A Bid-Validity Mechanism for Sequential Heat and Electricity Market Clearing: Online appendix

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***Abstract*—Coordinating the operation of units at the interface between heat and electricity systems, such as combined heat and power plants and heat pumps, is essential to reduce inefficiencies in each system and help achieve a cost-effective and efficient operation of the overall energy system. These energy systems are currently operated by sequential markets, which interface the technical and economic aspects of the systems. In that context, the companion paper to this online appendix introduces an electricity-aware heat unit commitment model, which seeks to optimize the operation of the heat system while accounting for the techno-economic interdependencies between heat and electricity markets. These interdependencies are represented by bid-validity constraints, which model the linkage between the heat and electricity outputs and costs of combined heat and power plants and heat pumps. This approach also constitutes a novel market mechanism for the coordination of heat and electricity systems, which defines heat bids conditionally on electricity prices. Additionally, a tractable reformulation of the resulting trilevel optimization problem as a mixed integer linear program is proposed in Proposition 1. Finally, it is shown on a case study that the proposed model yields a 23% reduction in total operating cost and a 6% reduction in wind curtailment compared to a traditional decoupled unit commitment model. This online appendix provides the detailed formulations of the work in the companion paper, as well as a proof of Proposition 1.**

***Index Terms*—Bid validity, Hierarchical optimization, Market mechanism, Multi-energy systems, Unit commitment.**

I. Heat transfer dynamics in district heating networks

A. Heat flow constraints

The district heating network (DHN) comprises supply and return pipelines, as illustrated in Figure 1.

The control strategies in DHNs can adjust both mass flow rates, as well as supply and return temperatures within desired levels, in order to optimally exploit the energy storage capacity of DHNs. Mass flow rates and temperatures in supply and return networks and at each node are constrained by their upper and lower bounds, which are adjusted at each hour depending on loads and outside temperatures, such that

$$\underline{mf}^{\mathrm{S}}_{pt} \le mf^{\mathrm{S}}_{pt} \le \overline{mf}^{\mathrm{S}}_{pt},\ \underline{mf}^{\mathrm{R}}_{pt} \le mf^{\mathrm{R}}_{pt} \le \overline{mf}^{\mathrm{R}}_{pt}, \forall p \in \mathcal{I}^{\mathrm{P}}, t \in \mathcal{T} \tag{1a}$$

$$\underline{T}^{\mathrm{S}}_{pt} \le T^{\mathrm{S,in}}_{pt} \le \overline{T}^{\mathrm{S}}_{pt},\ \underline{T}^{\mathrm{R}}_{pt} \le T^{\mathrm{R,in}}_{pt} \le \overline{T}^{\mathrm{R}}_{pt}, \forall p \in \mathcal{I}^{\mathrm{P}}, t \in \mathcal{T} \tag{1b}$$

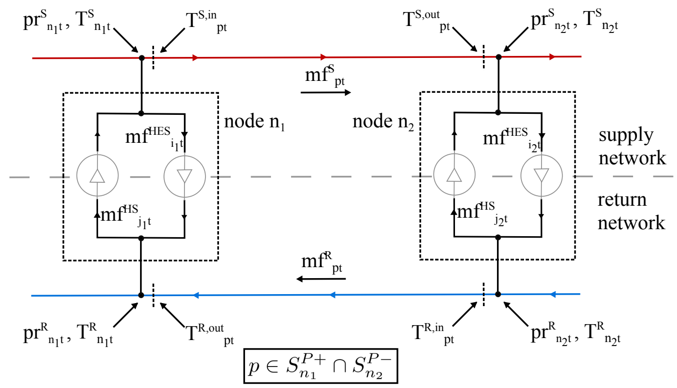

Figure 1: General structure of nodes and double-pipe DHN

$$\underline{T}^{\mathrm{S}}_{pt} \le T^{\mathrm{S,out}}_{pt} \le \overline{T}^{\mathrm{S}}_{pt},\ \underline{T}^{\mathrm{R}}_{pt} \le T^{\mathrm{R,out}}_{pt} \le \overline{T}^{\mathrm{R}}_{pt}, \forall p \in \mathcal{I}^{\mathrm{P}}, t \in \mathcal{T} \tag{1c}$$

$$\underline{T}^{\mathrm{S}}_{nt} \le T^{\mathrm{S}}_{nt} \le \overline{T}^{\mathrm{S}}_{nt},\ \underline{T}^{\mathrm{R}}_{nt} \le T^{\mathrm{R}}_{nt} \le \overline{T}^{\mathrm{R}}_{nt}, \forall n \in \mathcal{I}^{\mathrm{N}}, t \in \mathcal{T}. \tag{1d}$$

At each node n, Heat Exchanger Stations (HES) are modeled as heat loads. Their heat consumption is proportional to the mass flow rates through them and the difference of temperatures, between supply and demand networks at this node, such that

$$L^{\mathrm{H}}_{jt} = c\,mf^{\mathrm{HES}}_{jt}\left(T^{\mathrm{S}}_{nt} - T^{\mathrm{R}}_{nt}\right)\ \forall n \in \mathcal{I}^{\mathrm{N}}, i \in \mathcal{I}^{\mathrm{HES}}_{n}, t \in \mathcal{T}. \tag{2}$$

Similarly, the heat production of Heat Stations (HS) located at each node represent heat production units, such as CHPs, heat-only units, and heat pumps (HP), is defined as

$$Q_{jt} = c\,mf^{\mathrm{HS}}_{jt}\left(T^{\mathrm{S}}_{nt} - T^{\mathrm{R}}_{nt}\right) \quad j \in \mathcal{I}^{\mathrm{HS}}, t \in \mathcal{T}, \tag{3}$$

and bounded by

$$\underline{Q}_{j} u^{0}_{jt} \le Q_{jt} \le \overline{Q}_{j} u^{0}_{jt}\ \forall j \in \mathcal{I}^{\mathrm{HS}}, t \in \mathcal{T}. \tag{4}$$

The mass flow rates through these units are also bounded, such that

$$\underline{mf}^{\mathrm{HES}}_{jt} \le mf^{\mathrm{HES}}_{jt} \le \overline{mf}^{\mathrm{HES}}_{jt}\ \forall n \in \mathcal{I}^{\mathrm{N}}, j \in \mathcal{I}^{\mathrm{HES}}_{n}, t \in \mathcal{T} \tag{5a}$$

$$\underline{mf}^{\mathrm{HS}}_{jt} \le mf^{\mathrm{HS}}_{jt} \le \overline{mf}^{\mathrm{HS}}_{jt}\ \forall n \in \mathcal{I}^{\mathrm{N}}, j \in \mathcal{I}^{\mathrm{HS}}_{n}, t \in \mathcal{T}. \tag{5b}$$

This limits their energy production and consumption.

Furthermore, in order to ensure that mass flows through each HES from the supply to the return network, pressure differences between supply and return networks at each node must be greater than a threshold, such that

$$pr^{\mathrm{S}}_{nt} - pr^{\mathrm{R}}_{nt} \geq \underline{\Delta pr}_{nt}, \forall n \in \mathcal{I}^{\mathrm{N}}, t \in \mathcal{T} \tag{6a}$$

$$\underline{pr}^{\mathrm{S}}_{nt} \leq pr^{\mathrm{S}}_{nt} \leq \overline{pr}^{\mathrm{S}}_{nt}, \forall n \in \mathcal{I}^{\mathrm{N}}, t \in \mathcal{T} \tag{6b}$$

$$\underline{pr}^{\mathrm{R}}_{nt} \leq pr^{\mathrm{R}}_{nt} \leq \overline{pr}^{\mathrm{R}}_{nt}, \forall n \in \mathcal{I}^{\mathrm{N}}, t \in \mathcal{T}. \tag{6c}$$

A water pump is located at each HS in order to ensure mass flowing from the return to the supply network. The power consumption of the pump is proportional to the pressure difference between supply and return networks at this node, such that

$$L^{\mathrm{pump}}_{jt} = \frac{mf^{\mathrm{HS}}_{jt}\left(pr^{\mathrm{S}}_{nt} - pr^{\mathrm{R}}_{nt}\right)}{\rho \eta^{\mathrm{pump}}_{j}} \quad \forall n \in \mathcal{I}^{\mathrm{N}}, j \in S^{\mathrm{HS}}_{n}, t \in \mathcal{T}. \tag{7}$$

Furthermore, nodal balance equations must be met to ensure i) the continuity of mass flows and ii) the mixing of temperatures at each node. These two sets of equations ensure the balance of thermal energy at each node. Firstly, for incompressible water flows, net mass flow rates at each node are equal to zero, such that

$$\sum_{p \in \mathcal{I}^{\mathrm{P-}}_{n}} mf^{\mathrm{S}}_{pt} + \sum_{j \in \mathcal{I}^{\mathrm{HS}}_{n}} mf^{\mathrm{HS}}_{jt} = \sum_{p \in \mathcal{I}^{\mathrm{P+}}_{n}} mf^{\mathrm{S}}_{pt} + mf^{\mathrm{HES}}_{nt} \tag{8a}$$

$$, \forall n \in \mathcal{I}^{\mathrm{N}}, t \in \mathcal{T} \tag{8b}$$

$$\sum_{p \in \mathcal{I}^{\mathrm{P-}}_{n}} mf^{\mathrm{R}}_{pt} + \sum_{j \in \mathcal{I}^{\mathrm{HS}}_{n}} mf^{\mathrm{HS}}_{jt} = \sum_{p \in \mathcal{I}^{\mathrm{P+}}_{n}} mf^{\mathrm{R}}_{pt} + mf^{\mathrm{HES}}_{nt} \tag{8c}$$

$$, \forall n \in \mathcal{I}^{\mathrm{N}}, t \in \mathcal{T}. \tag{8d}$$

Secondly, supply and return temperatures at each node are expressed as a mix of the outlet temperatures of the pipes arriving at this node, such that

$$T^{\mathrm{S}}_{nt} \sum_{p \in \mathcal{I}^{\mathrm{P-}}_{n}} mf^{\mathrm{S}}_{pt} = \sum_{p \in \mathcal{I}^{\mathrm{P-}}_{n}} mf^{\mathrm{S}}_{pt} T^{\mathrm{S,out}}_{pt}\ , \ \forall n \in \mathcal{I}^{\mathrm{N}}, t \in \mathcal{T} \tag{9a}$$

$$T^{\mathrm{R}}_{nt} \sum_{p \in \mathcal{I}^{\mathrm{P+}}_{n}} mf^{\mathrm{R}}_{pt} = \sum_{p \in \mathcal{I}^{\mathrm{P+}}_{n}} mf^{\mathrm{R}}_{pt} T^{\mathrm{R,out}}_{pt}, \forall n \in \mathcal{I}^{\mathrm{N}}, t \in \mathcal{T}. \tag{9b}$$

Note that for nodes with a single pipe arriving or departing, Equations (9a) and (9b) can be replaced by the linear constraints

$$T^{\mathrm{S}}_{nt} = T^{\mathrm{S,out}}_{pt}, \forall n \in \tilde{\mathcal{I}}^{\mathrm{N-}}, p \in \mathcal{I}^{\mathrm{P-}}_{n} t \in \mathcal{T} \tag{10a}$$

$$T^{\mathrm{R}}_{nt} = T^{\mathrm{R,out}}_{pt}, \forall n \in \tilde{\mathcal{I}}^{\mathrm{N+}}, p \in \mathcal{I}^{\mathrm{P+}}_{n}, t \in \mathcal{T}. \tag{10b}$$

