Electricity-Aware Bid Format for Heat and Electricity Market Coordination

Lesia Mitridati, *Member, IEEE*, Pascal Van Hentenryck, *Member, IEEE*, and Jalal Kazempour, *Senior Member, IEEE,*

***Abstract*—Coordination between heat and electricity markets is essential to achieve a cost-effective and efficient operation of the energy system. In the current sequential market practice, the heat market is cleared before the electricity market, and has no insight into the impacts of heat dispatch on the electricity market. While preserving this sequential practice, this paper introduces an electricity-aware bid format for the coordination of heat and electricity systems. This novel market mechanism defines heat bids conditionally on day-ahead electricity prices. Prior to clearing heat and electricity markets, the proposed bid selection mechanism selects the valid bids which minimize the heat system operating cost, while anticipating heat and electricity market clearing. This mechanism is modeled as a trilevel optimization problem, which we recast as a mixed-integer linear program using a lexicographic function. We use a realistic case study based on the Danish electricity and heat system, and show that the proposed bid selection mechanism yields a 4.5% reduction in total operating cost of heat and electricity systems compared to the existing market-clearing procedure, while reducing the financial losses of combined heat and power plants and heat pumps due to invalid bids by up to 20.3 million euros.**

I. Introduction

Exploiting potential synergies between electricity and other energy systems has been identified as a key solution to achieve a sustainable energy future [1]–[3]. In particular, an coordinated electricity and heat system operation is central in a sustainable energy system due to the strong technical and economic interdependencies between these two systems [4]–[6]. In several countries, such as Denmark, energy markets operate heat and electricity systems in the day-ahead stage sequentially and independently [3]. The main shortcomings of this market framework is that the myopic dispatch of assets at the interface of electricity and heat systems, including Combined Heat and Power (CHP) units and large-scale Heat Pumps (HPs) may limit their operational flexibility in the electricity market to facilitate integration of additional renewable energy production [7], [8]. This challenge raises an important research question: How to improve the dispatch of CHPs and HPs in the heat market in order to achieve a cost-effective and efficient operation of the overall energy system?

This question has been extensively addressed in the literature by proposing a drastic change in the current market design, i.e., moving towards *fully integrated* approaches based on electricity and heat production co-optimization [7], [9]–[12]. These studies showed significant gains in terms of operating cost for the overall energy system and penetration of renewable energy in the power system. However, heat and electricity markets are currently operated by independent entities, who cover vastly different geographical areas, and have different (and sometimes conflicting) objectives and rules [3]. Therefore, jointly dispatching heat and electricity production in an integrated market platform can only realistically be considered as an *ideal benchmark*, due to the disruptive organizational and regulatory changes it would require. Additionally, while the overall operating costs may be decreased, the operating costs of one individual sector may be drastically increased, as has previously been observed in the literature [13], [14]. Due to the aforementioned reasons, and based on extensive talks with regulators and heat market operators, we are convinced that a fully integrated market is not a feasible and realistic framework for sector coordination.

By opposition with these fully integrated operational approaches, the novel heat market-clearing procedure introduced in [14] claims to provide a *soft coordination* between heat and electricity systems while respecting the sequential clearing of their respective markets. This coordination is achieved by allowing the heat market clearing to model the heat production costs of CHPs and HPs as a function of day-ahead electricity prices while anticipating the impact of heat dispatch on the electricity market clearing as equilibrium constraints. Although the resulting hierarchical heat market-clearing procedure respects the sequential heat and electricity market-clearing framework, it results in increased computational complexity for the heat market operator, and pricing issues arising from non-convexities. Therefore, in practice, this approach still requires important regulatory changes.

Other approaches in the literature have focused on enhancing bid formats and bidding mechanisms to better represent and harness the operational flexibility of assets in the markets and improve the coordination between multiple trading floors. Several studies introduced extended bid formats for demand response, aiming at harnessing operational flexibility from prosumers by providing a better representation of their complex preferences [15], [16]. Similarly, the price-region bid format introduced in [17] provides a unified framework to enhance the representation of the techno-economic characteristics of non-conventional flexible assets in electricity markets. In particular, this approach was applied to model the flexibility of a district-heating utility and represent its price curve as a covex function of electricity prices. Besides, the authors in [18] introduced *bid-validity* constraints in the unit commitment problem of gas-fired power plants to better account for the interdepen-

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dencies between electricity and natural gas systems in their electricity bids. The main appeal of these methods is that they remain compatible with existing market frameworks without major regulatory changes.

To tackle the above challenges, this paper makes several contributions to the state-of-the-art. The first contribution is to introduce an *electricity-aware bid format* in the day-ahead heat market, inspired by the concept of bid-validity conditions introduced in [18]. This format enables CHPs and HPs to design bids for the heat market, which are *conditioned* on day-ahead electricity prices. Contrary to traditional bid formats, e.g., price-quantity bids, this novel bid format provides an explicit representation of the complex techno-economic characteristics of CHPs and HPs, which create interdependencies between heat and electricity markets. The second contribution is to develop an *electricity-aware bid selection* mechanism which commits heat producers and selects their *valid* bids, i.e., those bids that ensure *cost recovery* of heat producers prior to their participation in heat and electricity markets, by anticipating the sequential clearing of heat and electricity markets. This approach does not require any major regulatory changes as both the heat and electricity market-clearing procedures remains unchanged, and solely relies on information exchange between these trading platforms to achieve coordination. The third contribution of this paper pertains to computational tractability: We formulate the proposed electricity-aware bid selection mechanism as a tractable single-level mixed-integer linear program (MILP) using a lexicographic optimization approach to represent the sequential clearing of heat and electricity markets. Finally, the last contribution of this paper concerns a thorough numerical evaluation of the proposed mechanism on two substantial case studies. The first illustrative case study shows that the proposed electricity-aware mechanism can achieve 77.6% of the so-called *value of coordination* achieved by the fully integrated mechanism, while maintaining a sequential heat and electricity market framework. The second case study, based on the realistic electricity and heat systems in Denmark, provides additional insights on how the proposed mechanism is able to ensure cost recovery for CHPs and HPs in the heat market, while reducing the operating cost of the overall energy system by 4.5% and wind curtailment by 0.4% compared to the decoupled mechanism.

