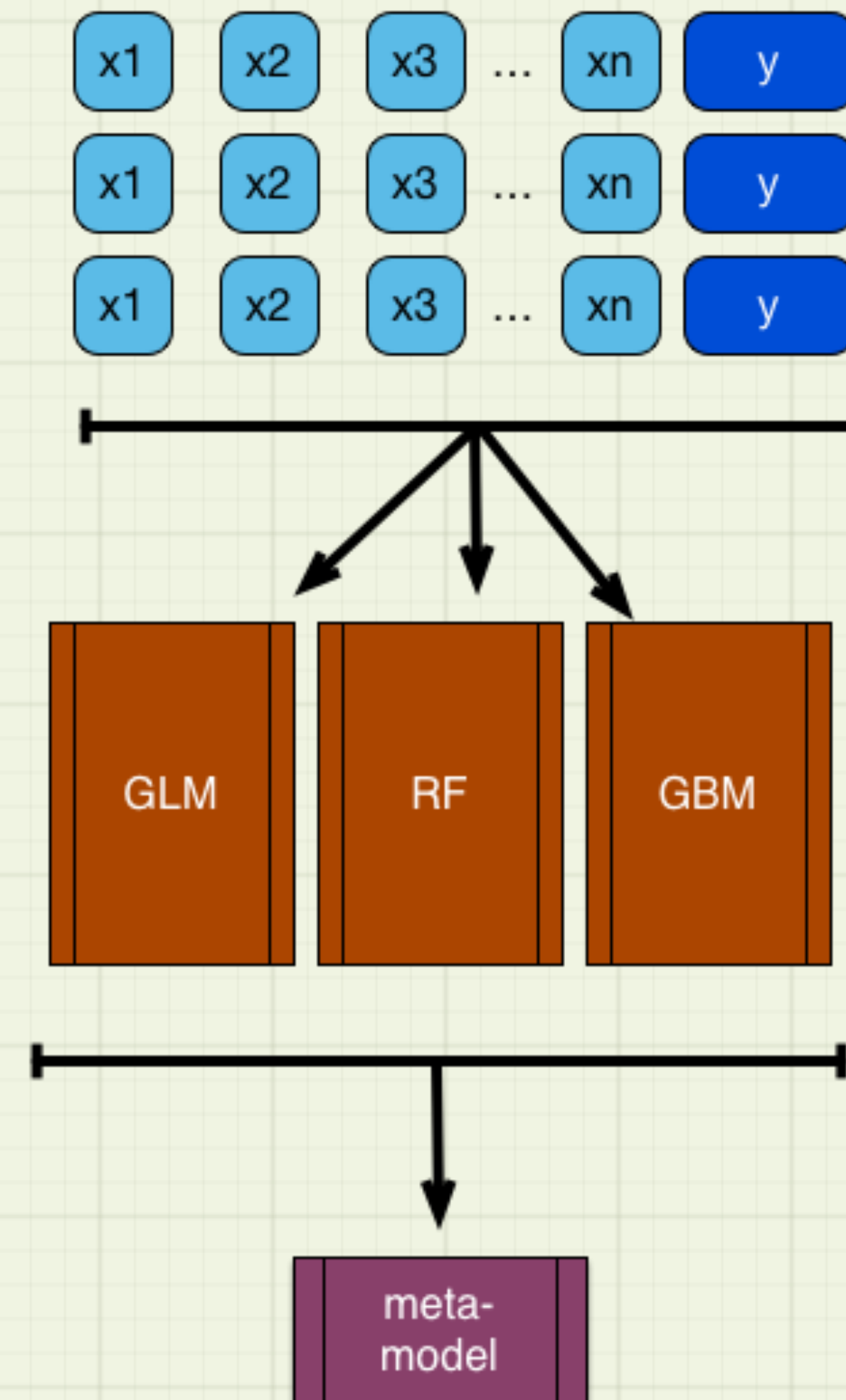
# Ensemble Learning

- Boosting (AdaBoost 1996, Gradient Boosting 1999)
- Bagging (1996)
- Random Forest (1995)
- Several diverse (usually) low-complexity learners that vote on outcome (sum, weighted sum)
- Different algorithms: different ways of getting diversity