The main challenge is to model the temperature dynamics in the pipelines, and express the outlet temperatures and time delays. The inlet temperatures of the pipes are defined as

$$T^{\mathrm{S,in}}_{pt} = T^{\mathrm{S}}_{nt} \ \forall n \in \mathcal{I}^{\mathrm{N}}, p \in \mathcal{I}^{\mathrm{P+}}_{n}, t \in \mathcal{T} \tag{11a}$$

$$T^{\mathrm{R,in}}_{pt} = T^{\mathrm{R}}_{nt} \ \forall n \in \mathcal{I}^{\mathrm{N}}, p \in \mathcal{I}^{\mathrm{P-}}_{n}, t \in \mathcal{T}. \tag{11b}$$

Reference [1] provides an exact solution of the heat propagation equation, such that:

$$T^{\mathrm{S,out}}_{pt} = T^{\mathrm{S,in}}_{p(t-\Phi^{\mathrm{S}}_{pt})} e^{-\frac{2\mu_p}{c\rho R_p}\Phi^{\mathrm{S}}_{pt}}, \forall p \in \mathcal{I}^{\mathrm{P}}, t \in \mathcal{T} \tag{12a}$$

$$T^{\mathrm{R,out}}_{pt} = T^{\mathrm{S,in}}_{p(t-\Phi^{\mathrm{R}}_{pt})} e^{-\frac{2\mu_p}{c\rho R_p}\Phi^{\mathrm{R}}_{pt}}, \forall p \in \mathcal{I}^{\mathrm{P}}, t \in \mathcal{T} \tag{12b}$$

which expresses the outlet temperature of the pipelines, in function of the inlet temperature at times $t - \Phi^{\mathrm{S}}_{pt}$ and $t - \Phi^{\mathrm{R}}_{pt}$. The variable time delays Φ^{S}_{pt} and Φ^{R}_{pt} represent the time it takes to the hot water to travel along the pipelines, such that

$$\Phi^{\mathrm{S}}_{pt} = \min\left\{\phi \in \{0, ..., N_p\}, \text{s.t.} \sum_{k=t-\phi}^{t} \frac{mf^{\mathrm{S}}_{pk}}{\pi R^2_p \rho}\Delta t \geq L_p\right\}$$

$$, \forall p \in \mathcal{I}^{\mathrm{P}}, t \in \mathcal{T} \tag{13a}$$

$$\Phi^{\mathrm{R}}_{pt} = \min\left\{\phi \in \{0, ..., N_p\}, \text{s.t.} \sum_{k=t-\phi}^{t} \frac{mf^{\mathrm{R}}_{pk}}{\pi R^2_p \rho}\Delta t \geq L_p\right\}$$

$$, \forall p \in \mathcal{I}^{\mathrm{P}}, t \in \mathcal{T}, \tag{13b}$$

where the maximum time delays N_p can be computed using the pipelines' physical characteristics. These variable time delays can be discretized using a binary expansion, such that

$$\Phi^{\mathrm{S}}_{pt} = \sum_{k=0}^{N_p} u^{\Phi^{\mathrm{S}}}_{pkt}, \forall p \in \mathcal{I}^{\mathrm{P}}, t \in \mathcal{T} \tag{14a}$$

$$\Phi^{\mathrm{R}}_{pt} = \sum_{k=0}^{N_p} u^{\Phi^{\mathrm{R}}}_{pkt}, \forall p \in \mathcal{I}^{\mathrm{P}}, t \in \mathcal{T} \tag{14b}$$

$$u^{\Phi^{\mathrm{S}}}_{p\phi t}, u^{\Phi^{\mathrm{R}}}_{p\phi t} \in \{0, 1\}, \forall p \in \mathcal{I}^{\mathrm{P}}, t \in \mathcal{T}, \tag{14c}$$

where $u^{\Phi^{\mathrm{S}}}_{p\phi t}$ and $u^{\Phi^{\mathrm{R}}}_{p\phi t}$ are auxiliary variables defined as

$$-M^0\left(1 - u^{\Phi^{\mathrm{S}}}_{p\phi t}\right) \leq \sum_{k=t-\phi}^{t} \frac{mf^{\mathrm{S}}_{pk}}{\pi R^2_p \rho}\Delta t - L_p$$

$$\leq M^0 u^{\Phi^{\mathrm{S}}}_{p\phi t} - \epsilon, \ \forall p \in \mathcal{I}^{\mathrm{P}}, \phi \in \{0, ..., N_p\}, t \in \mathcal{T} \tag{15a}$$

$$-M^0\left(1 - u^{\Phi^{\mathrm{R}}}_{p\phi t}\right) \leq \sum_{k=t-\phi}^{t} \frac{mf^{\mathrm{R}}_{pk}}{\pi R^2_p \rho}\Delta t - L_p$$

$$\leq M^0 u^{\Phi^{\mathrm{R}}}_{p\phi t} - \epsilon, \ \forall p \in \mathcal{I}^{\mathrm{P}}, \phi \in \{0, ..., N_p\}, t \in \mathcal{T}, \tag{15b}$$

where M^0 is a big enough constant and ϵ a small enough constant. Furthermore, (12) can be approximated using a Taylors' series expansion, such that

$$T^{\mathrm{S,out}}_{pt} = T^{\mathrm{S,in}}_{p(t-\Phi^{\mathrm{S}}_{pt})}\left(1 - \frac{2\mu_p}{c\rho R_p}\sum_{k=0}^{N_p} u^{\Phi^{\mathrm{S}}}_{pkt}\right) \ \forall p \in \mathcal{I}^{\mathrm{P}}, t \in \mathcal{T} \tag{16a}$$

$$T^{\mathrm{R,out}}_{pt} = T^{\mathrm{S,in}}_{p(t-\Phi^{\mathrm{R}}_{pt})}\left(1 - \frac{2\mu_p}{c\rho R_p}\sum_{k=0}^{N_p} u^{\Phi^{\mathrm{R}}}_{pkt}\right) \ \forall p \in \mathcal{I}^{\mathrm{P}}, t \in \mathcal{T}. \tag{16b}$$

The variable time indexes in (16) can be simplified using the auxiliary binary variables $u^{\Phi^{\mathrm{S}}}_{p\phi t} \in \{0, 1\}$ and $u^{\Phi^{\mathrm{R}}}_{p\phi t} \in \{0, 1\}$, such that

$$T_{pt}^{\text{S,out}} = T_{pt}^{\text{S,in}} \left(1 - u_{p0t}^{\Phi^{\text{S}}}\right) + \sum_{k=1}^{N_p} T_{p(t-k)}^{\text{S,in}} e^{-\frac{2\mu_p}{c\rho R_p}k} \left(u_{p(k-1)t}^{\Phi^{\text{S}}} - u_{pkt}^{\Phi^{\text{S}}}\right), \forall p \in \mathcal{I}^{\text{P}}, t \in \mathcal{T} \quad (17a)$$

$$T_{pt}^{\text{R,out}} = T_{pt}^{\text{R,in}} \left(1 - u_{p0t}^{\Phi^{\text{R}}}\right) + \sum_{k=1}^{N_p} T_{p(t-k)}^{\text{R,in}} e^{-\frac{2\mu_p}{c\rho R_p}k} \left(u_{p(k-1)t}^{\Phi^{\text{R}}} - u_{pkt}^{\Phi^{\text{R}}}\right), \forall p \in \mathcal{I}^{\text{P}}, t \in \mathcal{T}. \quad (17b)$$

Although Equations (17) are bilinear, the product of a binary and continuous variable can be exactly linearized using its McCormick envelope [2].

Additionally, pressure loss due to friction along the pipes can be expressed as

$$pr_{n_1t}^{\text{S}} - pr_{n_2t}^{\text{S}} = \nu_p \left(m\!f_{pt}^{\text{S}}\right)^2,\; pr_{n_2t}^{\text{R}} - pr_{n_1t}^{\text{R}} = \nu_p \left(m\!f_{pt}^{\text{R}}\right)^2 \quad \forall n_1, n_2 \in \mathcal{I}^{\text{N}}, p \in S_{n_1}^{\text{P+}} \cap S_{n_2}^{\text{P-}}, t \in \mathcal{T}. \quad (18a)$$

B. Convex relaxation approach

The feasible space defined by the aforementioned heat flow constraints is non-convex due to the bilinear and quadratic constraints representing the pressure loss, heat production and consumption in the DHN, and temperature mixing. Firstly, the bilinear terms in Equations (2), (3), (7), and (9) can be linearized using a McCormick relaxation approach [2]. Each bilinear term is approximated by a set of linear upper and lower bounding functions. The McCormick relaxation of a bilinear function $f : (x, y) \mapsto xy$ defines its convex envelope over the domain $[x^l, x^u]\text{x}[y^l, y^u]$ [3], such that the product $z = xy$ can be approximated by the linear following under and over-estimators:

$$z \geq xy^u + x^u y - x^u y^u \quad (19a)$$

$$z \geq xy^l + x^l y - x^l y^l \quad (19b)$$

$$z \leq xy^u + x^l y - x^l y^u \quad (19c)$$

$$z \leq xy^l + x^u y - x^u y^l. \quad (19d)$$

Note that, when the upper and lower bounds of a variable are not explicitly stated in the problem, they must be estimated. This can be achieved by maximizing or minimizing each variable subject to the constraints of the problem to find its bounds. As the tightness of these bounds has a great impact on the tightness of the McCormick envelopes, using these values provides a tighter relaxation on the problem. Additionally, for the product of a binary and a continuous variable, this McCormick relaxation is exact.

Finally, the constraints (18a) representing pressure loss in pipelines can be linearized using an outer approximation approach [4], [5], [6]. We introduce a set of feasible points $\left(pr_{n_1t}^{\text{S,l}}, pr_{n_2t}^{\text{S,l}}\right)$ for $l = 1, ..., L$ and use a Taylor's series expansion to linearize (18a) around these points, such that

$$m\!f_{pt}^{\text{S}} \leq \frac{\sqrt{pr_{n_1t}^{\text{S,l}} - pr_{n_2t}^{\text{S,l}}}}{2\sqrt{\mu_p}} + \frac{\left(pr_{n_1t}^{\text{S}} - pr_{n_2t}^{\text{S}}\right)}{2\sqrt{\mu_p \left(pr_{n_1t}^{\text{S,l}} - pr_{n_2t}^{\text{S,l}}\right)}}, \forall n_1, n_2 \in \mathcal{I}^{\text{N}}, p \in S_{n_1}^{\text{P+}} \cap S_{n_2}^{\text{P-}}, t \in \mathcal{T}, l \in \{1, ..., L\}. \quad (20)$$

As a result of these convex relaxations, the feasible space defined by the aforementioned heat flow constraints is represented by a convex set $\mathcal{F}^{\text{H}}$.