The remainder of this paper is organized as follows. Section II provides the required preliminaries and describes the techno-economic interdependencies between heat and electricity systems and the current status-quo in the markets that operate them. Section III introduces the proposed electricity-aware heat bid format and bid selection mechanism., and discussed the desirable properties of the resulting heat and electricity markets. Section IV details the mathematical formulation of the proposed electricity-aware heat bid selection mechanism. Section V presents numerical results. Finally, Section VI concludes and discusses potential extensions of this work.

II. Challenges in heat and electricity markets

A. Existing day-ahead market framework

In many European countries, including Nordic countries, energy systems are operated by competitive auction-based markets which interface the physical and economic aspects of each system. In particular, the day-ahead heat and electricity markets, respectively denoted by indexes $m \in \{\mathrm{H}, \mathrm{E}\}$ operate on the principles of *energy exchanges*, in which market participants in a given market zone submit bids, implicitly embed the techno-economic characteristics of each market participant. In each day-ahead energy market m, the market participants $j \in \mathcal{I}_z^m$ in a given market zone $z \in \mathcal{Z}^m$ submit a set of bids $b \in \mathcal{B}^m$ for each hour of the following day $t \in \mathcal{T}$, in the form of independent non-decreasing price-quantity pairs $\left(c_{jbt}^m, \overline{s}_{jbt}^m\right)$. Furthermore, market participants may choose to self-commit a minimum volume of their production (or consumption) $\underline{s}_{jt}^x$, which will be dispatched regardless of the equilibrium price in their market zone. By convention, we consider that the quantity parameters $\overline{s}_{jbt}^m$, and $\underline{s}_{jt}^m$) take positive values for generation offers, and negative values for consumption bids.

Contrary to energy markets operating on the principles of *energy pools*, such as North American markets, the physical flows within each market zone are neglected in the market clearing [19]. Furthermore, the market clearing in interconnected zones is coupled by the optimal allocation of the available transfer capacity (ATC), computed by the Transmission System Operators (TSOs) as the expected maximum energy that can be transferred between two market zones without leading to network constraints violation within either market zone [20], [21]. All market zones are coupled via the optimal allocation of the ATC, leading to uniform market prices, obtained as the marginal prices in each market zone.

The day-ahead heat and electricity markets are currently cleared sequentially and independently, with the day-head heat market being cleared before the electricity market.

B. Interdependencies and inefficiencies

In the sequential market framework described above, CHPs and large-scale HPs participating in both heat and electricity markets create implicit interdependencies between these independently-operated markets. Indeed, the techno-economic characteristics of these units link their bids in heat and electricity markets.

Firstly, the heat-driven dispatch of CHPs and HPs strongly constrains their bids in the day-ahead electricity market [22]. Indeed, the heat $\boldsymbol{Q_{jt}}$ and electricity $\boldsymbol{P_{jt}}$ outputs of CHPs and HPs are strongly linked [23]. Indeed, HPs $j \in \mathcal{I}^{\mathrm{HP}}$ produce heat from electricity at a fixed ratio COP_j, such that $\boldsymbol{Q_{jt}} = -\mathrm{COP}_j \boldsymbol{P_{jt}}$[1], and CHPs $j \in \mathcal{I}^{\mathrm{CHP}}$ produce heat and electricity at varying ratios, such that $r_j \boldsymbol{Q_{jt}} \leq \boldsymbol{P_{jt}} \leq \overline{F}_j - \rho_j \boldsymbol{Q_{jt}}$, where $\overline{F}_j$ represents their maximum fuel intake, r_j their minimum heat-to-power ratio and ρ_j their maximum relative heat-to-power efficiency. As a result, in the day-ahead electricity markets, CHPs and HPs must adjust the minimum $\underline{\boldsymbol{P}}_{\boldsymbol{jt}}$ and maximum $\overline{\boldsymbol{P}}_{\boldsymbol{jt}}$ electricity dispatch to ensure feasibility with respect to their fixed heat dispatch. In countries with high penetration of CHPs and HPs, such as Denmark, this lack of flexibility may impact day-ahead electricity prices, and limit

[1]By convention, $\boldsymbol{P_{jt}} \leq 0$ represents a power consumption.

the penetration of renewable energy sources [14]. Therefore, an electricity-myopic heat dispatch may lead to an inefficient dispatch in both heat and electricity systems.

