Appendix A
Proof of Proposition 1

The equilibrium problem representing the sequential heat and electricity market clearing in (7)-(8) can be expressed in a compact manner as:

$$\boldsymbol{x}^{\mathbf{H}} \in \text{ sol. of } \begin{Bmatrix} \min\limits_{\boldsymbol{x}^{\mathbf{H}} \geq \mathbf{0}} & c^{\mathrm{H}^\top} \boldsymbol{x}^{\mathbf{H}} \\ \text{s.t.} & A^{\mathrm{H}} \boldsymbol{x}^{\mathbf{H}} + B^{\mathrm{H}} \boldsymbol{z} \geq b^{\mathrm{H}} \end{Bmatrix} \tag{13a}$$

$$\boldsymbol{x}^{\mathbf{E}} \in \text{ sol. of } \begin{Bmatrix} \min\limits_{\boldsymbol{x}^{\mathbf{E}} \geq 0} & c^{\mathrm{E}^\top} \boldsymbol{x}^{\mathbf{E}} \\ \text{s.t.} & A^{\mathrm{E}} \boldsymbol{x}^{\mathbf{E}} + B^{\mathrm{E}} \boldsymbol{x}^{\mathbf{H}} \geq b^{\mathrm{E}} \end{Bmatrix} \tag{13b}$$

Similarly, the proposed lexicographic optimization problem (9) is formulated in a compact manner as (11d).

This lexicographic optimization problem can be solved in two steps:

1) Find the optimal heat dispatch cost such that:

$$\Theta^{\mathrm{H}^*} = \min_{\boldsymbol{x}^{\mathbf{H}}, \boldsymbol{x}^{\mathbf{E}} \geq \mathbf{0}} c^{\mathrm{H}^\top} \boldsymbol{x}^{\mathbf{H}} \tag{14a}$$

$$\text{s.t.} \quad (7\text{b}) - (7\text{g}) \tag{14b}$$

$$(8\text{b}) - (8\text{i}) \tag{14c}$$

2) Find an optimal heat and electricity dispatch such that:

$$\{x^{\mathrm{H}^*}, x^{\mathrm{E}^*}\} \in \underset{\boldsymbol{x}^{\mathbf{H}}, \boldsymbol{x}^{\mathbf{E}} \geq \mathbf{0}}{\operatorname{argmin}} \; c^{\mathrm{E}^\top} \boldsymbol{x}^{\mathbf{E}} \tag{15a}$$

$$\text{s.t.} \quad (7\text{b}) - (7\text{g}) \tag{15b}$$

$$(8\text{b}) - (8\text{i}) \tag{15c}$$

$$c^{\mathrm{H}^\top} \boldsymbol{x}^{\mathbf{H}} \leq \Theta^{\mathrm{H}^*} \tag{15d}$$

Due to Constraints (15b) and (15d), for any optimal solution $\{x^{\mathrm{H}^*}, x^{\mathrm{E}^*}\}$ to (14)-(15), x^{H^*} is also an optimal solution to (13a), and x^{E^*} is an optimal solution to (13b) with $\boldsymbol{x}^{\mathbf{H}}$ fixed to x^{H^*}.

Appendix B
Proof of Proposition 2

We consider the following approximation of the lexicographic optimization problem (11d), with $\gamma \in]0, 1[$:

$$\min_{\boldsymbol{x}^{\mathbf{H}}, \boldsymbol{x}^{\mathbf{E}} \geq \mathbf{0}} \gamma c^{\mathrm{H}^\top} \boldsymbol{x}^{\mathbf{H}} + (1-\gamma)\, c^{\mathrm{E}^\top} \boldsymbol{x}^{\mathbf{E}} \tag{16a}$$

$$\text{s.t.} \quad A^{\mathrm{H}} \boldsymbol{x}^{\mathbf{H}} + B^{\mathrm{H}} \boldsymbol{z} \geq b^{\mathrm{H}} \tag{16b}$$

$$A^{\mathrm{E}} \boldsymbol{x}^{\mathbf{E}} + B^{\mathrm{E}} \boldsymbol{x}^{\mathbf{H}} \geq b^{\mathrm{E}}, \tag{16c}$$