II. Interdependencies between heat and electricity bids

The bids of HPs and CHPs in heat and electricity markets must reflect the strong interdependencies between their heat and electricity outputs and costs. Firstly, for each time step of the following day, CHPs and HPs submit at set of bids $\mathcal{B}^{\text{H}} = \{b_1, .., b_{N^{\text{H}}}\}$ in the heat market in the form of independent price-quantity pairs $(\alpha_{jbt}^{\text{H}}, \overline{s}_{jbt}^{\text{H}})$. For simplicity, it is assumed that CHPs and HPs offer their total heat production $\overline{Q}_j$. Additionally, two sets of bids are considered for the CHPs, which split their feasible operating regions into two areas. The first set of bids $\mathcal{B}^{\text{H}^0} = \{b_1, .., b_{N^{\text{H}^0}}\}$ covers the area where the lower electricity production bound is defined by the minimum fuel consumption, such that

$$\sum_{k=1}^{N^{\text{H}^0}} \overline{s}_{jbt}^{\text{H}} = \frac{F_j}{r_j \rho_j^{\text{E}} + \rho_j^{\text{H}}}, \forall j \in \mathcal{I}^{\text{CHP}}, t \in \mathcal{T}. \quad (21)$$

And the second set of bids $\mathcal{B}^{\text{H}^1} = \{b_{N^{\text{H}^0}+1}, .., b_{N^{\text{H}}}\}$ covers the area where the lower electricity production bound is defined by the minimum heat-to-power ratio, such that

$$\sum_{k=N^{\text{H}^0}+1}^{N^{\text{H}}} \overline{s}_{jb_kt}^{\text{H}} = \overline{Q}_j - \frac{F_j}{r_j \rho_j^{\text{E}} + \rho_j^{\text{H}}}, \forall j \in \mathcal{I}^{\text{CHP}}, t \in \mathcal{T}. \quad (22)$$

Furthermore, for each heat bid $b_k \in \mathcal{B}^{\text{H}}$ of a CHP, the heat market receives as inputs set of associated electricity bids $\mathcal{B}_k^{\text{E}} = \{\tilde{b}_{k,1}, ..., \tilde{b}_{k,N^{\text{E}}}\}$, in the form of independent price-quantity pairs $(\alpha_{j\tilde{b}_{k,l}t}^{\text{E}}, \overline{s}_{j\tilde{b}_{k,l}t}^{\text{E}})$. These quantities, which are independent on the heat dispatch Q_{jt}, cover the whole feasible operating region of the CHP such that

$$\sum_{l=1}^{N^{\text{E}}} \overline{s}_{j\tilde{b}_{k,l}t}^{\text{E}} \geq \frac{\overline{F}_j - \rho_j^{\text{H}} \sum_{m=1}^{\text{k-1}} \overline{s}_{jb_mt}^{\text{H}}}{\rho_j^{\text{E}}}, \forall j \in \mathcal{I}^{\text{CHP}}, t \in \mathcal{T}, k \in \{1, ..., N^{\text{H}}\}, \quad (23)$$

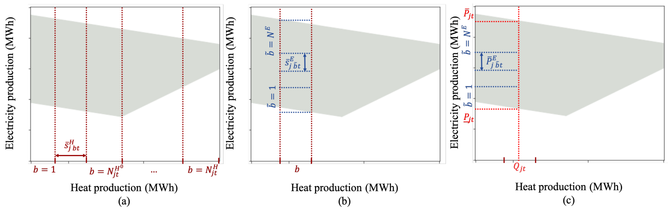

Figure 2: Bids of a CHP in the (a) day-ahead heat market, and (b) day-ahead electricity market, illustrated for a given heat dispatch Q_{jt} and last selected bid b_1.

and the first bid is large enough to cover the minimum electricity production, i.e.,

$$\bar{s}^{\mathrm{E}}_{j\tilde{b}_{k,1}t} \geq \frac{\underline{F}_j - \rho^{\mathrm{H}}_j \sum_{m=1}^{\text{k-1}} \bar{s}^{\mathrm{H}}_{jb_m t}}{\rho^{\mathrm{E}}_j}$$
$$,\forall j \in \mathcal{I}^{\mathrm{CHP}}, t \in \mathcal{T}, k \in \{1, ..., N^{\mathrm{H}^0}\} \tag{24a}$$
$$\bar{s}^{\mathrm{E}}_{j\tilde{b}_{k,1}t} \geq r_j \sum_{m=1}^{\text{k}} \bar{s}^{\mathrm{H}}_{jb_m t}$$
$$,\forall j \in \mathcal{I}^{\mathrm{CHP}}, t \in \mathcal{T}, k \in \{N^{\mathrm{H}^0} + 1, ..., N^{\mathrm{H}}\}. \tag{24b}$$

These bid quantities are illustrated schematically in Figure 2 (a).

Furthermore,these bids must reflect the marginal heat production cost $\dot{\Gamma}^{\mathrm{H}}_{jt}$ of CHPs and HPs. Due to the linkage between their heat and electricity outputs and costs, CHPs and HPs must anticipate their day-ahead electricity dispatch P_{jt} and electricity locational marginal prices (LMP) $\lambda^{\mathrm{E}}_{nt}$ in order to accurately compute their marginal heat production cost. As the heat-to-power ratio of HPs is considered fixed and given by their COP, the marginal heat production cost of HPs is proportional to the foreseen electricity LMPs, such that

$$\dot{\Gamma}^{\mathrm{H}}_{jt} = \frac{\lambda^{\mathrm{E}}_{nt}}{\mathrm{COP}_j}, \forall n \in \mathcal{I}^{\mathrm{N}}, j \in \mathcal{I}^{\mathrm{HP}}_n, t \in \mathcal{T}, \tag{25}$$

Additionally, as the heat-to-power ratio of CHPs is variable, their marginal heat production cost must be computed at the optimal heat-to-power ratio, for a given heat production and expected electricity LMPs. Therefore, for low electricity LMPs the marginal heat production cost of CHPs represents the incremental heat production cost at the minimum heat-to-power ratio, and for high electricity LMPs it represents the opportunity loss of producing an extra unit of heat at the maximum heat-to-power ratio, such that

$$\dot{\Gamma}^{\mathrm{H}}_{jt} = \begin{cases} \lambda^{\mathrm{E}}_{nt} \dfrac{\rho^{\mathrm{H}}_j}{\rho^{\mathrm{E}}_j} \text{ if } \lambda^{\mathrm{E}}_{nt} \geq \dfrac{C_j}{\rho^{\mathrm{E}}_j} \text{ or } Q_{jt} \geq \dfrac{\overline{F}_j}{(\rho^{\mathrm{H}}_j + r_j \rho^{\mathrm{E}}_j)} \\ C_j \left(\rho^{\mathrm{H}}_j + r_j \rho^{\mathrm{E}}_j\right) - \lambda^{\mathrm{E}}_{nt} r_j \text{ otherwise} \end{cases}$$
$$,\forall n \in \mathcal{I}^{\mathrm{N}}, j \in \mathcal{I}^{\mathrm{CHP}}_n, t \in \mathcal{T}. \tag{26}$$

Based on their last heat bid selected b and heat dispatch Q_{jt} in the heat market, HPs and CHPs must adjust their bids in the day-ahead electricity market. Indeed, the minimum electricity consumption of HPs, defined as

$$L^{\mathrm{HP}}_{jt} = \frac{Q_{jt}}{\mathrm{COP}_j}, \forall j\ in\mathcal{I}^{\mathrm{HP}}, t \in \mathcal{T} \tag{27}$$

is considered inflexible in the electricity market. Additionally, CHPs offer their minimum electricity output, defined as

$$\underline{P}_{jt} = \frac{\underline{F}_j - \rho^{\mathrm{H}}_j Q_{jt}}{\rho^{\mathrm{E}}_j} \left(u_{jb_1t} - u_{jb_{N^{\mathrm{H}^0}}t}\right) + r_j Q_{jt} u_{jb_{N^{\mathrm{H}^0}+1}t}$$
$$,\forall j \in \mathcal{I}^{\mathrm{CHP}}, t \in \mathcal{T}, \tag{28}$$

at any price. Furthermore, the maximum electricity outputs of CHPs is limited by their heat dispatch, such that

$$\overline{P}_{jt} = \frac{\overline{F}_j - \rho^{\mathrm{H}}_j Q_{jt}}{\rho^{\mathrm{E}}_j} u_{jb_1t}, \forall j \in \mathcal{I}^{\mathrm{CHP}}, t \in \mathcal{T}. \tag{29}$$

As a result, CHPs adjust their electricity quantity bids, such that if $b_k \in \mathcal{B}^{\mathrm{H}}$ is not the last selected bid, the quantities offered $\overline{P}_{j\tilde{b}_{k,l}t}$ are all equal to zero, i.e.,

$$\overline{P}_{j\tilde{b}_{k,l}t} = \bar{s}^{\mathrm{E}}_{j\tilde{b}_{k,l}t} \left(u_{jb_kt} - u_{jb_{k+1}t}\right)$$
$$,\forall j \in \mathcal{I}^{\mathrm{CHP}}, t \in \mathcal{T}, k \in \{1, ..., N^{\mathrm{H}}\}, l \in \{2, ..., N^{\mathrm{E}} - 1\}. \tag{30}$$

In addition, the quantity of the first electricity bid $\tilde{b}_{k,1}$, associated with the last selected heat bid b_k, is reduced by the minimum electricity output, such that

$$\overline{P}_{j\tilde{b}_{k,1}t} = \left[\bar{s}^{\mathrm{E}}_{j\tilde{b}_{k,1}t} - \underline{P}_{jt}\right] \left(u_{jb_kt} - u_{jb_{k+1}t}\right)$$
$$,\forall j \in \mathcal{I}^{\mathrm{CHP}}, t \in \mathcal{T}, k \in \{1, ..., N^{\mathrm{H}}\}. \tag{31}$$

Finally, the quantity of the last electricity bid $\tilde{b}_{k,N^{\mathrm{E}}}$, associated with the last selected heat bid b_k, is defined as the remaining electricity production, such that

$$\overline{P}_{j\tilde{b}_{k,N^{\mathrm{E}}}t} = \left[\overline{P}_{jt} - \sum_{l=1}^{N^{\mathrm{E}}-1} \bar{s}^{\mathrm{E}}_{j\tilde{b}_{k,l}t}\right] \left(u_{jb_kt} - u_{jb_{k+1}t}\right)$$
$$,\forall j \in \mathcal{I}^{\mathrm{CHP}}, t \in \mathcal{T}, k \in \{1, ..., N^{\mathrm{H}}\}. \tag{32}$$

These bid quantities are illustrated schematically in Figure 2 (b). In the proposed electricity-aware heat unit commitment (UC) problem, these new bounds and adjusted bids are modelled endogenously as outputs of the heat market clearing.

III. Formulation of the Electricity-Aware Heat UC Problem

This section presents in details the mathematical formulation of the electricity-aware heat UC problem proposed in the companion paper.