In addition, in order to ensure their profitability, the bids of CHPs and HPs in the day-ahead heat market must reflect their marginal heat production cost. However, the marginal production costs of CHPs and HPs are intrinsically dependent on the electricity market prices. Indeed, as HPs $j \in \mathcal{I}_z^{\mathrm{HP}}$ located in electricity market zone $z \in \mathcal{Z}^{\mathrm{E}}$ produce heat from electricity purchased in the day-ahead electricity market, their marginal heat production cost $\dot{\Gamma}_{jt}^{\mathrm{H}}$ is linearly dependent on the electricity market prices $\boldsymbol{\lambda}_{\boldsymbol{zt}}^{\mathrm{E}}$, such that

$$\dot{\Gamma}_{jt}^{\mathrm{H}} = \mathrm{COP}_j \boldsymbol{\lambda}_{\boldsymbol{zt}}^{\mathbf{E}}, \forall j \in \mathcal{I}_z^{\mathrm{HP}}, t \in \mathcal{T}, \tag{1}$$

Similarly, the variable heat production cost $c_j(\boldsymbol{P_{jt}} + \rho_j \boldsymbol{Q_{jt}}) - \boldsymbol{\lambda}_{\boldsymbol{zt}}^{\mathbf{E}} \boldsymbol{P_{jt}}$ of CHPs $j \in \mathcal{I}_z^{\mathrm{CHP}}$ located in electricity market zone $z \in \mathcal{Z}^{\mathrm{E}}$ represents their total variable production costs minus revenues from electricity sales. Therefore, their marginal heat production cost represents the incremental variable heat production cost at the optimal heat-to-power ratio, which depends on the day-ahead electricity price [3], [13], [14], such that

$$\dot{\Gamma}_{jt}^{\mathrm{H}} = \begin{cases} \rho_j \boldsymbol{\lambda}_{\boldsymbol{zt}}^{\mathbf{E}}, \text{ if } \boldsymbol{\lambda}_{\boldsymbol{zt}}^{\mathrm{E}} \geq c_j \\ -r_j \boldsymbol{\lambda}_{\boldsymbol{zt}}^{\mathbf{E}} + c_j(\rho_j + r_j), \text{ otherwise} \end{cases}, \forall j \in \mathcal{I}_z^{\mathrm{CHP}}, t \in \mathcal{T}. \tag{2}$$

For low electricity prices, their marginal heat production cost represents the increase in variable production cost for producing one extra unit of heat and r_j extra units of electricity, and for high electricity prices it represents the opportunity loss for producing one extra unit of heat and ρ_j less units of electricity. Therefore, the marginal heat production cost of CHPs can be expressed as a convex piece-wise linear function of the electricity prices.

As day-ahead electricity market prices are unknown prior to the heat market clearing, CHPs and HPs must *anticipate* these prices in order to accurately compute their heat bids. In practice, each CHP and HP uses their own deterministic electricity price forecast to compute their *expected* marginal heat production costs. This bidding process is myopic to the impacts of the heat dispatch of CHPs and HPs on the electricity market prices, which, in turn, impact the marginal heat production costs of CHPs and HPs [24]. As a result, the bids of CHPs and HPs in the day-ahead heat markets may differ from their *realized* marginal heat production costs due to (i) forecast errors on the day-ahead electricity market prices, and/or (ii) the exercise of market power [24]. This may lead to financial losses for CHPs and HPs and decreased social welfare in both heat and electricity markets. Additionally, this bidding process makes market monitoring challenging in the current heat market. Therefore, making the bidding process in the heat market more efficient and transparent is essential.

III. Towards an electricity-aware heat market

A. Electricity-aware bid format

As discussed above, the existing price-quantity bid format in the day-ahead heat market lacks transparency and is unable to account for the complex interdependencies between heat and electricity markets, in particular, the interdependencies between day-ahead electricity prices and marginal heat production costs of CHPs and HPs.

One major challenge for coordinating the day-ahead heat and electricity dispatch is to respect the current sequential order and independent clearing of the markets, and preserve desirable market properties, including cost recovery, and merit-order dispatch. Therefore, our proposed approach is to introduce a novel electricity-aware bid format in the day-ahead heat dispatch, in the form of simple price-quantity bids $\{c_{jbt}^H, s_{jbt}^H\}$, *conditioned* on day-ahead electricity prices. This electricity-aware bid format will (i) improve the efficiency of heat and electricity dispatch, (ii) provide greater transparency and facilitate market monitoring, and (iii) preserve the desirable properties of the sequential and independent heat and electricity markets and ensure cost recovery for CHPs and HPs.

Ensuring cost recovery requires ensuring that a bid can be dispatched only if it is profitable, i.e., $c_{jbt}^H \geq \dot{\Gamma}_{jt}$. For a heat market participant $\forall j \in \mathcal{I}_z^{\mathrm{H}}$ located in electricity market zone $z \in \mathcal{Z}^{\mathrm{E}}$ whose marginal heat production cost at each time $t \in \mathcal{T}$ can be expressed as a convex piece-wise linear function of the day-ahead electricity prices, such that

$$\dot{\Gamma}_{jt}^{\mathrm{H}} = \max(\{a_{jkt} \boldsymbol{\lambda}_{\boldsymbol{zt}}^{\mathbf{E}} + b_{jkt} : k = 1, ..., K\}), \tag{3}$$

with $k \in \{1, ..., K\}$ the number of affine pieces, and $a_{jkt}, b_{jkt} \in \mathbb{R}$ their fixed affine parameters. As a result, this cost-recovery condition can be recast as

$$\boldsymbol{\lambda}_{\boldsymbol{zt}}^{\mathrm{E}} \leq \frac{c_{jbt}^{\mathrm{H}} - b_{jkt}}{a_{jkt}}, \forall k \in \{1, ..., K\} \text{ s.t. } a_{jkt} > 0 \tag{4a}$$

$$\boldsymbol{\lambda}_{\boldsymbol{zt}}^{\mathrm{E}} \geq \frac{c_{jbt}^{\mathrm{H}} - b_{jkt}}{a_{jkt}}, \forall k \in \{1, ..., K\} \text{ s.t. } a_{jkt} < 0. \tag{4b}$$