where $\boldsymbol{y}^{\mathbf{E}}$ is obtained as the dual variable associated with constraint (16c) [18]. As a result, problem (10) can be approximated by the following linear bilevel optimization problem:

$$\min_{\substack{\boldsymbol{z} \in \{0,1\}^{\mathrm{N}}, \boldsymbol{x}^{\mathbf{H}}, \boldsymbol{x}^{\mathbf{E}} \geq \mathbf{0} \\ \boldsymbol{y}^{\mathbf{H}}, \boldsymbol{y}^{\mathbf{E}} \geq \mathbf{0}}} \gamma c^{0^\top} \boldsymbol{z} + \gamma c^{\mathrm{H}^\top} \boldsymbol{x}^{\mathbf{H}} + (1-\gamma)\, c^{\mathrm{E}^\top} \boldsymbol{x}^{\mathbf{E}} \tag{17a}$$

$$\text{s.t.} \quad \boldsymbol{z} \in \mathcal{Z}^{\mathrm{UC}} \tag{17b}$$

$$A^{\mathrm{bid}} \boldsymbol{z} + \frac{1}{(1-\gamma)} B^{\mathrm{bid}} \boldsymbol{y}^{\mathbf{E}} \geq b^{\mathrm{bid}} \tag{17c}$$

$$\{\boldsymbol{x}^{\mathbf{H}}, \boldsymbol{y}^{\mathbf{E}}\} \in \text{ primal and dual sol. of (16).} \tag{17d}$$

Besides, by strong duality of the lower-level problem (17d), problem (17) is equivalent to (12).

It remains to show that problem (12) is an asymptotic approximation to problem (11), i.e., as $\gamma \to 1$ the solutions to problem (12) become optimal solutions to problem (11). By introducing the auxiliary variables $\tilde{\boldsymbol{y}}^{\mathbf{H}} = \dfrac{\boldsymbol{y}^{\mathrm{H}}}{\gamma}$, and $\tilde{\boldsymbol{y}}^{\mathbf{E}} = \dfrac{\boldsymbol{y}^{\mathrm{E}}}{1-\gamma}$, problem (12) is equivalent to:

$$\min_{\substack{\boldsymbol{z} \in \{0,1\}^N, \boldsymbol{x}^{\mathbf{H}} \geq \mathbf{0} \\ \boldsymbol{x}^{\mathbf{E}} \geq \mathbf{0}, \boldsymbol{y}^{\mathbf{H}}, \boldsymbol{y}^{\mathbf{E}}}} \gamma c^{0^\top} \boldsymbol{z} + \gamma c^{\mathrm{H}^\top} \boldsymbol{x}^{\mathbf{H}} + (1-\gamma)\, c^{\mathrm{E}^\top} \boldsymbol{x}^{\mathbf{E}} \tag{18a}$$

$$\text{s.t.} \quad \boldsymbol{z} \in \mathcal{Z}^{\mathrm{UC}} \tag{18b}$$

$$A^{\mathrm{bid}} \boldsymbol{z} + B^{\mathrm{bid}} \tilde{\boldsymbol{y}}^{\mathbf{E}} \geq b^{\mathrm{bid}} \tag{18c}$$

$$A^{\mathrm{H}} \boldsymbol{x}^{\mathbf{H}} + B^{\mathrm{H}} \boldsymbol{z} \geq b^{\mathrm{H}} \tag{18d}$$

$$A^{\mathrm{E}} \boldsymbol{x}^{\mathbf{E}} + B^{\mathrm{E}} \boldsymbol{x}^{\mathbf{H}} \geq b^{\mathrm{E}} \tag{18e}$$

$$\tilde{\boldsymbol{y}}^{\mathbf{H}^\top} A^{\mathrm{H}} + \frac{(1-\gamma)}{\gamma} \tilde{\boldsymbol{y}}^{\mathbf{E}^\top} B^{\mathrm{E}} \leq c^{\mathrm{H}^\top} \tag{18f}$$

$$\tilde{\boldsymbol{y}}^{\mathbf{E}^\top} A^{\mathrm{E}} \leq c^{\mathrm{E}^\top} \tag{18g}$$

$$\tilde{\boldsymbol{y}}^{\mathbf{H}^\top} \left(b^{\mathrm{H}} - B^{\mathrm{H}} \boldsymbol{z}\right) - c^{\mathrm{H}^\top} \boldsymbol{x}^{\mathbf{H}} \geq \frac{(1-\gamma)}{\gamma} \left(c^{\mathrm{E}^\top} \boldsymbol{x}^{\mathbf{E}} - \tilde{\boldsymbol{y}}^{\mathbf{E}^\top} b^{\mathrm{E}}\right). \tag{18h}$$