A. Upper-level: Heat UC

As explained in the companion paper, the upper-level problem represents the heat UC problem, which can be formulated as follows:

$$\min \sum_{t\in\mathcal{T}}\sum_{j\in\mathcal{I}^{\text{HS}}}\left(\alpha^0_{jt}u^0_{jt}+r_{jt}+\sum_{k=1}^{N^{\text{H}}}\alpha^{\text{H}}_{jb_kt}Q_{jb_kt}\right) \tag{33a}$$

$$\text{s.t.}\quad r_{jt}\ge\alpha^{\uparrow}_{jh}\left(u^0_{jt}-\sum_{k=t-h}^{t}u^0_{jk}\right),\forall j\in\mathcal{I}^{\text{HS}},t\in\mathcal{T},h\in\Phi^u_{jt} \tag{33b}$$

$$r_{jt}\ge 0,\forall j\in\mathcal{I}^{\text{HS}},t\in\mathcal{T} \tag{33c}$$

$$u^0_{jt}=u^{0,\text{init}}_j,\forall j\in\mathcal{I}^{\text{HS}},t\in\Phi^{u,\text{init}}_j \tag{33d}$$

$$\sum_{k=t-\underline{\Phi}^{\uparrow}_{jt}+1}^{t}v^{\uparrow}_{jk}\le u^0_{jt},\forall j\in\mathcal{I}^{\text{HS}},t\in\mathcal{T}\setminus\Phi^{u,\text{init}}_j \tag{33e}$$

$$\sum_{k=t-\underline{\Phi}^{\downarrow}_{jt}+1}^{t}v^{\uparrow}_{jk}\le 1-u^0_{j(t-\underline{\Phi}^{\downarrow}_{jt})},\forall j\in\mathcal{I}^{\text{HS}},t\in\mathcal{T}\setminus\Phi^{u,\text{init}}_j \tag{33f}$$

$$v^{\uparrow}_{jt}-v^{\downarrow}_{jt}=u^0_{jt}-u^0_{j(t-1)},\forall j\in\mathcal{I}^{\text{HS}},t\in\mathcal{T} \tag{33g}$$

$$u_{jb_kt}\le u_{jb_{k-1}t},\forall j\in\mathcal{I}^{\text{HS}},t\in\mathcal{T},k\in\{1,...,N^{\text{H}}\} \tag{33h}$$

$$v^{\uparrow}_{jt},v^{\downarrow}_{jt},u^0_{jt}\in\{0,1\},\forall j\in\mathcal{I}^{\text{HS}},t\in\mathcal{T} \tag{33i}$$

$$u_{jb_kt}\in\{0,1\},\forall j\in\mathcal{I}^{\text{HS}},t\in\mathcal{T},k\in\{1,...,N^{\text{H}}\} \tag{33j}$$

$$\text{Equations (13)} \tag{33k}$$

$$\left(\alpha^{\text{H}}_{jb_{k-1}t}-M_j\right)\left(u_{jb_{k-1}t}-u_{jb_kt}\right)\ge\dot{\Gamma}^{\text{H}}_{jt}-M_j$$
$$,\forall j\in\mathcal{I}^{\text{CHP}}\cup\mathcal{I}^{\text{HP}},t\in\mathcal{T},k\in\{1,..,N^{\text{H}}\} \tag{33l}$$

$$\{Q_{jb_kt},\lambda^{\text{E}}_{nt}\}\in\ \text{primal and dual sol. of (34)} \tag{33m}$$

Equations (33b) and (33c) model the start-up cost depending on the time the units have been offline. Indeed, the expression $\left(u^0_{jt}-\sum_{k=t-h}^{t}u^0_{jk}\right)$ is one when unit j becomes online after it has been turned off for h time periods. Equation (33d) fixes the initial minimum up- and down-time of the units. Equations (33e) and (33f) enforce the minimum up- and down-time, respectively. Equations (33g) and (33h) state the relationship between the binary variables for the on-off, start-up, and shut-down statuses of each unit. Equation (33h) ensures that a bid is selected only if the previous bid has been selected. Equations (33k) defines the (discrete) time delays in all the pipelines, as previously discussed. Furthermore, Equation (33l) represents the bid-validity constraints for CHPs and HPs, which ensure that the prices of the last selected bids are greater than the marginal heat production cost of these units, as expressed in Equations (25) and (26). Finally, Equation (33m) represents the feedback from the primal and dual solutions of the middle- and lower-level problems.

B. Middle-level: Heat market clearing

As explained in the companion paper, the middle-level problem represents the heat market-clearing problem, which can be formulated as follows:

$$\min \sum_{t\in\mathcal{T}}\sum_{j\in\mathcal{I}^{\text{HS}}}\sum_{k=1}^{N^{\text{H}}}\alpha^{\text{H}}_{jb_kt}Q_{jb_kt} \tag{34a}$$

$$\text{s.t.}\quad \overline{s}^{\text{H}}_{jb_kt}u_{jb_{k+1}t}\le Q_{jb_kt}\le\overline{s}^{\text{H}}_{jbt}u_{jb_kt}$$
$$,\forall j\in\mathcal{I}^{\text{HS}},t\in\mathcal{T},k\in\{1,...,N^{\text{H}}-1\} \tag{34b}$$

$$Q_{jN^{\text{H}}_{jt}t}\le\overline{s}^{\text{H}}_{jbt}u_{jN^{\text{H}}_{jt}t},\forall j\in\mathcal{I}^{\text{HS}},t\in\mathcal{T} \tag{34c}$$

$$Q_{jt}=\sum_{k=1}^{N^{\text{H}}}Q_{jb_kt},\forall j\in\mathcal{I}^{\text{HS}},t\in\mathcal{T} \tag{34d}$$

$$\{Q_{jt},m\!f^{\text{P}}_{pt},m\!f^{\text{HS}}_{jt},m\!f^{\text{HES}}_{jt}$$
$$,T^{\text{in}}_{pt},T^{\text{out}}_{pt},T^{\text{S}}_{nt},T^{\text{R}}_{nt},pr_{nt}\}\in\mathcal{F}^{\text{H}} \tag{34e}$$

$$\text{Equations (27)}-\text{(32)} \tag{34f}$$

$$\{\lambda^{\text{E}}_{nt}\}\in\text{dual sol. of (35).} \tag{34g}$$

Equations (34b) and (34d) represent the heat production bounds of all heat bids, for the given commitment variables u_{jbt}. Equation (34e) represents the convex relaxation of the heat flow constraints as described in Section I, where the discrete time delays, Φ^{S}_{pt} and Φ^{R}_{pt}, and auxiliary variables, $u^{\Phi^{\text{S}}}_{pt}$ and $u^{\Phi^{\text{R}}}_{pt}$, are fixed. Furthermore, (34f) computes the adjusted bids of CHPs and HPs in the electricity market, as described in Section II.

C. Lower-level: Electricity market clearing

As explained in the companion paper, the lower-level problem represents the electricity market-clearing problem, which can be formulated as follows:

$$\min \sum_{t\in\mathcal{T}}\left[\sum_{j\in\mathcal{I}^{\text{E}}}\sum_{l=1}^{N^{\text{E}}}\alpha^{\text{E}}_{j\tilde{b}_lt}P_{j\tilde{b}_lt}+\sum_{j\in\mathcal{I}^{\text{CHP}}}\sum_{k=1}^{N^{\text{H}}}\sum_{l=1}^{N^{\text{E}}}\alpha^{\text{E}}_{j\tilde{b}_{k,l}t}P_{j\tilde{b}_{k,l}t}\right] \tag{35a}$$

$$\text{s.t.}\quad \sum_{j\in\mathcal{I}^{\text{CHP}}_n}\left(\underline{P}_{jt}+\sum_{k=1}^{N^{\text{H}}}\sum_{l=1}^{N^{\text{E}}}P_{j\tilde{b}_{k,l}t}\right)+\sum_{j\in\mathcal{I}^{\text{E}}_n}\left(\underline{s}^{\text{E}}_{jt}+\sum_{l=1}^{N^{\text{E}}}P_{j\tilde{b}_lt}\right)$$
$$=L^{\text{E}}_{nt}+\sum_{j\in\mathcal{I}^{\text{HP}}_n}L^{\text{HP}}_{jt}+\sum_{j\in\mathcal{I}^{\text{HS}}_n}L^{\text{pump}}_{jt}$$
$$+\sum_{m\in\mathcal{I}^{\text{N}}_n}B_{mn}(\theta_{mt}-\theta_{nt}),\forall n\in\mathcal{I}^{\text{N}},t\in\mathcal{T} \tag{35b}$$

$$B_{mn}\left(\theta_{mt}-\theta_{nt}\right)\le\overline{f}_{mn},\forall n\in\mathcal{I}^{\text{N}},m\in\mathcal{I}^{\text{N}}_n,t\in\mathcal{T} \tag{35c}$$

$$0\le P_{j\tilde{b}_{k,l}t}\le\overline{s}^{\text{E}}_{j\tilde{b}_lt},\forall j\in\mathcal{I}^{\text{E}},t\in\mathcal{T},l\in\{1,...,N^{\text{E}}\} \tag{35d}$$

$$0\le P_{j\tilde{b}_{k,l}t}\le\overline{P}_{j\tilde{b}_{k,l}t}$$
$$,\forall j\in\mathcal{I}^{\text{CHP}},t\in\mathcal{T},k\in\{1,...,N^{\text{H}}\},l\in\{1,...,N^{\text{E}}\} \tag{35e}$$

Equation (35b) represents the power balance equation at each node of the power system. Equation (35c) bounds the power flow between two nodes. As the focus of this study is to model the flexibility provided by DHNs and the interactions between heat and electricity systems, the power transmission network is modeled using the standard linearized DC power flow. More advanced modeling approaches, including second order cone relaxations of AC power flow equations, could be investigated in future works. Finally, Equations (35d) and (35e) represent the maximum production of each bid of the power plants and CHPs, respectively. Note that the inflexible electricity consumption L_{jt}^{HP}, L_{jt}^{pump} of HPs and water pumps, the bid quantities $\overline{P}_{j\tilde{b}_{k,t}t}$ of CHPs, and the minimum electricity production $\underline{P}_{jt}$ of CHPs are treated as fixed parameters, which are passed on from the middle-level problem (34). Finally, the electricity LMPs $\lambda_{nt}^{\mathrm{E}}$ are defined as the dual variables of the nodal balance constraints (35b).

IV. Reformulation of the electricity-aware heat UC problem as a single-level optimization problem

A. Compact trilevel formulation

Based, on the mathematical formulation (33), the proposed electricity-aware heat UC problem can be formulated in a compact form as

$$\min_{\substack{\boldsymbol{z}\in\{0,1\}^N \\ \boldsymbol{x}^{\mathbf{H}},\boldsymbol{y}^{\mathbf{E}}\geq\mathbf{0}}} \; c^{0^\top}\boldsymbol{z} + c^{\mathrm{H}^\top}\boldsymbol{x}^{\mathbf{H}} \tag{36a}$$

$$\text{s.t.} \quad \boldsymbol{z}\in\mathcal{Z}^{\mathrm{UC}} \tag{36b}$$

$$A^{\mathrm{UC}}\boldsymbol{z} + B^{\mathrm{UC}}\boldsymbol{y}^{\mathbf{E}} \geq \; b^{\mathrm{UC}} \tag{36c}$$

$$\boldsymbol{x}^{\mathbf{H}},\boldsymbol{y}^{\mathbf{E}} \in \text{primal and dual sol. of}$$

$$\min_{\boldsymbol{x}^{\mathbf{H}},\boldsymbol{y}^{\mathbf{E}}\geq\mathbf{0}} \; c^{\mathrm{H}^\top}\boldsymbol{x}^{\mathbf{H}} \tag{36d}$$

$$\text{s.t.} \; A^{\mathrm{H}}(\boldsymbol{z})\,\boldsymbol{x}^{\mathbf{H}} + B^{\mathrm{H}}\boldsymbol{z} \geq b^{\mathrm{H}} \tag{36e}$$

$$\boldsymbol{y}^{\mathbf{E}} \in \text{dual sol. of}$$

$$\min_{\boldsymbol{x}^{\mathbf{E}}\geq\mathbf{0}} c^{\mathrm{E}^\top}\boldsymbol{x}^{\mathbf{E}} \tag{36f}$$

$$\text{s.t.} \; A^{\mathrm{E}}\boldsymbol{x}^{\mathbf{E}} + B^{\mathrm{E}}\boldsymbol{x}^{\mathbf{H}} \geq b^{\mathrm{E}}, \tag{36g}$$

where (36a) represents the heat system operating cost, and Equation (36b) the feasible set $\mathcal{Z}^{\mathrm{UC}}$ of UC and time-delay variables. Equation (36c) represents bid validity constraints. Equations (36d) and (36e) represent the objective and constraints of the heat market clearing, respectively. Additionally, Equations (36f) and (36g) represent the objective and constraints of the electricity market clearing, respectively. The set of decision variables $\boldsymbol{z}$ of the heat UC problem includes the commitment, start-up and shut-down variables, and start-up costs of each heat producer, and time delays in the pipelines of the DHN. The set of day-ahead heat market variables $\boldsymbol{x}^{\mathbf{H}}$ includes the dispatch of all bids, mass flow rates, temperatures, and pressures in the DHN, as well as the adjusted electricity bids of CHPs and HPs. The set of variables $\boldsymbol{x}^{\mathbf{E}}$ of the electricity market-clearing problem includes the dispatch of all bids, and the voltage angles at each node of the network. Furthermore, the vector $\boldsymbol{y}^{\mathbf{E}}$ represents the variables of the dual electricity market-clearing problem.