This leads us to define the bounds $\{\underline{\lambda}_{jbt}^{\mathrm{E}}, \underline{\lambda}_{jbt}^{\mathrm{E}}\}$ on the electricity prices, within which this bid is profitable as

$$\underline{\lambda}_{jbt}^{\mathrm{E}} = \max(\{\frac{c_{jbt}^{\mathrm{H}} - b_{jkt}}{a_{jkt}} : k = 1, ..., K \text{ s.t. } a_{jkt} < 0\} \cup \{\underline{\lambda}_z^{\mathrm{E}}\}) \tag{5a}$$

$$\overline{\lambda}_{jbt}^{\mathrm{E}} = \min(\{\frac{c_{jbt}^{\mathrm{H}} - b_{jkt}}{a_{jkt}} : k = 1, ..., K \text{ s.t. } a_{jkt} < 0\} \cup \{\overline{\lambda}_z^{\mathrm{E}}\}), \tag{5b}$$

where $\overline{\lambda}_z^{\mathrm{E}}$ and $\underline{\lambda}_z^{\mathrm{E}}$ are the upper and lower bounds on electricity prices in this market zone.

Therefore, as the marginal heat production cost of CHPs and HPs can be formulated as convex piece-wise linear functions of electricity prices, as detailed in (1)-(2), in order to ensure cost-recovery, their price-quantity bids in the heat market (c_{jbt}^H, s_{jbt}^H) should be defined over a range of electricity prices $\{\underline{\lambda}_{jbt}^E, \overline{\lambda}_{jbt}^E\}$ over which this bid is profitable. This new electricity-aware bid format is defined formally below.

Definition 1 (Electricity-aware heat bid format)**.** *For each heat market participant $j \in \mathcal{I}^H$ and time step $t \in \mathcal{T}$, an electricity-aware heat bid $b \in \mathcal{B}^H$ is defined as a price-quantity pair $\{c_{jbt}^H, s_{jbt}^H\}$ associated with a range $\{\underline{\lambda}_{jbt}^E, \overline{\lambda}_{jbt}^E\}$ of electricity prices over which this bid is considered valid and can be dispatched.*

B. Bid selection mechanism

We introduce an electricity-aware bid selection mechanism for the proposed electricity-aware bid format. This mechanism aims at selecting the *valid* bids of heat market participants which minimize the operating cost of the overall heat system while anticipating their impact on the participation of CHPs and HPs in the sequential heat and electricity markets. This electricity-aware bid selection mechanism is implemented by the heat system operator through enforcing the following linear *bid-validity conditions*:

$$\boldsymbol{\lambda^E_{zt}} - \overline{\lambda}^E_{jbt} \leq M\left(1 - \boldsymbol{u^{\mathrm{bid}}_{jbt}}\right), \forall z \in \mathcal{Z}^{\mathrm{E}}, j \in \mathcal{I}^{\mathrm{H}}_z, t \in \mathcal{T}, b \in \mathcal{B}^{\mathrm{H}}, \tag{6a}$$

$$\underline{\lambda}^E_{jbt} - \boldsymbol{\lambda^E_{zt}} \leq M\left(1 - \boldsymbol{u^{\mathrm{bid}}_{jbt}}\right), \forall z \in \mathcal{Z}^{\mathrm{E}}, j \in \mathcal{I}^{\mathrm{H}}_z, t \in \mathcal{T}, b \in \mathcal{B}^{\mathrm{H}}, \tag{6b}$$

with M a large-enough constant. These conditions enforce that a bid can only be selected, i.e., $\boldsymbol{u^{\mathrm{bid}}_{jbt}} = 1$, if electricity prices are within the bounds $\{\underline{\lambda}^E_{jbt}, \overline{\lambda}^E_{jbt}\}$. These bounds are directly computed by each heat market participant based on their marginal heat production costs, such that cost recovery is guaranteed within this range of prices, i.e., $\dot{\Gamma}^{\mathrm{H}}_{jt} \geq c^{\mathrm{H}}_{jbt}$.

Once the bid selection mechanism has selected the valid bids, CHPs and HPs can participate in the day-ahead heat market by solely submitting their valid bids, in the form of independent price-quantity pairs. Once their heat dispatch is fixed in the heat market, CHPs and HPs can then participate in the day-ahead electricity market, in which they adjust their minimum and maximum electricity outputs. This sequence of bid selection and market clearing is highlighted in Figure 1. As isolated heat networks may interface with the same electricity market zones, this electricity-aware bid selection mechanism must coordinate the bids of CHPs and HPs across multiple heat market zones. Therefore, the proposed mechanism is solved centrally, as illustrated in Figure 1. As a result, this electricity-aware mechanism coordinates the participation of these units in both heat and electricity markets, while the heat and electricity market-clearing mechanisms remain unchanged.