# Stacked Models

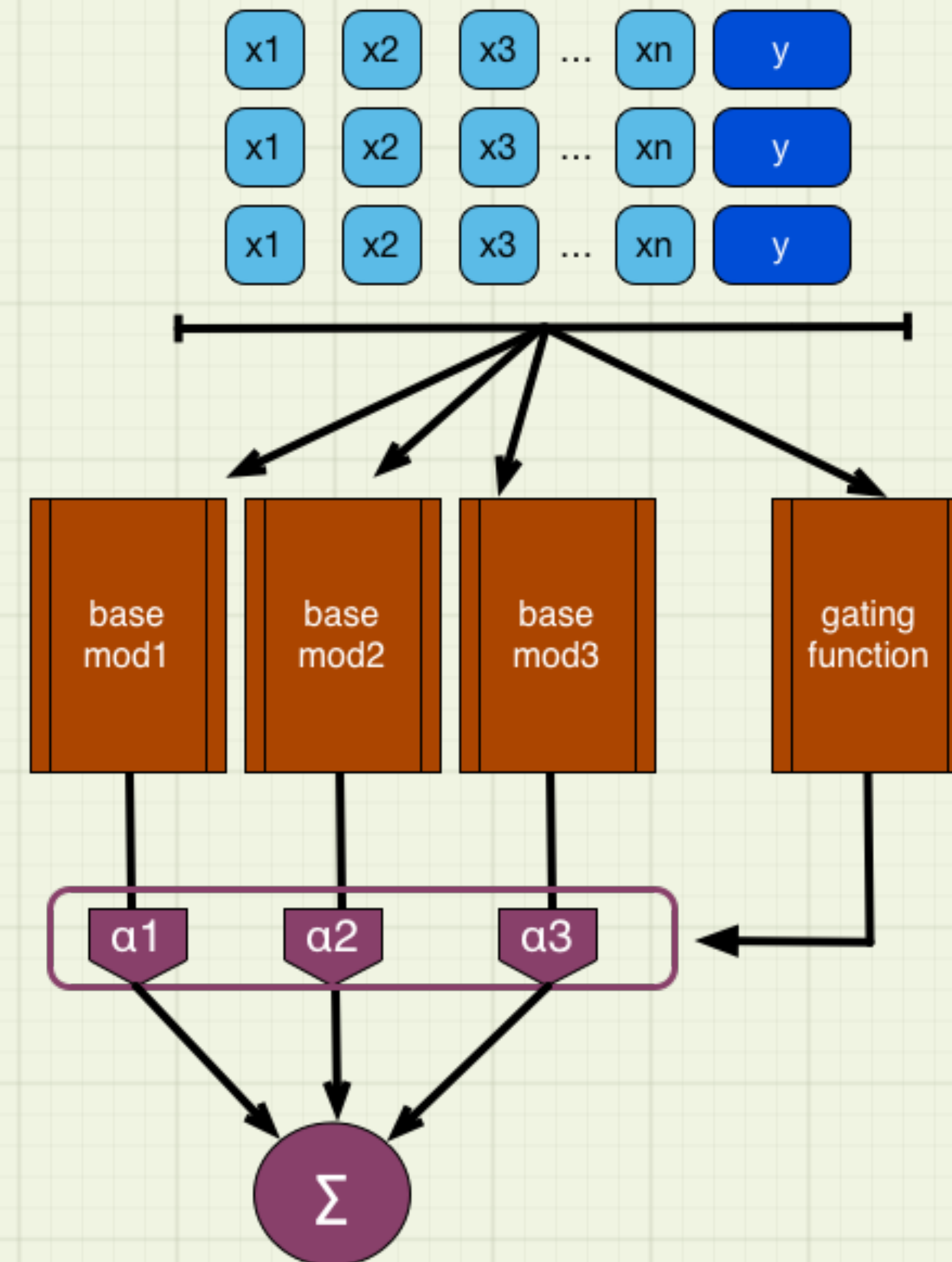
- Wolpert, 1992; Breiman, 1996
- Super Learner (van der Laan, 2007)
- Extension of hyper-parameter tuning
- Strong learners, higher-complexity meta-model





# Mixture of Experts

- Jacobs, 1991
- Data fusion, Sensor fusion
- Base models combined by weighted sum
- Weights are per-example (gating function)
- Often neural-net based



# Other "nested models"

**Any pre-model-fitting task that uses knowledge of outcome is a nested model**

- Variable treatment
- Hyperparameter tuning
- Variable selection/stepwise methods
- Y-aware dimension reduction
- Y-aware scaling
  - [http://www.win-vector.com/blog/2016/05/pcr\\_part2\\_yaware/](http://www.win-vector.com/blog/2016/05/pcr_part2_yaware/)

# Example 1: Super Learner

- H2OEnsemble (R)
  - <https://github.com/h2oai/h2o-3/tree/master/h2o-r/ensemble>
- Manages training of base models and meta-model to combine them
- Alternatives: SuperLearner, MLR (both in CRAN)



(Image: H2OEnsemble github)