B. Reformulation as a single-level optimization problem

The companion paper introduced the following proposition:

Proposition 1. *The trilevel optimization problem (36) can be asymptotically approximated by the following single-level problem:*

$$\min_{\substack{\boldsymbol{z}\in\{0,1\}^N,\boldsymbol{x}^{\boldsymbol{H}}\geq\mathbf{0} \\ \boldsymbol{x}^{\boldsymbol{E}}\geq\mathbf{0},\boldsymbol{y}^{\boldsymbol{H}},\boldsymbol{y}^{\boldsymbol{E}}}} \gamma c^{0^\top}\boldsymbol{z} + \gamma c^{H^\top}\boldsymbol{x}^{\boldsymbol{H}} + (1-\gamma)\,c^{E^\top}\boldsymbol{x}^{\boldsymbol{E}} \tag{37a}$$

$$\textit{s.t.} \quad \boldsymbol{z}\in\mathcal{Z}^{UC} \tag{37b}$$

$$A^{UC}\boldsymbol{z} + \frac{1}{(1-\gamma)}B^{UC}\boldsymbol{y}^{\boldsymbol{E}} \geq b^{UC} \tag{37c}$$

$$A^{H}(\boldsymbol{z})\,\boldsymbol{x}^{\boldsymbol{H}} + B^{H}\boldsymbol{z} \geq b^{H} \tag{37d}$$

$$A^{E}\boldsymbol{x}^{\boldsymbol{E}} + B^{E}\boldsymbol{x}^{\boldsymbol{H}} \geq b^{E} \tag{37e}$$

$$\boldsymbol{y}^{\boldsymbol{H}^\top}A^{H}(\boldsymbol{z}) + \boldsymbol{y}^{\boldsymbol{E}^\top}B^{E} \leq \gamma c^{H^\top} \tag{37f}$$

$$\boldsymbol{y}^{\boldsymbol{E}^\top}A^{E} \leq (1-\gamma)\,c^{E^\top} \tag{37g}$$

$$\boldsymbol{y}^{\boldsymbol{H}^\top}\left(b^{H} - B^{H}\boldsymbol{z}\right) + \boldsymbol{y}^{\boldsymbol{E}^\top}b^{E} \geq \gamma c^{H^\top}\boldsymbol{x}^{\boldsymbol{H}} + (1-\gamma)\,c^{E^\top}\boldsymbol{x}^{\boldsymbol{E}}. \tag{37h}$$

When the penalty factor γ tends to 1, the solutions of (37) converge to the solutions of (36).

The proof of this proposition is provided below.

C. Proof of Proposition 1

Firstly, by strong duality of the linear lower-level problem (36f)-(36g), the middle-level problem (36d)-(36g) is equivalent to

$$\boldsymbol{x}^{\mathbf{H}},\boldsymbol{y}^{\mathbf{E}} \in \arg\min_{\boldsymbol{x}^{\mathbf{H}},\boldsymbol{y}^{\mathbf{E}}\geq\mathbf{0}} \; c^{\mathrm{H}^\top}\boldsymbol{x}^{\mathbf{H}} \tag{38a}$$

$$\text{s.t.} \quad A^{\mathrm{H}}(\boldsymbol{z})\,\boldsymbol{x}^{\mathbf{H}} + \; B^{\mathrm{H}}\boldsymbol{z} \geq b^{\mathrm{H}} \tag{38b}$$

$$\boldsymbol{y}^{\mathbf{E}} \in \arg\min_{\boldsymbol{x}^{\mathbf{E}},\boldsymbol{y}^{\mathbf{E}}\geq\mathbf{0}} c^{\mathrm{E}^\top}\boldsymbol{x}^{\mathbf{E}} \tag{38c}$$

$$\text{s.t.} \quad A^{\mathrm{E}}\boldsymbol{x}^{\mathbf{E}} + B^{\mathrm{E}}\boldsymbol{x}^{\mathbf{H}} \geq b^{\mathrm{E}} \tag{38d}$$

$$\boldsymbol{y}^{\mathbf{E}^\top}A^{\mathrm{E}} \leq c^{\mathrm{E}^\top} \tag{38e}$$

$$\boldsymbol{y}^{\mathbf{E}^\top}\left(B^{\mathrm{E}} - B^{\mathrm{E}}\boldsymbol{x}^{\mathbf{H}}\right) \geq c^{\mathrm{E}^\top}\boldsymbol{x}^{\mathbf{E}}. \tag{38f}$$

Equation (38e) represents the dual constraints of the lower-level problem (36f)-(36g) (dual feasibility). Equation (38f) enforces equality of the primal and dual objective values of the lower-level problem (36f)-(36g) at optimality (strong duality).

Furthermore, the objective (38a) and constraints (38b) of the middle-level problem do not depend on the lower-level variables $\boldsymbol{x}^{\mathrm{E}}$, and $\boldsymbol{y}^{\mathrm{E}}$. Therefore, the solutions of middle-level optimization problem are not affected by the solutions of the lower-level problem. Problem (38) can thus be solved in two steps: (i) solve the heat market-clearing problem (38a)-(38b) and obtain the optimal solutions $\hat{\boldsymbol{x}}^{\mathrm{H}}$, (ii) solve the electricity market-clearing problem (38c)-(38f) with the variable $\boldsymbol{x}^{\mathrm{H}}$ fixed to $\hat{\boldsymbol{x}}^{\mathrm{H}}$ and obtain the optimal solutions $\hat{\boldsymbol{y}}^{\mathrm{E}}$. Therefore, the middle-level problem (38) can be reformulated using a lexicographic function as follows:

$$\boldsymbol{x}^{\mathbf{H}}, \boldsymbol{y}^{\mathbf{E}} \in \underset{\boldsymbol{x}^{\mathrm{H}}, \boldsymbol{x}^{\mathrm{E}}, \boldsymbol{y}^{\mathrm{E}} \geq \mathbf{0}}{\arg\min} < \quad c^{\mathrm{H}^{\top}} \boldsymbol{x}^{\mathbf{H}}, \quad c^{\mathrm{E}^{\top}} \boldsymbol{x}^{\mathbf{E}} > \tag{39a}$$

$$\text{s.t.} \quad A^{\mathrm{H}}(\boldsymbol{z})\, \boldsymbol{x}^{\mathbf{H}} + B^{\mathrm{H}} \boldsymbol{z} \geq b^{\mathrm{H}} \tag{39b}$$

$$A^{\mathrm{E}} \boldsymbol{x}^{\mathbf{E}} + B^{\mathrm{E}} \boldsymbol{x}^{\mathbf{H}} \geq b^{\mathrm{E}} \tag{39c}$$

$$\boldsymbol{y}^{\mathbf{E}^{\top}} A^{\mathrm{E}} \leq c^{\mathrm{E}^{\top}} \tag{39d}$$

$$\boldsymbol{y}^{\mathbf{E}^{\top}} \left(B^{\mathrm{E}} - B^{\mathrm{E}} \boldsymbol{x}^{\mathbf{H}}\right) \geq c^{\mathrm{E}^{\top}} \boldsymbol{x}^{\mathbf{E}} \tag{39e}$$

Any optimal solution $\hat{\boldsymbol{x}}^{\mathrm{H}}, \hat{\boldsymbol{x}}^{\mathrm{E}}, \hat{\boldsymbol{y}}^{\mathrm{E}}$ of Problem (39) satisfies the following property:

$$\hat{\boldsymbol{x}}^{\mathbf{H}} \in \underset{\boldsymbol{x}^{\mathbf{H}}}{\arg\min} \quad c^{\mathrm{H}^{\top}} \boldsymbol{x}^{\mathbf{H}} \tag{40a}$$

$$\text{s.t.} \quad A^{\mathrm{H}}(\boldsymbol{z})\, \boldsymbol{x}^{\mathbf{H}} + B^{\mathrm{H}} \boldsymbol{z} \geq b^{\mathrm{H}} \tag{40b}$$

and

$$\hat{\boldsymbol{x}}^{\mathbf{E}}, \hat{\boldsymbol{y}}^{\mathbf{E}} \in \underset{\boldsymbol{x}^{\mathbf{E}}, \boldsymbol{y}^{\mathbf{E}} \geq \mathbf{0}}{\arg\min} \quad c^{\mathrm{E}^{\top}} \boldsymbol{x}^{\mathbf{E}} \tag{41a}$$

$$\text{s.t.} \quad A^{\mathrm{E}} \boldsymbol{x}^{\mathbf{E}} + B^{\mathrm{E}} \hat{\boldsymbol{x}}^{\mathbf{H}} \geq b^{\mathrm{E}} \tag{41b}$$

$$\boldsymbol{y}^{\mathbf{E}^{\top}} A^{\mathrm{E}} \leq c^{\mathrm{E}^{\top}} \tag{41c}$$

$$\boldsymbol{y}^{\mathbf{E}^{\top}} \left(B^{\mathrm{E}} - B^{\mathrm{E}} \hat{\boldsymbol{x}}^{\mathbf{H}}\right) \geq c^{\mathrm{E}^{\top}} \boldsymbol{x}^{\mathbf{E}} \tag{41d}$$