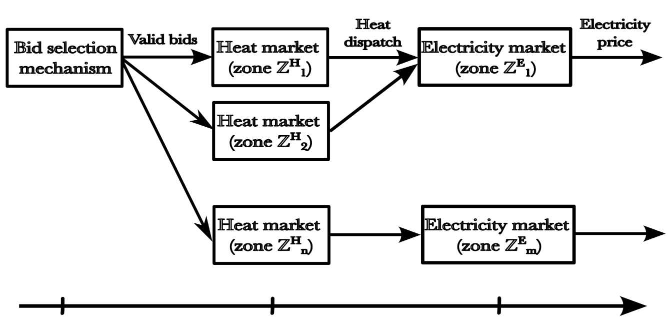

Fig. 1. Sequential bid selection mechanism and clearing of heat and electricity markets

The proposed bid selection mechanism is built upon a Stackelberg game with a single leader and multiple followers. The leader is the heat system operator solving a bid selection problem that determines the validity of the bids of CHPs and HPs by anticipating the reaction of the followers, i.e., the sequential heat and electricity market clearings. The action of the leader constrains the reaction of the followers, and, in return, the reaction of the followers impacts the objective function of the leader. In our case, the selected bids affect the bids submitted in the heat market. In return, the dispatch of CHPs and HPs in the heat market affects the operating cost of the heat system in the objective of the bid selection mechanism, with the day-ahead electricity prices impact the bid-validity constraints.

Under the assumption of perfect information on the input data of the heat and electricity markets, our proposed bid selection mechanism can be formulated as a hierarchical optimization problem, as illustrated in Figure 1. The upper-level problem represents the heat bid selection mechanism, which is constrained by an embedded (lower-level) optimization problem, representing the sequential and independent clearing of the heat and electricity markets, in which the heat market is cleared first.

The proposed electricity-aware heat bid format provides CHPs and HPs with greater flexibility to take into account their techno-economic characteristics and the linkage between their heat and electricity outputs in the current sequential market design. In addition, this electricity-aware approach can be implemented in the current sequential market framework without major regulatory and organizational changes by introducing a bid selection mechanism prior to the clearing of sequential heat and electricity markets, as described in this section.

C. Heat and electricity market properties

After the bid selection step, the sequential clearing of heat and electricity markets remains unchanged, and therefore the economic properties of the current sequential heat and electricity markets are still valid. In particular, day-ahead heat and electricity markets with price-quantity bids and uniform pricing are *budget balanced* [25]. Furthermore, as the bid selection mechanism may reject certain bids, the revenue of certain heat market participants may be reduced. However, as previously discussed, solely bids that do not guarantee cost recovery are discarded. Therefore, after the bid selection step, the heat and electricity markets guarantee *cost recovery* for all market participants with respect to the variable production costs announced. These two important properties guarantee that the proposed electricity-aware approach can be implemented in the current sequential market framework without major changes.

However, *incentive compatibility* can be guaranteed for these market mechanisms only under the assumption of perfect competition [26], [27]. Otherwise, individual market participants may manipulate market outcomes by submitting strategic bids. [28] show that incentive-compatibility can be achieved in the limit, as the number of agents tends to infinity. Furthermore, heat and electricity markets with uniform pricing may not be *efficient*. [29] show that a Nash equilibrium may not exist in the general case. However, these markets are efficient under the assumption of a perfect competition.

Finally, this mechanism relies on the exchange of information on the electricity market supply and demand curves. This information is solely exchanged between the electricity market operator and the heat system operator. Market participants in both sectors do not have access to it prior to the market clearings. Therefore, this proposal does not raise privacy issues for electricity market participants, or provide any additional strategic advantage to heat market participants. Furthermore, this framework is in line with recent regulatory changes, encouraging sector coordination and information exchange between market and system operators. For instance, the joint Federal Energy Regulatory Commission (FERC) and North American Electric Reliability Corporation (NERC) report on the 2011 polar vortex put an emphasis on increased information sharing between energy sectors to improve systems reliability and prevent extreme events [30], [31].

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IV. Mathematical formulations and solution method

A. Lower-level: Sequential heat and electricity market clearing

The lower-level problems of the bid validity mechanism represent the sequential and independent clearing of the day-ahead heat and electricity markets, once the valid heat bids have been selected. These two market-clearing problems are formulated below.

1) Day-ahead heat market problem: For given values of the status $\boldsymbol{u^0_{jt}}$ and selected bids $\boldsymbol{u^{\text{bid}}_{jbt}}$ of CHPs and HPs, the day-ahead heat market aims at minimizing the heat dispatch cost across all interconnected market zones over the set of decision variables Ω^{H} including the dispatch of all submitted bids $\boldsymbol{Q_{jbt}}$, as well as heat flows $\boldsymbol{f^{\text{H}}_{z\tilde{z}t}}$ between market zones of the district heating network. It can be formulated as a linear optimization problem as follows:

$$\min_{\Omega^{\text{H}}} \Theta^{\text{H}}(\boldsymbol{Q_{jbt}}) = \sum_{t\in\mathcal{T}}\sum_{j\in\mathcal{I}^{\text{H}}}\sum_{b\in\mathcal{B}^{\text{H}}} c^{\text{H}}_{jbt}\boldsymbol{Q_{jbt}} \tag{7a}$$

$$\text{s.t.} \quad \sum_{j\in\mathcal{I}^H_z} \boldsymbol{Q_{jt}} = L^{\text{H}}_{zt} + \sum_{\tilde{z}\in\mathcal{Z}^H_z} \boldsymbol{f^{\text{H}}_{z\tilde{z}t}}, \forall z\in\mathcal{Z}^{\text{H}}, t\in\mathcal{T} \tag{7b}$$