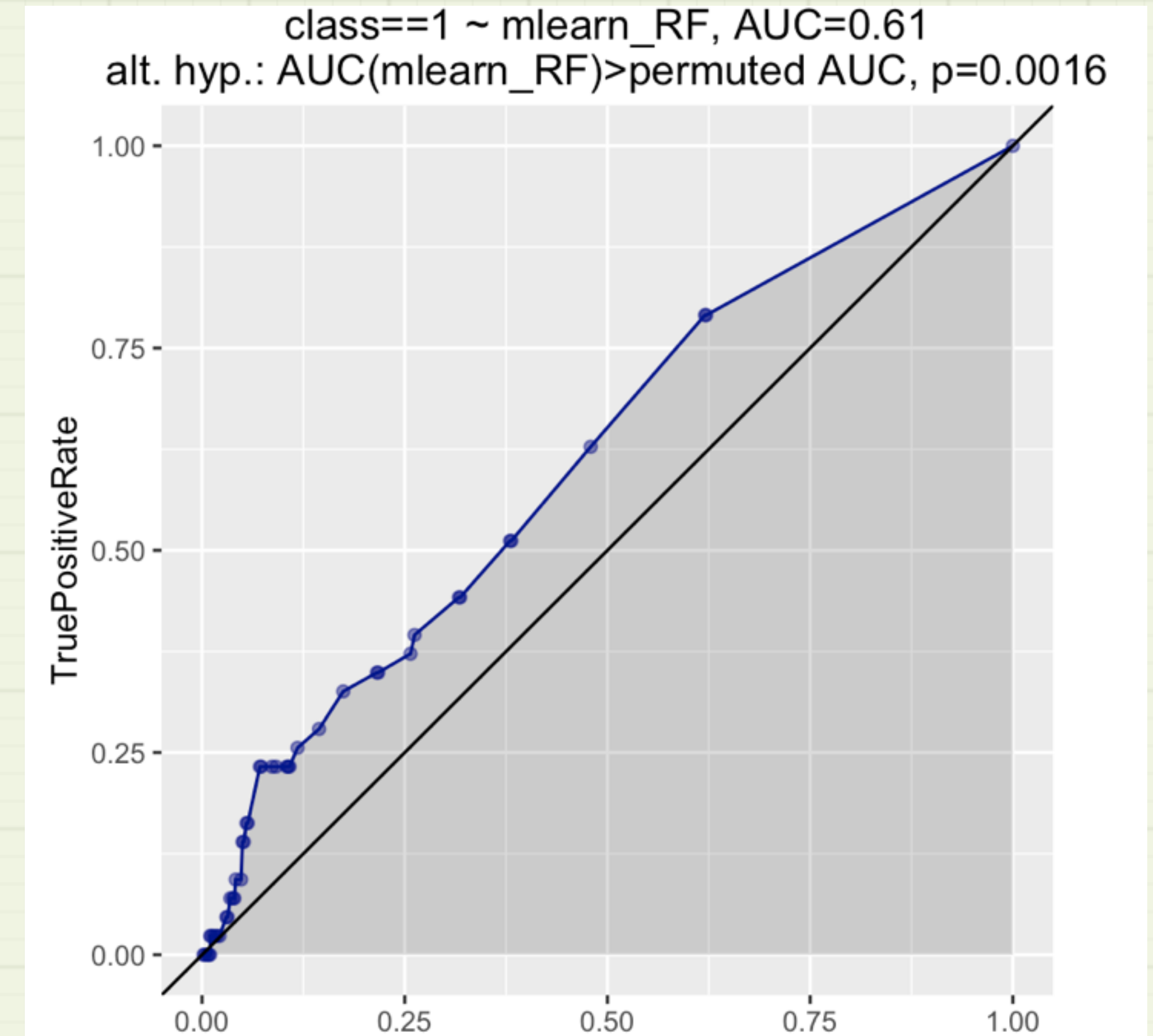
# The Prediction Task

- Predict whether a seismic event will occur in the next time interval
- 2584 rows, 19 features, ~6.5% target class prevalence
- <http://archive.ics.uci.edu/ml/datasets/seismic-bumps>



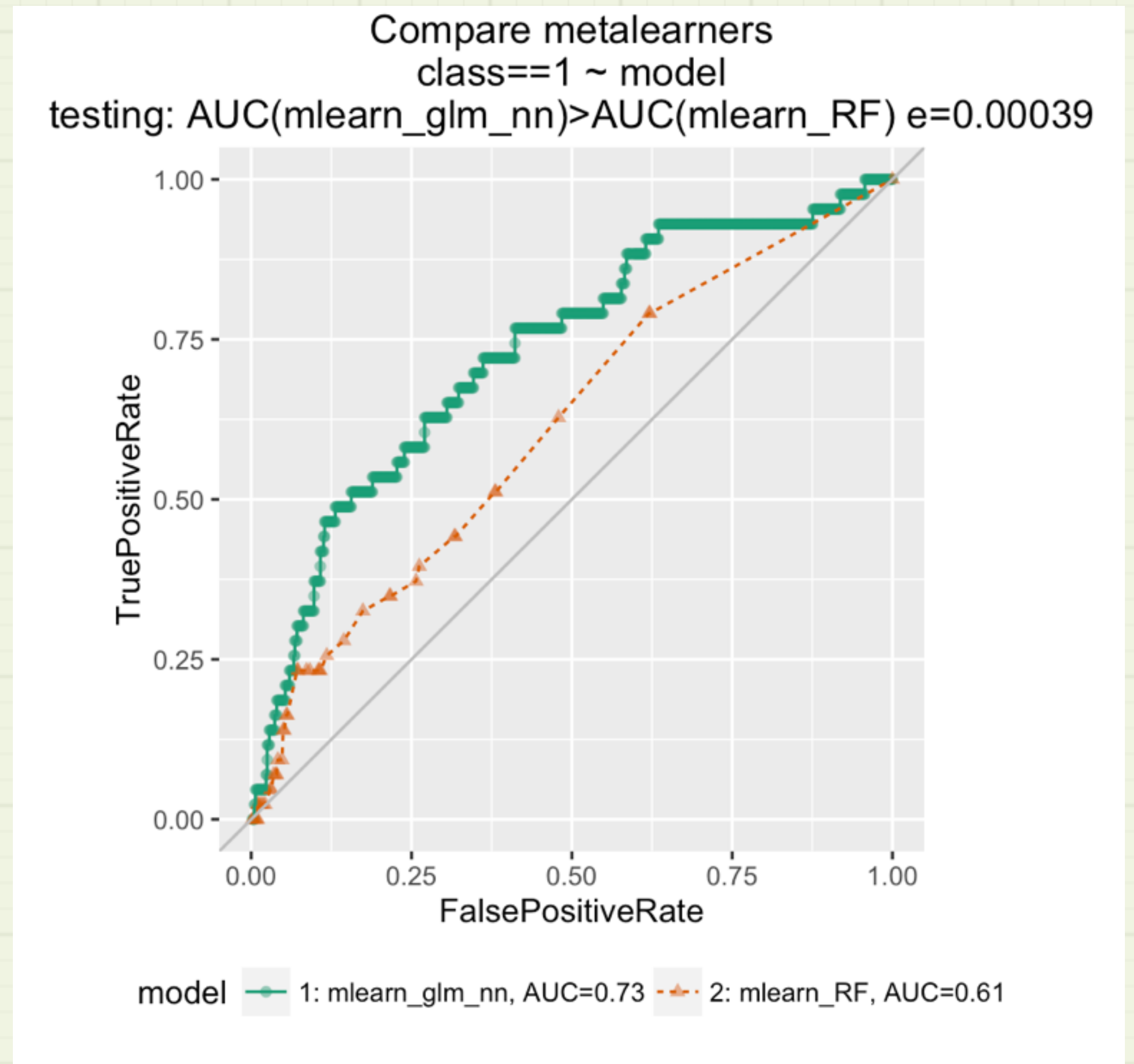
# How It Can Fail

- Base learners:
  - Random Forest
  - Gradient Boosting
  - Logistic Regression (Elastic Net)
  - Deep Learning NN
- Meta-learner: Random Forest
- Base learners: AUC in 0.65-0.71 range (test set)
- Ensemble: AUC 0.61 on test