For the value of the unit commitment variable $\boldsymbol{z}$ fixed to z^*, let us denote $\Theta(z^*)$ the optimal objective value to (11), and $\tilde{\Theta}(z^*)$ and $\{x^{\mathrm{H}^*}, x^{\mathrm{E}^*}, y^{\mathrm{H}^*}, y^{\mathrm{E}^*}\}$ the optimal objective and solutions to (18). As $\gamma \to 1$, (18f) and (18h) become

$$\tilde{\boldsymbol{y}}^{\mathbf{H}^\top} A^{\mathrm{H}} \leq c^{\mathrm{H}^\top} \tag{19a}$$

$$\tilde{\boldsymbol{y}}^{\mathbf{H}^\top} \left(b^{\mathrm{H}} - B^{\mathrm{H}} \boldsymbol{z}\right) \geq c^{\mathrm{H}^\top} \boldsymbol{x}^{\mathbf{H}}. \tag{19b}$$

Constraint (18d) guarantees that x^{H^*} is feasible to problem (14) with $\boldsymbol{z}$ fixed to z^*. Additionally, (19a) guarantees that y^{H^*} becomes feasible to the dual of problem (14) with $\boldsymbol{z}$ fixed to z^* when $\gamma \to 1$. Moreover, (19b) guarantees that x^{H^*} and y^{H^*}, together, satisfy the strong duality equation of problem (14) with $\boldsymbol{z}$ fixed to z^* when $\gamma \to 1$. Therefore, x^{H^*} and y^{H^*} approximate a primal and dual optimal solution to problem (14) with $\boldsymbol{z}$ fixed to z^* when $\gamma \to 1$. This implies that x^{H^*} and y^{H^*} become feasible solutions to (11) when $\gamma \to 1$.

Moreover, the combination of (18f) $\times x^{\mathrm{H}^*}$ and (18h) gives

$$\tilde{\boldsymbol{y}}^{\mathbf{H}^\top} \left(b^{\mathrm{H}} - B^{\mathrm{H}} \boldsymbol{z} - A^{\mathrm{H}} x^{\mathrm{H}^*}\right) \geq \frac{(1-\gamma)}{\gamma} \left(c^{\mathrm{E}^\top} \boldsymbol{x}^{\mathbf{E}} - \tilde{\boldsymbol{y}}^{\mathbf{E}^\top} \left(b^{\mathrm{E}} - B^{\mathrm{E}} x^{\mathrm{H}^*}\right)\right). \tag{20}$$

It follows from (20) and (18d) that, for any gamma $\gamma \in]0, 1[$:

$$\tilde{\boldsymbol{y}}^{\mathbf{E}^\top} \left(b^{\mathrm{E}} - B^{\mathrm{E}} x^{\mathrm{H}^*}\right) \geq c^{\mathrm{E}^\top} \boldsymbol{x}^{\mathbf{E}}. \tag{21}$$

Constraints (18e) and (18g) guarantee that x^{E^*} and y^{E^*} are feasible primal and dual solutions to problem (15) with $\boldsymbol{x}^{\mathrm{H}}$ fixed to x^{H^*}. Additionally, (21) guarantees that x^{E^*} and y^{E^*}, together, satisfy the strong duality equation of problem (15). Therefore, x^{E^*} and y^{E^*} are the primal and dual optimal solutions to problem (15) with $\boldsymbol{x}^{\mathrm{H}}$ fixed to x^{H^*} for any $\gamma \in]0, 1[$.

In summary, x^{H^*} is a feasible solution to (14), which converges towards an optimal solution when $\gamma \to 1$, and y^{E^*} is an optimal dual solution of the lower-level problem for any $\gamma \in]0, 1[$. Hence, problem (12) always provides a feasible solution to problem (11), which converges towards the optimal solution when $\gamma \to 1$.