$$\boldsymbol{Q_{jt}} = \sum_{b\in\mathcal{B}^{\text{H}}} \boldsymbol{Q_{jbt}}, \forall j\in\mathcal{I}^{\text{H}}, t\in\mathcal{T} \tag{7c}$$

$$\underline{s}^{\text{H}}_j \boldsymbol{u^0_{jt}} \le \boldsymbol{Q_{jt}}, \forall j\in\mathcal{I}^{\text{H}}, t\in\mathcal{T} \tag{7d}$$

$$0 \le \boldsymbol{Q_{jbt}} \le \overline{s}^{\text{H}}_{jbt}\boldsymbol{u^{\text{bid}}_{jbt}}, \forall j\in\mathcal{I}^{\text{H}}, t\in\mathcal{T}, b\in\mathcal{B}^{\text{H}} \tag{7e}$$

$$-\underline{f}^{\text{H}}_{z\tilde{z}t} \le \boldsymbol{f^{\text{H}}_{z\tilde{z}t}} \le \overline{f}^{\text{H}}_{z\tilde{z}t}, \forall z\in\mathcal{Z}^{\text{H}}, \tilde{z}\in\mathcal{Z}^{\text{H}}_z, t\in\mathcal{T} \tag{7f}$$

$$\boldsymbol{f^{\text{H}}_{z\tilde{z}t}} = -\boldsymbol{f^{\text{H}}_{\tilde{z}zt}}, \forall z\in\mathcal{Z}^{\text{H}}, \tilde{z}\in\mathcal{Z}^{\text{H}}_z, t\in\mathcal{T}. \tag{7g}$$

This problem aims at minimizing the operating cost of the heat system for given valid bids submitted by the heat market participants (7a), subject to constraints ensuring the heat balance in each market zone of the district heating network is respected (7b), defining the total heat production $\boldsymbol{Q_{jt}}$ of each market participant as the sum of its bids dispatched (7c), enforcing lower bounds on the heat production of all market participants based on their minimum capacity $\underline{s}^{\text{H}}_j$ and commitment status (7d), setting upper bounds on the dispatch of all heat bids based on the quantity $\overline{s}^{\text{H}}_{jbt}$ and validity $\boldsymbol{u^{\text{bid}}_{jbt}}$ of these bids (7e), [2], such that only valid bids selected in the upper-level (10) can be dispatched, defining upper and lower bounds ($\overline{f}^{\text{H}}_{z\tilde{z}t}$, $\underline{f}^{\text{H}}_{z\tilde{z}t}$) on the heat flow between heat market zones (7f), and linking the directed heat flows between market zones (7g).

2) Day-ahead electricity market: Once the heat market has been cleared, the day-ahead electricity market is cleared for the fixed values of the heat dispatch of CHPs and HPs $\boldsymbol{Q_{jt}}$. It is assumed that for any value of the heat dispatch, the electricity market clearing problem is feasible and bounded. It can be formulated as a linear optimization problem as follows:

$$\min_{\Omega^{\text{E}}} \Theta^{\text{E}}(\boldsymbol{P_{jbt}}) = \sum_{j\in\mathcal{I}^{\text{E}}}\sum_{b\in\mathcal{B}^{\text{E}}} c^{\text{E}}_{jbt}\boldsymbol{P_{jbt}} \tag{8a}$$

$$\text{s.t.} \quad \sum_{j\in\mathcal{I}^{\text{E}}_z} \boldsymbol{P_{jt}} = L^{\text{E}}_{zt} + \sum_{\tilde{z}\in\mathcal{Z}^{\text{E}}_z} \boldsymbol{f^{\text{E}}_{z\tilde{z}t}}, \forall z\in\mathcal{Z}^{\text{E}}, t\in\mathcal{T} \tag{8b}$$

$$\boldsymbol{P_{jt}} = \sum_{b\in\mathcal{B}^{\text{E}}} \boldsymbol{P_{jbt}}, \forall j\in\mathcal{I}^{\text{E}}, t\in\mathcal{T} \tag{8c}$$

$$\underline{\boldsymbol{P}}_{\boldsymbol{jt}} \le \boldsymbol{P_{jt}} \le \overline{\boldsymbol{P}}_{\boldsymbol{jt}}, \forall j\in\mathcal{I}^{\text{E}}, t\in\mathcal{T} \tag{8d}$$

$$0 \le \boldsymbol{P_{jbt}} \le \overline{s}^{\text{E}}_{jbt}, \forall j\in\mathcal{I}^{\text{E}}, b\in\mathcal{B}^{\text{E}}, t\in\mathcal{T} \tag{8e}$$

$$-\underline{f}^{\text{E}}_{z\tilde{z}t} \le \boldsymbol{f^{\text{E}}_{\tilde{z}zt}} \le \overline{f}^{\text{E}}_{z\tilde{z}t}, \forall z\in\mathcal{Z}^{\text{E}}, \tilde{z}\in\mathcal{Z}^{\text{E}}_z t\in\mathcal{T} \tag{8f}$$

$$\boldsymbol{f^{\text{E}}_{z\tilde{z}t}} = -\boldsymbol{f^{\text{E}}_{\tilde{z}zt}}, \forall z\in\mathcal{Z}^{\text{E}}, \tilde{z}\in\mathcal{Z}^{\text{E}}_z, t\in\mathcal{T} \tag{8g}$$