# Issue: Meta-model complexity

- High-complexity meta-model can overfit
  - "Explain" unexplainable variance in training data
- Low-complexity meta-models have bounded excess generalization error
- Base models correlated
- Prefer non-negative weighted sum, or ridge regression
- $AUC(RF) = 0.61$ ;  $AUC(glm) = 0.73$





# Example 2: Effects/Impact Coding

- For categorical variables with many levels.
  - $K$  levels =  $K-1$  indicator vars
- Re-encode the categorical variable as a few numerical variables.

ZIP	PriceK	...	SoldInMon
94127	\$1,122	...	Yes
94117	\$1,296	...	No
94127	\$1,370	...	Yes
94118	\$1,325	...	No

# The Prediction Task

- Predict whether a listed house will sell within a month
- Input: ZIP, list price
- 1000 houses, 100 zip codes
- $P(\text{sold})$  weakly dependent on price
- $P(\text{sold})$  independent of ZIP

# Impact Model

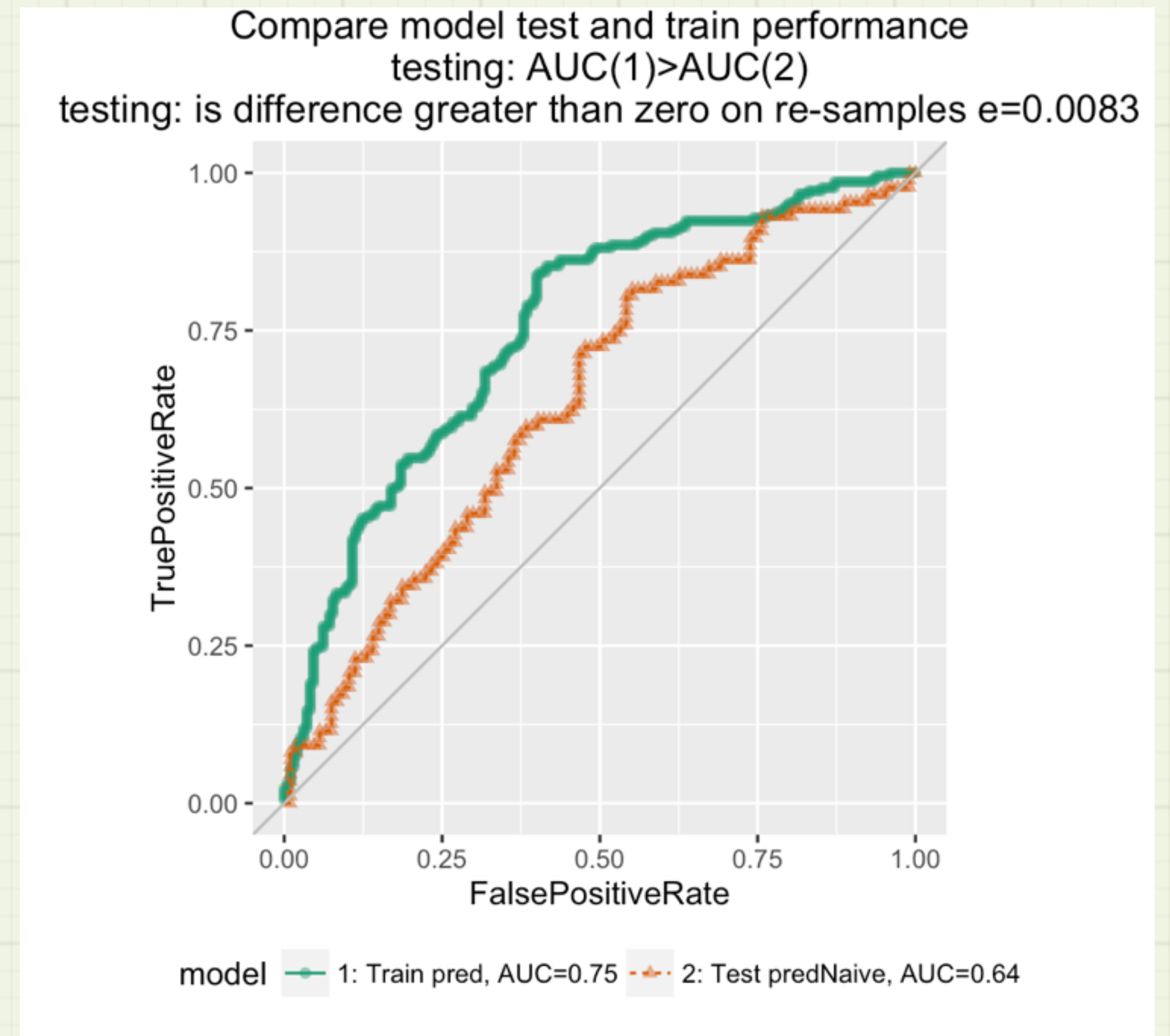
ZIP	P(SoldIn Mon)	Impact
94117	0.6	0.2
94118	0.34	-0.06
94121	0.16	-0.24
94127	0.72	0.32
<b>94132</b>	<b>1.0</b>	<b>0.6</b>
...	...	...
Overall	0.4	0

ZIP	N_SoldIn Month	N_NotSold InMonth	LogDiff	IsRare
94117	60	40	0.41	No
94118	68	132	-0.66	No
94121	8	42	-1.6	No
94127	108	42	0.94	No
<b>94132</b>	<b>1</b>	<b>0</b>	<b>1E+06</b>	<b>Yes</b>
...	...	...		