In addition, by strong duality of Problem (41), any feasible solution $\tilde{\boldsymbol{x}}^{\mathbf{E}}$, $\tilde{\boldsymbol{y}}^{\mathbf{E}}$ is optimal. Therefore, $\tilde{\boldsymbol{x}}^{\mathbf{E}}$ is an optimal solution of the primal formulation of the lower-level problem (36f)-(36g), such that

$$\tilde{\boldsymbol{x}}^{\mathbf{E}} \in \underset{\boldsymbol{x}^{\mathbf{E}} \geq \mathbf{0}}{\arg\min} \quad c^{\mathrm{E}^{\top}} \boldsymbol{x}^{\mathbf{E}} \tag{42a}$$

$$\text{s.t.} \quad A^{\mathrm{E}} \boldsymbol{x}^{\mathbf{E}} + B^{\mathrm{E}} \hat{\boldsymbol{x}}^{\mathbf{H}} \geq b^{\mathrm{E}}, \tag{42b}$$

and $\tilde{\boldsymbol{y}}^{\mathbf{E}}$ is an optimal solution of the dual formulation of the lower-level problem (36f)-(36g), such that

$$\tilde{\boldsymbol{y}}^{\mathbf{E}} \in \underset{\boldsymbol{y}^{\mathrm{E}} \geq \mathbf{0}}{\arg\max} \quad \boldsymbol{y}^{\mathbf{E}^{\top}} \left(B^{\mathrm{E}} - B^{\mathrm{E}} \hat{\boldsymbol{x}}^{\mathbf{H}}\right) \tag{43a}$$

$$\text{s.t.} \quad \boldsymbol{y}^{\mathbf{E}^{\top}} A^{\mathrm{E}} \leq c^{\mathrm{E}^{\top}}. \tag{43b}$$

As Problem (42) is a relaxation of Problem (41) and the set of solutions $\tilde{\boldsymbol{x}}^{\mathbf{E}}$, $\tilde{\boldsymbol{y}}^{\mathbf{E}}$ is feasible to Problem (41), then problem (39) can be approximated by the following linear program (LP), with $\gamma \in]0, 1[$:

$$\min_{\boldsymbol{x}^{\mathbf{H}}, \boldsymbol{x}^{\mathbf{E}} \geq \mathbf{0}} \quad \gamma c^{\mathrm{H}^{\top}} \boldsymbol{x}^{\mathbf{H}} + (1-\gamma)\, c^{\mathrm{E}^{\top}} \boldsymbol{x}^{\mathbf{E}} \tag{44a}$$

$$\text{s.t.} \quad A^{\mathrm{H}}(\boldsymbol{z})\, \boldsymbol{x}^{\mathbf{H}} + B^{\mathrm{H}} \boldsymbol{z} \geq b^{\mathrm{H}} \tag{44b}$$

$$A^{\mathrm{E}} \boldsymbol{x}^{\mathbf{E}} + B^{\mathrm{E}} \boldsymbol{x}^{\mathbf{H}} \geq b^{\mathrm{E}}, \tag{44c}$$

where $\boldsymbol{y}^{\mathbf{E}}$ is obtained as the dual variable associated with constraint (44c) [7]. As a result, Problem (36) can be approximated by the following linear bilevel optimization problem:

$$\min_{\substack{\boldsymbol{z} \in \{0,1\}^{\mathrm{N}}, \boldsymbol{x}^{\mathbf{H}}, \boldsymbol{x}^{\mathbf{E}} \geq \mathbf{0} \\ \boldsymbol{y}^{\mathbf{H}}, \boldsymbol{y}^{\mathbf{E}} \geq \mathbf{0}}} \quad \gamma c^{0^{\top}} \boldsymbol{z} + \gamma c^{\mathrm{H}^{\top}} \boldsymbol{x}^{\mathbf{H}} + (1-\gamma)\, c^{\mathrm{E}^{\top}} \boldsymbol{x}^{\mathbf{E}} \tag{45a}$$

$$\text{s.t.} \quad \boldsymbol{z} \in \mathcal{Z}^{\mathrm{UC}} \tag{45b}$$

$$A^{\mathrm{UC}} \boldsymbol{z} + \frac{1}{(1-\gamma)} B^{\mathrm{UC}} \boldsymbol{y}^{\mathbf{E}} \geq b^{\mathrm{UC}} \tag{45c}$$

$$\boldsymbol{x}^{\mathbf{H}}, \boldsymbol{y}^{\mathbf{E}} \text{ primal and dual sol. of (44).} \tag{45d}$$

And, by strong duality of the lower-level problem (45d), Problem (45) is equivalent to Problem (37).

It remains to show that Problem (37) is an asymptotic approximation to Problem (36), i.e., as $\gamma \to 1$ the solutions to Problem (37) become optimal solutions to Problem (36). Firstly by introducing the auxiliary variables $\tilde{\boldsymbol{y}}^{\mathrm{H}} = \dfrac{\boldsymbol{y}^{\mathrm{H}}}{\gamma}$, and $\tilde{\boldsymbol{y}}^{\mathrm{E}} = \dfrac{\boldsymbol{y}^{\mathrm{E}}}{1-\gamma}$, Problem (37) is equivalent to

$$\min_{\substack{\boldsymbol{z} \in \{0,1\}^{N}, \boldsymbol{x}^{\mathbf{H}} \geq \mathbf{0} \\ \boldsymbol{x}^{\mathbf{E}} \geq \mathbf{0}, \boldsymbol{y}^{\mathbf{H}}, \boldsymbol{y}^{\mathbf{E}}}} \quad \gamma c^{0^{\top}} \boldsymbol{z} + \gamma c^{\mathrm{H}^{\top}} \boldsymbol{x}^{\mathbf{H}} + (1-\gamma)\, c^{\mathrm{E}^{\top}} \boldsymbol{x}^{\mathbf{E}} \tag{46a}$$

$$\text{s.t.} \quad \boldsymbol{z} \in \mathcal{Z}^{\mathrm{UC}} \tag{46b}$$

$$A^{\mathrm{UC}} \boldsymbol{z} + B^{\mathrm{UC}} \tilde{\boldsymbol{y}}^{\mathrm{E}} \geq b^{\mathrm{UC}} \tag{46c}$$

$$A^{\mathrm{H}}(\boldsymbol{z})\, \boldsymbol{x}^{\mathbf{H}} + B^{\mathrm{H}} \boldsymbol{z} \geq b^{\mathrm{H}} \tag{46d}$$

$$A^{\mathrm{E}} \boldsymbol{x}^{\mathbf{E}} + B^{\mathrm{E}} \boldsymbol{x}^{\mathbf{H}} \geq b^{\mathrm{E}} \tag{46e}$$

$$\tilde{\boldsymbol{y}}^{\mathbf{H}^{\top}} A^{\mathrm{H}}(\boldsymbol{z}) + \frac{(1-\gamma)}{\gamma} \tilde{\boldsymbol{y}}^{\mathbf{E}^{\top}} B^{\mathrm{E}} \leq c^{\mathrm{H}^{\top}} \tag{46f}$$

$$\tilde{\boldsymbol{y}}^{\mathbf{E}^{\top}} A^{\mathrm{E}} \leq c^{\mathrm{E}^{\top}} \tag{46g}$$

$$\tilde{\boldsymbol{y}}^{\mathbf{H}^{\top}} \left(b^{\mathrm{H}} - B^{\mathrm{H}} \boldsymbol{z}\right) - c^{\mathrm{H}^{\top}} \boldsymbol{x}^{\mathbf{H}} \geq \frac{(1-\gamma)}{\gamma} \left(c^{\mathrm{E}^{\top}} \boldsymbol{x}^{\mathbf{E}} - \tilde{\boldsymbol{y}}^{\mathbf{E}^{\top}} b^{\mathrm{E}}\right). \tag{46h}$$

Let us denote $P(\hat{\boldsymbol{z}})$ and $\tilde{P}(\hat{\boldsymbol{z}})$ the optimal objective value of Problems (39) and (46), respectively, with the value of the UC variable $\boldsymbol{z}$ fixed to $\hat{\boldsymbol{z}}$. Let $\left\{\hat{\boldsymbol{x}}^{\mathrm{H}}, \hat{\boldsymbol{x}}^{\mathrm{E}}, \hat{\boldsymbol{y}}^{\mathrm{H}}, \hat{\boldsymbol{y}}^{\mathrm{E}}\right\}$ be the optimal solutions to $\tilde{P}(\hat{\boldsymbol{z}})$.

As $\gamma \to 1$, Equations (46f) and (46h) become the following

$$\tilde{\boldsymbol{y}}^{\mathbf{H}^{\top}} A^{\mathrm{H}}(\hat{\boldsymbol{z}}) \leq c^{\mathrm{H}^{\top}} \tag{47a}$$

$$\tilde{\boldsymbol{y}}^{\mathbf{H}^{\top}} \left(b^{\mathrm{H}} - B^{\mathrm{H}} \boldsymbol{z}\right) - \geq c^{\mathrm{H}^{\top}} \boldsymbol{x}^{\mathbf{H}}. \tag{47b}$$

Equation (46d) guarantees that $\hat{\boldsymbol{x}}^{\mathrm{H}}$ is feasible to Problem (40) with $\boldsymbol{z}$ fixed to $\hat{\boldsymbol{z}}$. Additionally, Equation (47a) guarantees that $\hat{\boldsymbol{y}}^{\mathrm{H}}$ becomes feasible to Problem (40) with $\boldsymbol{z}$ fixed to $\hat{\boldsymbol{z}}$ when $\gamma \to 1$. And, Equation (47b) guarantees that $\hat{\boldsymbol{x}}^{\mathrm{H}}$ and $\hat{\boldsymbol{y}}^{\mathrm{H}}$, together, satisfy the strong duality equation of Problem (40) with $\boldsymbol{z}$ fixed to $\hat{\boldsymbol{z}}$ when $\gamma \to 1$. Therefore, $\hat{\boldsymbol{x}}^{\mathrm{H}}$ and $\hat{\boldsymbol{y}}^{\mathrm{H}}$ approximate the primal and dual optimal solutions to Problem (40) with $\boldsymbol{z}$ fixed to $\hat{\boldsymbol{z}}$ when $\gamma \to 1$. This implies that $\hat{\boldsymbol{x}}^{\mathrm{H}}$ and $\hat{\boldsymbol{y}}^{\mathrm{H}}$ become feasible solutions to $P(\hat{\boldsymbol{z}})$ when $\gamma \to 1$.