$$\overline{\boldsymbol{P}}_{\boldsymbol{jt}} = \begin{cases} \sum_{b\in\mathcal{B}^{\text{E}}} \overline{s}^{\text{E}}_{jbt}, \forall j\in\mathcal{I}^{\text{E}}\setminus\mathcal{I}^{\text{H}}, t\in\mathcal{T} \\ -\dfrac{\boldsymbol{Q_{jt}}}{\text{COP}_j}, \forall j\in\mathcal{I}^{\text{HP}}, t\in\mathcal{T} \\ \overline{F}_j - \rho_j\boldsymbol{Q_{jt}}, \forall j\in\mathcal{I}^{\text{CHP}}, t\in\mathcal{T} \end{cases} \tag{8h}$$

$$\underline{\boldsymbol{P}}_{\boldsymbol{jt}} = \begin{cases} \underline{s}^{\text{E}}_{jt}, \forall j\in\mathcal{I}^{\text{E}}\setminus\mathcal{I}^{\text{H}}, t\in\mathcal{T} \\ -\dfrac{\boldsymbol{Q_{jt}}}{\text{COP}_j}, \forall j\in\mathcal{I}^{\text{HP}}, t\in\mathcal{T} \\ r_j\boldsymbol{Q_{jt}}, \forall j\in\mathcal{I}^{\text{CHP}}, t\in\mathcal{T} \end{cases} \tag{8i}$$

This problem aims at minimizing the electricity dispatch cost, based on the set of decision variables Ω^{E} including the dispatch $\boldsymbol{P_{jbt}}$ of all electricity bids, and the power flow $\boldsymbol{f^{\text{E}}_{z\tilde{z}t}}$ between electricity market zones. This market clearing aims at minimizing the electricity dispatch cost (8a), subject to constraints ensuring the power balance in each electricity market zone is respected (8b), defining the total electricity production $\boldsymbol{P_{jt}}$ of each market participant as the sum of its bids dispatched (8c), enforcing upper and lower bounds ($\overline{\boldsymbol{P}}_{\boldsymbol{jt}}$, $\underline{\boldsymbol{P}}_{\boldsymbol{jt}}$) on the self-committed electricity dispatch of all market participants (8d), enforcing upper bounds $\overline{s}^{\text{E}}_{jbt}$ on the dispatch of all electricity bids based on their quantity bid (8e), setting upper and lower bounds ($\overline{f}^{\text{E}}_{z\tilde{z}t}$, $\underline{f}^{\text{E}}_{z\tilde{z}t}$) on the power exchanged between electricity market zones (8f), and linking the directed power exchanged between these market zones (8g). Additionally, (8h) and (8i) define the upper and lower

[2]As the bids submitted are required to be non-decreasing, a bid b cannot be dispatched if previous bids are not fully be dispatched.

bounds on the electricity dispatch as their maximum bids $\sum_{b\in\mathcal{B}^{\mathrm{E}}} \overline{s}^{\mathrm{E}}_{jbt}$ and self-committed power $\underline{s}^{\mathrm{E}}_{jt}$ for conventional electricity market participants, and as functions of their heat dispatch (fixed in the heat market-clearing problem (7)) for CHPs and HPs. Finally, the electricity zonal prices $\boldsymbol{\lambda^{\mathrm{E}}_{zt}}$ are defined as the dual variables of the balance equations (8b).

3) Lower-level problem formulation: These sequential market clearings can be modeled as an equilibrium problem (7)-(8), where the electricity market-clearing problem takes as input the optimal solutions of the heat market-clearing problem. In this formulation, both the heat (7) and electricity (8) market-clearing problems are lower-level problems for the upper-level problem (10). However, approaches to reformulate and solve the resulting bilevel optimization problem, relying on replacing the lower-level problems by their KKT conditions or the strong duality theorem would introduce non-convex bilinear terms, making the solution of the whole electricity-aware bid selection mechanism computationally challenging.

We observe that, for a fixed solution of the bid selection mechanism, the heat market clearing (7) is cleared first and does not depend on the outcomes of the electricity market clearing. Therefore, its objective is optimized in priority, regardless of the solutions of the electricity market clearing. As a result, and in order to circumvent the limitations of the aforementioned approaches, we propose a novel formulation of the sequential heat and electricity market clearings as a linear lexicographic optimization problem, such that:

$$\min_{\Omega^{\mathrm{H}}\cup\Omega^{\mathrm{E}}} \ < \Theta^{\mathrm{H}}(\boldsymbol{Q_{jbt}}), \Theta^{\mathrm{E}}(\boldsymbol{P_{jbt}}) > \tag{9a}$$
$$\text{s.t.} \quad \text{Eqs. (7b)} - \text{(7g)} \tag{9b}$$
$$\text{Eqs. (8b)} - \text{(8i)}. \tag{9c}$$

The aim of this multi-objective optimization problem is to minimize the heat and electricity dispatch costs, ranked in a lexicographic order (9a), subject to constraints on the heat and electricity dispatch (9b)-(9c). In this approach, one first minimizes the heat dispatch cost $\Theta^{\mathrm{H}}(\boldsymbol{Q_{jbt}})$, then holding $\Theta^{\mathrm{H}}(\boldsymbol{Q_{jbt}})$ constant, minimizes the electricity dispatch cost $\Theta^{\mathrm{E}}(\boldsymbol{Q_{jbt}})$.