(Alternative model: count sold/not sold)

# How It Can Fail

- Use training data to effects code
- Train a logistic regression model to predict
- $AUC(\text{Train}) = 0.75$
- $AUC(\text{Test}) = 0.64$
- Overfit



# Issue: Nested Model Bias

```
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)   9.01213    1.06276   8.480  < 2e-16 ***
## log(price)  -0.73513    0.08693  -8.456  < 2e-16 ***
## zip_impact    0.98732    0.12683   7.785 6.99e-15 ***
```

- Effects model can memorize the training data
  - Rare levels
  - “Houses in 94132 always sell in a month”
- Full model may overestimate the value of effects-coded variable
  - Overfit



# Hidden Degrees of Freedom

- High-complexity impact model posing as a low-complexity single variable.
- Same thing can happen when naively stacking models

## True Complexity of ZIP variable

```
mod1 = glm(sold~zip, data=train,  
family=binomial)  
## [1] "Complexity (degrees of freedom): 96"  
## [1] "% deviance explained: 20.8%"  
## [1] "p-value on Chi^2 Test on model:  
0.0725"
```

## Apparent Complexity of ZIP (zip\_impact)

```
mod2 = glm(sold~zip_impact, data=train,  
family=binomial)  
## [1] "Complexity (degrees of freedom): 1"  
## [1] "% deviance explained: 7%"  
## [1] "p-value on Chi^2 Test on model:  
3.68e-10"
```