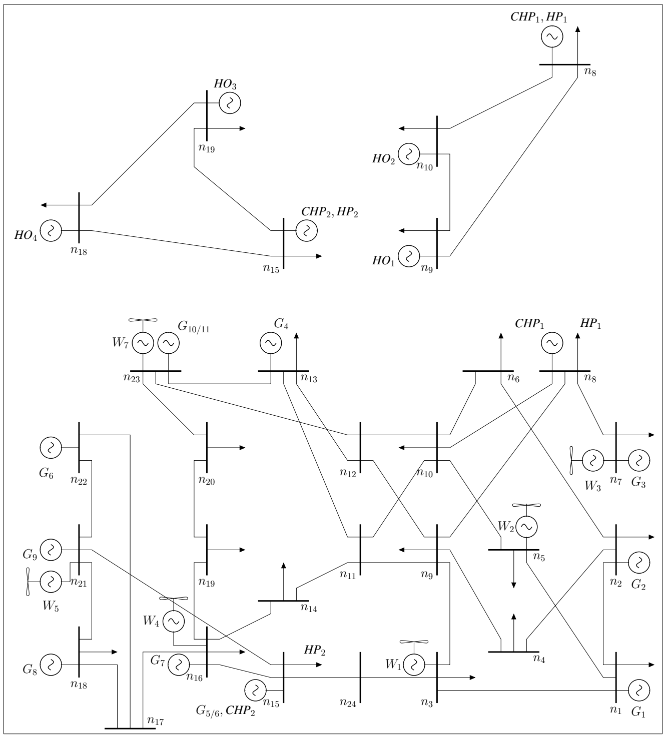

Figure 3: Modified IEEE 24-Bus reliability test system with 6 wind farms and two isolated 3-node district heating systems

Moreover, combining Equation (46h) and Equation (46f) $\times \hat{\boldsymbol{x}}^{\mathrm{H}}$ gives

$$\tilde{\boldsymbol{y}}^{\mathbf{H}^{\top}} \left(b^{\mathrm{H}} - B^{\mathrm{H}} \boldsymbol{z} - A^{\mathrm{H}} (\boldsymbol{z}) \hat{\boldsymbol{x}}^{\mathrm{H}} \right) \geq \frac{(1-\gamma)}{\gamma} \left(c^{\mathrm{E}^{\top}} \boldsymbol{x}^{\mathbf{E}} - \tilde{\boldsymbol{y}}^{\mathbf{E}^{\top}} \left(b^{\mathrm{E}} - B^{\mathrm{E}} \hat{\boldsymbol{x}}^{\mathrm{H}} \right) \right). \tag{48}$$

It follows from Equation (46d) that, for any gamma $\gamma \in [0, 1]$:

$$\tilde{\boldsymbol{y}}^{\mathbf{E}^{\top}} \left(b^{\mathrm{E}} - B^{\mathrm{E}} \hat{\boldsymbol{x}}^{\mathrm{H}} \right) \geq c^{\mathrm{E}^{\top}} \boldsymbol{x}^{\mathbf{E}}. \tag{49}$$

Equations (46e) and (46g) guarantee that $\hat{\boldsymbol{x}}^{\mathrm{E}}$ and $\hat{\boldsymbol{y}}^{\mathrm{E}}$ are feasible to Problem (41) with $\boldsymbol{x}^{\mathrm{H}}$ fixed to $\hat{\boldsymbol{x}}^{\mathrm{H}}$. Additionally, Equation (49) guarantees that $\hat{\boldsymbol{x}}^{\mathrm{E}}$ and $\hat{\boldsymbol{y}}^{\mathrm{E}}$, together, satisfy the strong duality equation of Problem (41) with $\boldsymbol{x}^{\mathrm{H}}$ fixed to $\hat{\boldsymbol{x}}^{\mathrm{H}}$. Therefore, $\hat{\boldsymbol{x}}^{\mathrm{E}}$ and $\hat{\boldsymbol{y}}^{\mathrm{E}}$ are the primal and dual optimal solutions to Problem (41) with $\boldsymbol{x}^{\mathrm{H}}$ fixed to $\hat{\boldsymbol{x}}^{\mathrm{H}}$ for any $\gamma \in [0, 1]$.

In summary, $\hat{\boldsymbol{x}}^{\mathrm{H}}$ is a feasible solution to $P(\hat{\boldsymbol{z}})$, which converges towards the optimal solution when $\gamma \to 1$, and $\hat{\boldsymbol{y}}^{\mathrm{E}}$ is the optimal solution of the lower-level problem with respect to $\hat{\boldsymbol{x}}^{\mathrm{H}}$ for any $\gamma \in [0, 1]$. Hence, Problem (37) always provides a feasible solution to Problem (33), which converges towards the optimal solution when $\gamma \to 1$.

V. Case Study Setup

In the associated paper, a 24-bus power system and two isolated 3-node district heating systems, respectively connected to nodes (n_8, n_9, n_{10}) and (n_{15}, n_{18}, n_{19}) of the power system are considered, as illustrated in Figure 3 .

A modified version of the 24-bus IEEE Reliability Test System composes the integrated energy system. It consists of 12 thermal power plants, 6 wind farms, 2 extraction CHPs, and 2 HPs. Data for power generation, costs, transmission, and loads for the 24-bus IEEE Reliability Test System is derived from [8]. Additionally, 6 wind farms of 250MW each are installed at buses n_3, n_5, n_7, n_{16}, n_{21}, n_{23}. Additionally, the transmission capacity of the lines connecting the node pairs (n_{15}, n_{21}), (n_{14}, n_{16}) and (n_{13}, n_{23}) is reduced to 400MW, 250MW and 250MW, respectively, in order to introduce congestion in the transmission network. Additionally, spatially and temporally correlated profiles of wind power generation at six locations are derived from [9], and [10], and illustrated in Figure 4.

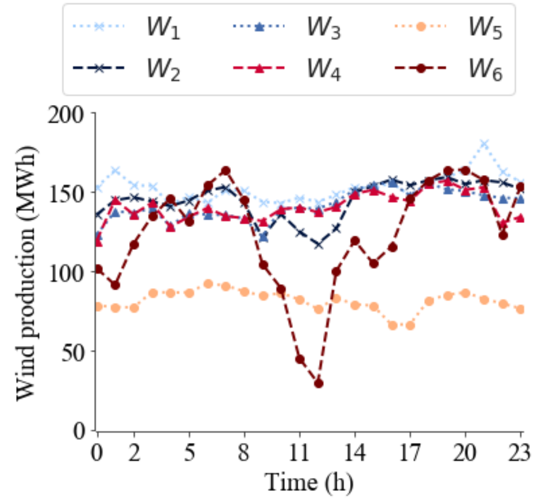

Figure 4: Wind power generation profiles

Two isolated 3-node DHNs connected to the power system, and representative of the Danish energy system, are considered. These two DHNs have similar load profiles, generation costs, energy mix, and network data. The size of each district-heating network is scaled to be compatible with the power system to which it is connected. Two isolated 3-node DHNs connected to the power system, and representative of the Danish energy system, are considered. Each DHN comprises one CHP, one waste incinerator heat-only unit, one heat-only peak boiler, and 1 large-scale HP. The techno-economic characteristics of these units and DHNs are derived from [11], [12], [13] and representative of the greater Copenhagen area. The heat loads in both networks are derived from [14] and representative of heat load profiles in the greater Copenhagen area. Heat loads are illustrated in Figure 5. Heat generation data is summarized in Table I. District heating transmission networks data is derived from [13], and provided in the online appendix of that paper. Although these two DHNs are similar, the main difference between them, which is expected to lead to different UC and dispatch, is related to their interdependencies with the power system, at different nodes.

For simplicity, thermal power plants, wind producers, and heat-only units offer their total production in a single price-quantity bid, as well as a their minimum production at any price. These prices and quantities are derived from [8]. Additionally, at each time step, for each HP, 5 heat bids $\{b_1, ..., b_5\}$ of equal sizes $\overline{s}^{\mathrm{H}}_{jb_kt}$ are derived. The prices of these

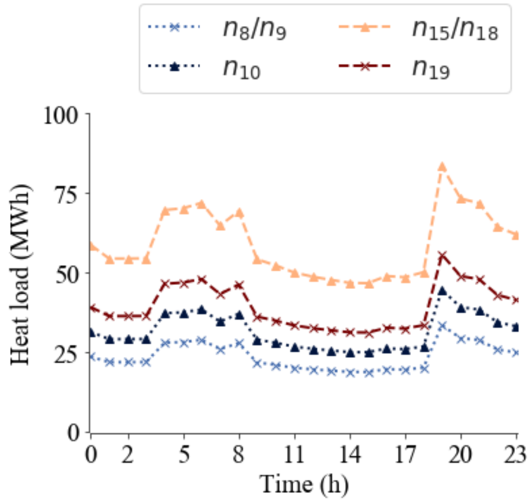

Figure 5: Heat loads at each node of the DHNs

Table I: Heat generation data in isolated DHNs 1 and 2.

Type	CHP$_1$ CHP	HO$_1$ waste	HO$_2$ peak	HP$_1$ HP	CHP$_2$ CHP	HO$_3$ waste	HO$_4$ peak	HP$_2$ HP
$\boldsymbol{\rho}^{\mathrm{E}}$	2.4	-	-	-	2.4	-	-	-
$\boldsymbol{\rho}^{\mathrm{H}}$	0.25	-	-	-	0.25	-	-	-
$\boldsymbol{r}$	0.6	-	-	-	0.6	-	-	-
$\overline{\boldsymbol{Q}}$	100	100	100	10	200	200	200	20
$\underline{\boldsymbol{Q}}$	0	0	0	0	0	0	0	0
$\overline{\boldsymbol{F}}$	250	-	-	-	500	-	-	-
$\underline{\boldsymbol{F}}$	25	-	-	-	50	-	-	-
COP	-	-	-	2.5	-	-	-	2.5
$\boldsymbol{\Phi}^{\uparrow}$	2	2	1	1	2	2	1	1
$\boldsymbol{\Phi}^{\downarrow}$	2	2	1	1	2	2	1	1
$\lvert\boldsymbol{\Phi}^{u}\rvert$	2	2	1	1	2	2	1	1
$\boldsymbol{C}$	10.5	13.5	30	0	10.5	13.5	30	0
$\boldsymbol{C}^{\uparrow,0}$	100	100	25	-	100	100	25	-
$\boldsymbol{C}^{\uparrow,1}$	5	5	2	0	5	5	2	0
$\boldsymbol{C}^{0}$	100	250	100	100	100	250	100	100

bids, denoted $\hat{\lambda}^{\mathrm{E}}_{nt}$, are computed based on the average day- and night-time electricity LMPs $\hat{\lambda}^{\mathrm{E}}_{nt}$ arising from solving the DC optimal power flow on the 24-bus IEEE Reliability Test System in [8], such that

$$\alpha^{\mathrm{H}}_{jb_kt} = \frac{\left(\hat{\lambda}^{\mathrm{E}}_{nt} + k - 1\right)}{\mathrm{COP}_j} \\ ,\forall n \in \mathcal{I}^{\mathrm{N}}, j \in \mathcal{I}^{\mathrm{HP}}_n, t \in \mathcal{T}, k \in \{1, ..., 5\}. \tag{50}$$

Additionally, 5 heat bids $\{b_1, ..., b_5\}$ are derived for each CHP. The heat bids are split between two areas of the feasible operating regions. The first set of bids $\mathcal{B}^{\mathrm{H}^0} = \{b_1\}$ covers the area where the lower electricity production bound is defined by the minimum fuel consumption, and the second set of bids $\mathcal{B}^{\mathrm{H}^1} = \{b_2, b_3, b_4, b_5\}$ of equal sizes covers the area where the lower electricity production bound is defined by the minimum heat-to-power ratio. The price of each heat bid is computed as follows

$$\alpha^{\mathrm{H}}_{jb_1t} = \frac{\left(\hat{\lambda}^{\mathrm{E}}_{nt}\right)\rho^{\mathrm{H}}_j}{\rho^{\mathrm{E}}_j}, \forall n \in \mathcal{I}^{\mathrm{N}}, j \in \mathcal{I}^{\mathrm{CHP}}_n, t \in \mathcal{T} \tag{51a}$$

$$\alpha^{\mathrm{H}}_{jb_kt} = \begin{cases} \dfrac{\left(\hat{\lambda}^{\mathrm{E}}_{nt} + k - 1\right)\rho^{\mathrm{H}}_j}{\rho^{\mathrm{E}}_j} & \text{if } \hat{\lambda}^{\mathrm{E}}_{nt} \geq \dfrac{C_j}{\rho^{\mathrm{E}}_j} \\ C_j\left(\rho^{\mathrm{H}}_j + r_j\rho^{\mathrm{E}}_j\right) - r_j\left(\hat{\lambda}^{\mathrm{E}}_{nt} - k + 1\right) & \text{otherwise} \end{cases} \\ ,\forall n \in \mathcal{I}^{\mathrm{N}}, j \in \mathcal{I}^{\mathrm{CHP}}_n, t \in \mathcal{T}, k \in \{2, ..., 5\}, \tag{51b}$$

where C_j is a fixed cost parameter. Furthermore, 5 heat bids $\{b_1, ..., b_5\}$ of equal sizes are derived for each heat-only unit, covering their entire maximum heat output $\overline{Q}_j$. For simplicity, the prices of these bids are all equal to

$$\alpha^{\mathrm{H}}_{jb_kt} = C_j, \forall j \in \mathcal{I}^{\mathrm{HO}}, t \in \mathcal{T}, k \in \{1, ..., 5\}. \tag{52}$$