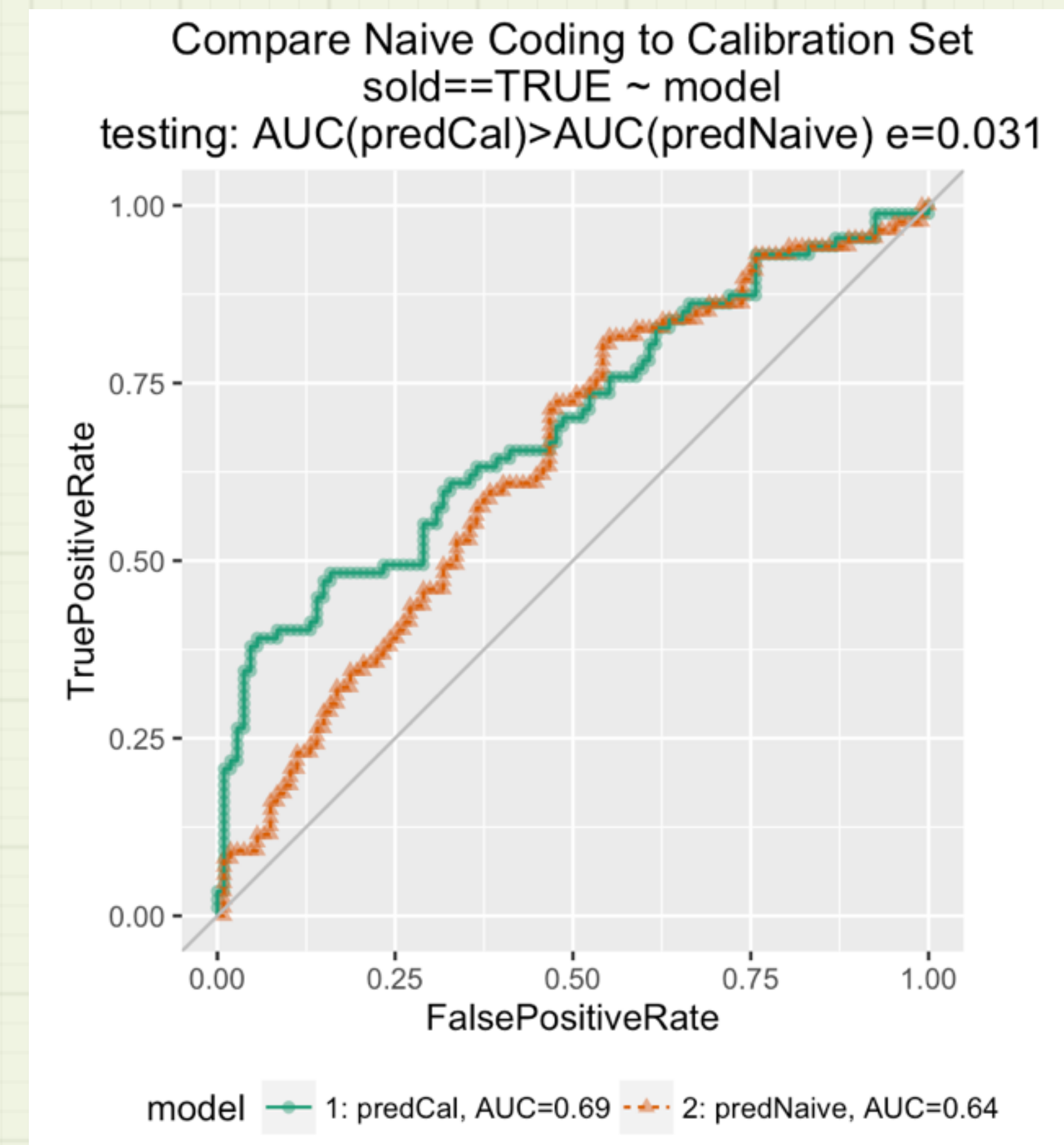


# Why Do We Not Always Notice?

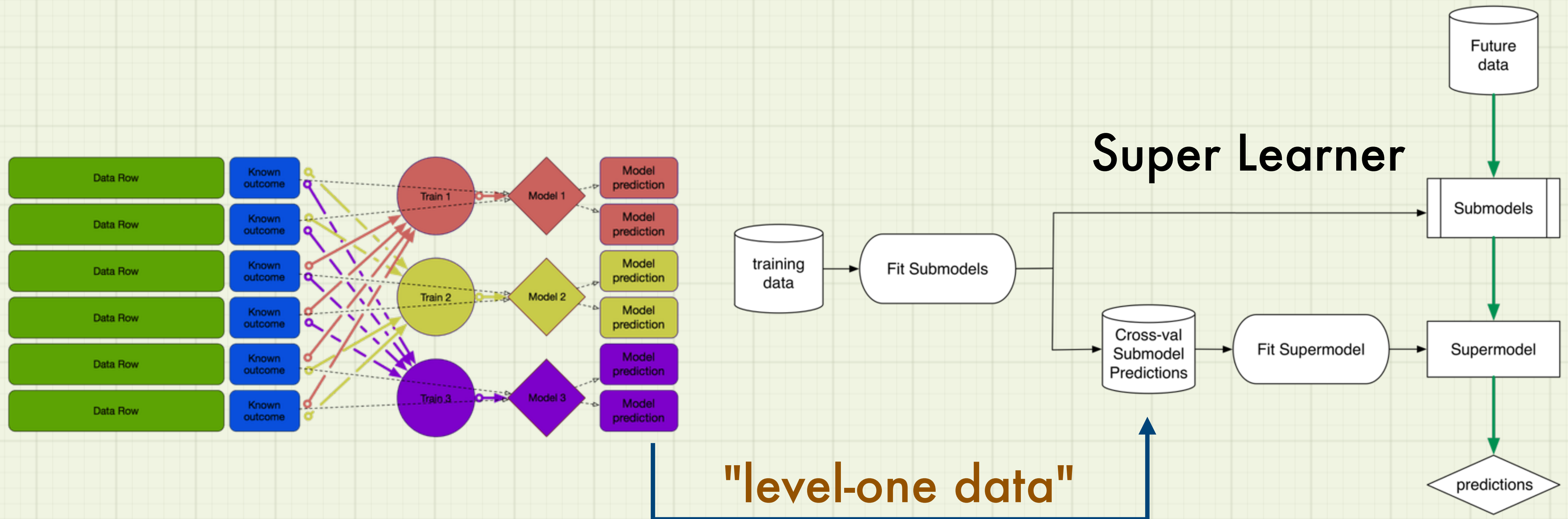
1. Problem not as obvious with ensembles
  - Lower complexity base models, simple model combination
  - Stacked learning: Higher-complexity models
2. We have noticed, we just ignore it.
  - Random Forests do tend to overfit

# Solution 1: Train Base models on Different Data

- Split training set into train/calibration
- Fit effects coding on calibration
- Train model on train
- $AUC(\text{Train}) = 0.71$ ,  $AUC(\text{Test})=0.69$ 
  - (vs 0.64 naive coding)



# Solution 2: Simulate training models on different data

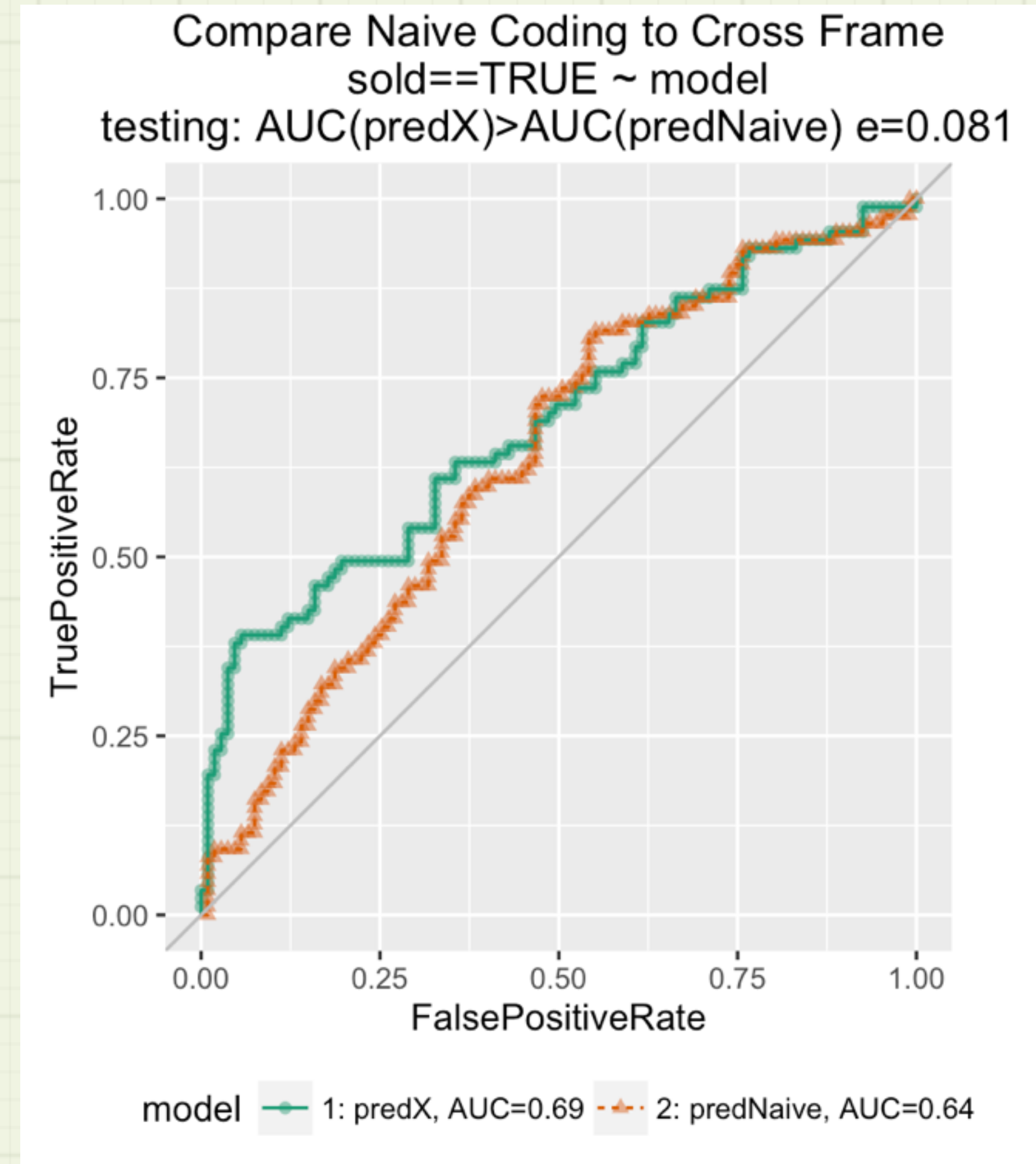




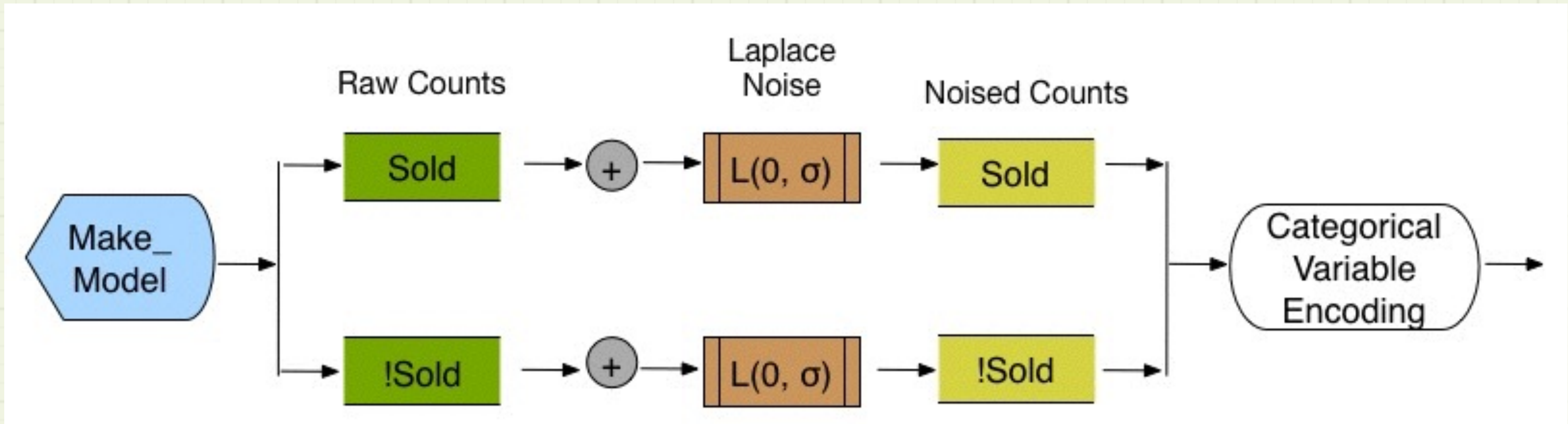
# Impact Coding: CrossFrames

Data preparation package  
vtreat

- Fits impact code on training data
- Creates "CrossFrame" impact coding for fitting model
- $AUC(\text{Train}) = 0.7$ ,  $AUC(\text{Test})=0.69$ 
  - (vs 0.64 naive coding)