In addition, no-load costs for CHPs are derived based on the same average electricity LMPs, and the minimum electricity outputs of these units when $Q_{jt} = 0$, such that

$$\alpha^0_{jt} = C^0_j + \left(C_j\rho^{\mathrm{E}}_j - \hat{\lambda}^{\mathrm{E}}_{nt}\right)\frac{\underline{F}_j}{\rho^{\mathrm{E}}_j}, \forall n \in \mathcal{I}^{\mathrm{N}}, j \in \mathcal{I}^{\mathrm{CHP}}_n, t \in \mathcal{T}, \tag{53}$$

where C^0_j is a fixed cost parameter. For all other heat generators, these no-load costs are constant, such that

$$\alpha^0_{jt} = C^0_j, \forall j \in \mathcal{I}^{\mathrm{HP}} \cup \mathcal{I}^{\mathrm{HO}}, t \in \mathcal{T}. \tag{54}$$

Moreover, for all heat generators, at each time period $h \in \Phi^u_{jt}$, the distinct start-up costs are defined as piecewise linear functions, such that

$$\alpha^{\uparrow}_{jh} = C^{\uparrow,1}_j h + C^{\uparrow,0}_j, \forall j \in \mathcal{I}^{\mathrm{HS}}, t \in \mathcal{T}, h \in \{1, ..., |\Phi^u_{jt}|\}. \tag{55}$$

Finally, for each heat bid b_k, 5 electricity bids $\{\tilde{b}_{k,1}, ..., \tilde{b}_{k,5}\}$ of equal sizes $\overline{s}^{\mathrm{E}}_{j\tilde{b}_{k,l}t}$ are derived. The size of these bids cover the entire feasible operating region of the CHPs, as described in Section I.

VI. Nomenclature

$\mathcal{T}$	Set of time steps in optimization period
$\mathcal{I}^{\mathrm{N}}$	Set of nodes in heat and electricity networks
$\mathcal{I}^{\mathrm{N}}$	Set of nodes connected to node n
$\mathcal{I}^{\mathrm{P+}}_n$	Set of pipelines starting at node n
$\mathcal{I}^{\mathrm{P-}}_n$	Set of pipelines ending at node n
$\mathcal{I}^{\mathrm{HS}}$	Set of heat stations (HS) in heat system
$\mathcal{I}^{\mathrm{HS}}_n$	Set of HS located at node n
$\mathcal{I}^{\mathrm{HES}}$	Set of heat exchanger stations (HES) in heat system
$\mathcal{I}^{\mathrm{HES}}_n$	Set of HES located at node n
$\mathcal{I}^{\mathrm{HP}}$	Set of HPs in heat system
$\mathcal{I}^{\mathrm{HP}}_n$	Set of HPs located at node n
$\mathcal{I}^{\mathrm{CHP}}$	Set of CHPs in heat system
$\mathcal{I}^{\mathrm{CHP}}_n$	Set of CHPs located at node n
$\mathcal{I}^{\mathrm{HO}}$	Set of heat only units in heat system
$\mathcal{I}^{\mathrm{HO}}_n$	Set of heat only units located at node n
$\mathcal{I}^{\mathrm{E}}$	Set of power plants in power system
$\mathcal{I}^{\mathrm{E}}_n$	Set of power plants located at node n
Φ^u_j	Set of counts of time periods with distinct start-up costs of unit j
$\Phi^{u,\mathrm{init}}_j$	Set of initial up- or down-time periods of unit j
$\mathcal{B}^{\mathrm{H}}$	Set of bids in heat market
$\mathcal{B}^{\mathrm{E}}$	Set of bids in electricity market

Table II: Sets of indexes in heat and electricity systems and markets

$\underline{\Phi}^{\uparrow}_{jt}$	Minimum up-time of unit j at time t (h)
$\underline{\Phi}^{\downarrow}_{jt}$	Minimum down-time of unit j at time t (h)
$u^{0,\text{init}}_{j}$	Initial on/off state of unit j
Δt	Time intervals (s)
$\underline{F}_j$	Minimum fuel consumption of CHP j (Wh)
$\overline{F}_j$	Maximum fuel consumption of CHP j (Wh)
ρ^{E}_j	Electricity efficiency ratio of CHP j
ρ^{H}_j	Heat efficiency ratio of CHP j
r_j	Minimum power-to-heat ratio of CHP j
COP_j	Coefficient of performance of HP j
$\underline{Q}_j$	Minimum heat production of HS j (Wh)
$\overline{Q}_j$	Maximum heat production of HS j (Wh)
$\overline{s}^{\text{H}}_{jbt}$	Quantity of heat bid b of HS j at time t (Wh)
$\alpha^{\text{H}}jbt$	Price of heat bid b of HS j at time t (Wh)
R_p	Radius of pipeline p (m)
L_p	Length of pipeline p (m)
N_p	maximum time delays in pipeline p (h)
μ_p	Thermal loss coefficient in pipeline p (J.m^{-2}.s^{-2}.K^{-1})
ρ	Density of water (kg.m^{-3})
ν_p	Pressure loss coefficient in pipeline p
c	Specific heat capacity of water (J.kg^{-1}.K^{-1})
η^{pump}_j	Efficiency of water pump at heat station j
$\underline{mf}^{\text{HS}}_{jt}$	Minimum mass flow rate at HS j at time t (kg.s^{-1})
$\overline{mf}^{\text{HS}}_{jt}$	Maximum mass flow rate at HS j at time t (kg.s^{-1})
$\underline{mf}^{\text{HES}}_{jt}$	Minimum mass flow rate through HES j at time t (kg.s^{-1})
$\overline{mf}^{\text{HES}}_{jt}$	Maximum mass flow rate through HES j at time t (kg.s^{-1})
$\underline{mf}^{\text{S}}_{pt}$	Minimum mass flow rate through supply pipeline p at time t (kg.s^{-1})
$\overline{mf}^{\text{S}}_{pt}$	Maximum mass flow rate through supply pipeline p at time t (kg.s^{-1})
$\underline{mf}^{\text{R}}_{pt}$	Minimum mass flow rate through return pipeline p at time t (kg.s^{-1})
$\overline{mf}^{\text{R}}_{pt}$	Maximum mass flow rate through return pipeline p at time t (kg.s^{-1})
$\underline{T}^{\text{S}}_{nt}$	Minimum temperature in supply networks at node n at time t (K)
$\overline{T}^{\text{S}}_{nt}$	Maximum temperature in supply networks at node n at time t (K)
$\underline{T}^{\text{R}}_{nt}$	Minimum temperature in return network at node n at time t (K)
$\overline{T}^{\text{R}}_{nt}$	Maximum temperature in return network at node n at time t (K)
$\underline{pr}^{\text{S}}_{nt}$	Minimum pressure in supply network at node n at time t (pa)
$\overline{pr}^{\text{S}}_{nt}$	Maximum pressure in supply network at node n at time t (pa)
$\underline{pr}^{\text{R}}_{nt}$	Minimum pressure in return network at node n at time t (pa)
$\overline{pr}^{\text{R}}_{nt}$	Maximum pressure in return network at node n at time t (pa)
$\underline{\Delta pr}_{nt}$	Minimum pressure difference at node n at time t (pa)
L^{H}_{jt}	Heat load at HES j at node n and time t (Wh)
$\overline{s}^{\text{E}}_{j\tilde{b}t}$	Quantity of electricity bid $\tilde{b}$ of power plant j at time t (Wh)
$\alpha^{\text{E}}_{j\tilde{b}t}$	Price of electricity bid $\tilde{b}$ of power plant j at time t (Wh)
L^{E}_{nt}	Electricity load at node n at time t (Wh)
B_{nm}	Susceptance of line connecting nodes n and m (S)
$\overline{f}_{nm}$	Maximum flow in line connecting nodes n and m (Wh)

Table III: Parameters in heat UC, heat and electricity market-clearing problems

u^{0}_{jt}	Commitment variable of of unit j at time step t, in $\{0,1\}$
u_{jbt}	Commitment variable of bid b of unit j at time step t, in $\{0,1\}$
$v^{\uparrow}_{jt}$	Start-up variable of unit j at time step t, in $\{0,1\}$
$v^{\downarrow}_{jt}$	Shut-down variable of unit j at time step t, in $\{0,1\}$
r_{jt}	Start-up cost of unit j at time step t
Φ^{S}_{pt}	Time delay of water exiting supply pipeline p at time step t
Φ^{R}_{pt}	Time delay of water exiting return pipeline p at time step t
$u^{\Phi^{\text{S}}}_{pt}$	Auxiliary binary variable in supply pipeline p at time step t
$u^{\Phi^{\text{R}}}_{pt}$	Auxiliary binary variable in return pipeline p at time step t
Q_{jbt}	Dispatch of heat bid b of unit j at time step t (Wh)
Q_{jt}	Total heat production of unit j at time step t (Wh)
mf^{S}_{pt}	Mass flow rate in supply pipeline p at time step t (kg.s^{-1})
mf^{R}_{pt}	Mass flow rate in return pipeline p at time step t (kg.s^{-1})
mf^{HES}_{jt}	Mass flow rate through HES j at time step t (kg.s^{-1})
mf^{HS}_{jt}	Mass flow rate through HS j at time step t (kg.s^{-1})
T^{S}_{nt}	Supply temperature at node n at time step t (K)
T^{R}_{nt}	Return temperature at node n at time step t (K)
$T^{\text{S,in}}_{pt}$	Inlet temperature of supply pipeline p at time step t (K)
$T^{\text{S,out}}_{pt}$	Outlet temperature of supply pipeline p at time step t (K)
$T^{\text{R,in}}_{pt}$	Inlet temperature of return pipeline p at time step t (K)
$T^{\text{R,out}}_{pt}$	Outlet temperature of return pipeline p at time step t (K)
L^{HP}_{jt}	Electricity consumption of HP j at time t (Wh)
L^{pump}_{jt}	Electricity consumption of water pump j at time t (Wh)
$\underline{P}_{jt}$	Minimum electricity production of CHP j at time t (Wh)
$\overline{P}_{jt}$	Maximum electricity production of CHP j at time t (Wh)
$\overline{P}_{j\tilde{b}t}$	Adjusted quantity of electricity bid $\tilde{b}$ of CHP j at time t (Wh)
$P_{j\tilde{b}t}$	Dispatch of bid $\tilde{b}$ of power plant or CHP j at time step t (Wh)
P_{jt}	Total electricity production of unit j at time step t (Wh)
θ_{nt}	Voltage angle at node n at time t (rad)
λ^{E}_{nt}	Electricity LMP at node n at time step t (\$/Wh)

Table IV: Primal and dual decision variables of the heat and electricity market–clearing problems

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