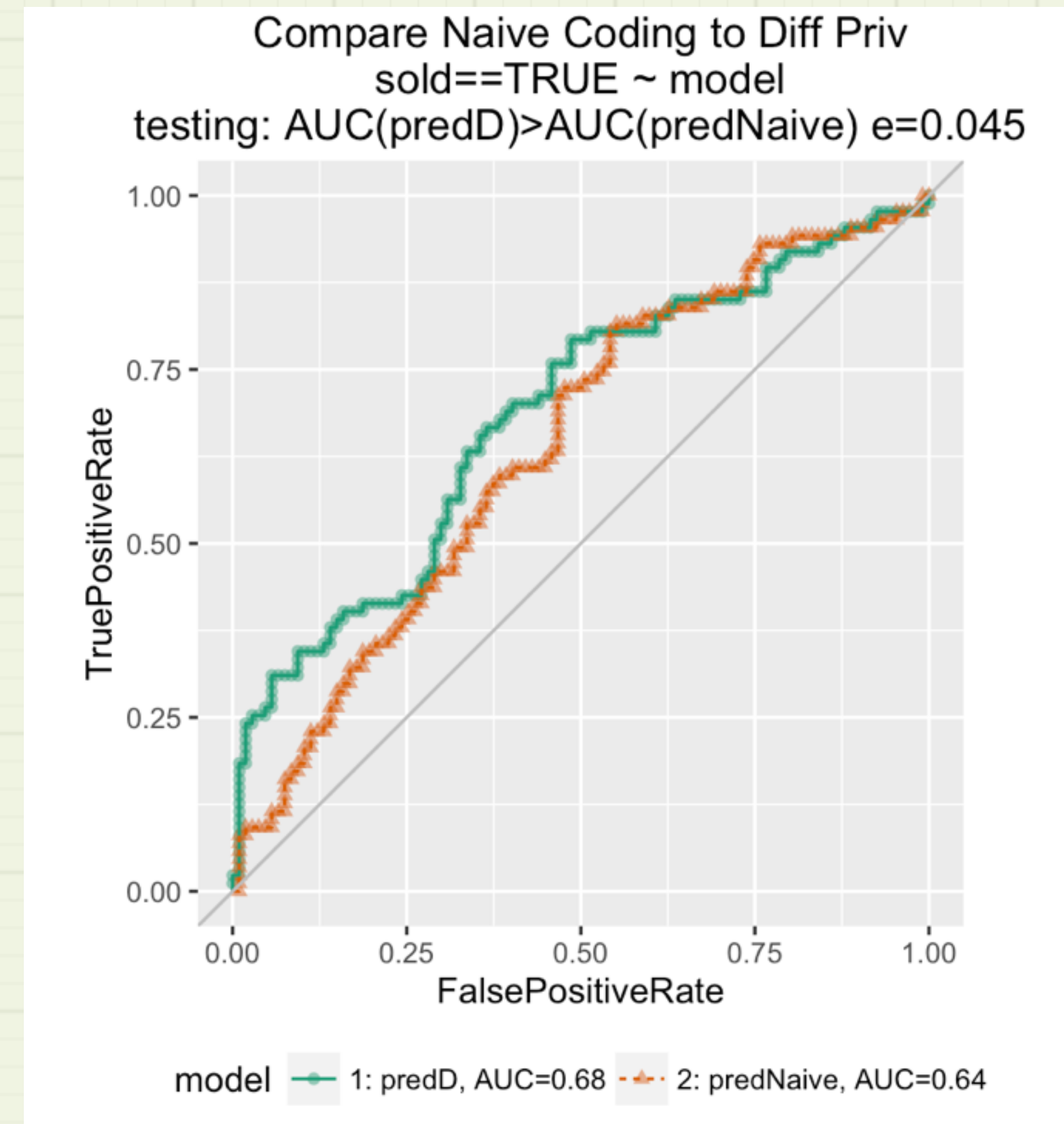
# Solution 3: Noise the Observations



Add noise to training data before passing to effects coding

# ZIP code example

- Add synthetic observations to training data
- Use noisy training data to effects code, clean training data to model
- $AUC(\text{Train}) = 0.71$ ,  $AUC(\text{Test})=0.68$ 
  - (vs 0.64 naive coding)





# Differential Privacy

- Bilenko, Misha. “Big Learning made Easy — with Counts!” *Machine Learning Blog* <http://blogs.technet.com/b/machinelearning/archive/2015/02/17/big-learning-made-easy-with-counts.aspx>
- Dwork, Cynthia, *et.al.* “The reusable holdout: Preserving validity in adaptive data analysis”, *Science*, vol 349, no 6248 pp 636-638, August 2015.
  - Abstract: <https://www.sciencemag.org/content/349/6248/636.abstract>
- <http://www.win-vector.com/blog/2015/11/our-differential-privacy-mini-series/>

# What about Smoothing/Regularization?

- Obscures rare values — should work, right?
- All three methods outperform Laplace smoothing/regularization
- The meta-model can sometimes undo smoothing
  - Performs no better than naive effects coding

# Summary

- Training data reuse increases nested model bias.
- High-complexity models (base or meta) increase nested model bias.
- Use (simulated) out-of-sample data for meta-model training.
- Prefer lower-complexity models when feasible.

# Nested Model Workflow

1. Use training data to train base models
2. Create (simulated) out-of-sample base model features
  - Calibration set; cross-frame; Laplacian noise
3. Train meta-model on out-of-sample features

	Base Model	Meta Model
Ensembles	(Usually) Low comp.	Sum (oblivious/"no complexity")
Stacked models	Arbitrary (High comp.)	Arbitrary (prefer Lower comp.)
Impact Coding	Impact code (High comp) Numerical var (Low comp)	Arbitrary (High comp)

	Base Model	Meta Model
Hyperparameter tuning	Arbitrary w/ varying $\rho$ (high comp.)	Pick best $\rho$
Variable selection (univariate models)	Numerical var (low comp.) Categorical var (high comp) Model algo (prefer lower comp)	Pick N best (low comp)  Exceeds threshold (low comp)
Stepwise methods	Arbitrary (High comp., varies with #vars)	Pick best (multiple times) (Very high complexity)



# More Information

- Examples from talk: <https://github.com/WinVector/NestedModelsTalk>
- Cross-frames for impact coding (`vtreat`)
  - <http://winvector.github.io/vtreathtml/vtreatCrossFrames.html>
  - `vtreat` on CRAN
- Breiman, Leo. "Stacked Regressions," *Machine Learning*, 24 p. 49-64 (1996). <http://statistics.berkeley.edu/sites/default/files/tech-reports/367.pdf>
- van der Laan, Alan, et.al. *Super Learner*. <http://biostats.bepress.com/ucbbiostat/paper222/>

# Thank You!