Proposition 1. *Any optimal solution to the proposed lexicographic optimization formulation* (9) *of the sequential heat and electricity market clearing is an optimal solution to the equilibrium problem between the heat* (7) *and electricity* (8) *market clearing problems.*

The proof of Proposition 1 is provided in the Appendix.

B. Upper level: Electricity-aware bid selection mechanism

The proposed electricity-aware bid selection mechanism in the upper-level problem seeks to minimize the operating cost of the overall heat system for each hour of the following day $t \in \mathcal{T}$, while anticipating the impact of its decisions on the heat market-clearing problem in the middle level and the electricity market-clearing problem in the lower level. The set of decision variables Ω^{UL} of this problem includes the commitment state $\boldsymbol{u^{0}_{jt}}$, start-up $\boldsymbol{v^{\mathrm{SU}}_{jt}}$, and shut-down $\boldsymbol{v^{\mathrm{SD}}_{jt}}$ states of all heat market participants, the validity $\boldsymbol{u^{\mathrm{bid}}_{jbt}}$ of all heat bids, which are defined as binary variables (10f), as well as the dispatch $\boldsymbol{Q_{jbt}}$ of all heat bids and the electricity market prices $\boldsymbol{\lambda^{\mathrm{E}}_{zt}}$ in all electricity market zones, which are defined as the solutions of the sequential heat and electricity market clearing in the lower-level problem (10g). It can be formulated as a hierarchical optimization problem as follows:

$$\min_{\Omega^{\mathrm{UL}}} \Omega^{\mathrm{UL}} = \sum_{t\in\mathcal{T}} \sum_{j\in\mathcal{I}^{\mathrm{H}}} \Big(c^{0}_{jt}\boldsymbol{u^{0}_{jt}} + c^{\mathrm{SU}}_{j}\boldsymbol{v^{\mathrm{SU}}_{jt}} + \sum_{k=1}^{B^{\mathrm{H}}} c^{\mathrm{H}}_{jbt}\boldsymbol{Q_{jbt}} \Big) \tag{10a}$$
$$\boldsymbol{v^{\mathrm{SU}}_{jt}} - \boldsymbol{v^{\mathrm{SD}}_{jt}} = \boldsymbol{u^{0}_{jt}} - \boldsymbol{u^{0}_{j(t-1)}}, \forall j \in \mathcal{I}^{\mathrm{H}}, t \in \mathcal{T} \tag{10b}$$
$$\boldsymbol{u^{\mathrm{bid}}_{jbt}} \leq \boldsymbol{u^{\mathrm{bid}}_{j(b-1)t}}, \forall j \in \mathcal{I}^{\mathrm{H}}, t \in \mathcal{T}, b \in \mathcal{B}^{\mathrm{H}}, b \in \mathcal{B}^{\mathrm{H}} \setminus \{1\} \tag{10c}$$
$$\boldsymbol{u^{\mathrm{bid}}_{j1t}} \leq \boldsymbol{u^{0}_{jt}}, \forall j \in \mathcal{I}^{\mathrm{H}}, t \in \mathcal{T}, b \in \mathcal{B}^{\mathrm{H}}, b \in \mathcal{B}^{\mathrm{H}} \tag{10d}$$
$$\text{Eqs. (6)} \tag{10e}$$
$$\boldsymbol{v^{\mathrm{SU}}_{jt}}, \boldsymbol{v^{\mathrm{SD}}_{jt}}, \boldsymbol{u^{0}_{jt}}, \boldsymbol{u^{\mathrm{bid}}_{jbt}} \in \{0,1\}, \forall j \in \mathcal{I}^{\mathrm{H}}, b \in \mathcal{B}^{\mathrm{H}}, t \in \mathcal{T} \tag{10f}$$
$$\{\boldsymbol{Q_{jbt}}, \boldsymbol{\lambda^{\mathrm{E}}_{zt}}\} \in \text{primal and dual solutions of (9).} \tag{10g}$$

This bid selection mechanism aims at minimizing the heat system operating cost, which includes start-up costs $c^{\mathbf{SU}}_{jt}$, no-load costs c^{0}_{jt}, and the cost of dispatching heat bids c^{H}_{jbt} (10a), subject to constraints stating the relationship between the binary variables for the on/off, start-up, and shut-down statuses of each unit (10b), ensuring that a bid can be selected only if the previous bids have been selected (10c), and if the unit is committed (10d), and enforcing that only valid bids are selected (10e).

C. Reformulation as Mixed Integer Linear Program (MILP)

The proposed bid selection mechanism (10) can be formulated in a compact manner as

$$\min_{\boldsymbol{z}\in\{0,1\}, \boldsymbol{x^{\mathrm{H}}}\geq 0, \boldsymbol{y^{\mathrm{E}}}} \ c^{\mathrm{bid}^\top}\boldsymbol{z} + c^{\mathrm{H}^\top}\boldsymbol{x^{\mathrm{H}}} \tag{11a}$$
$$\text{s.t.} \quad \boldsymbol{z} \in \mathcal{Z}^{\mathrm{bid}} \tag{11b}$$
$$hh \tag{11c}$$
$$\{\boldsymbol{x^{\mathrm{H}}}, \boldsymbol{y^{\mathrm{E}}}\} \in \text{ primal and dual sol. of:}$$
$$\left\{ \begin{array}{ll} \min\limits_{\boldsymbol{x^{\mathrm{H}}},\boldsymbol{x^{\mathrm{E}}}\geq \boldsymbol{0}} & < c^{\mathrm{H}^\top}\boldsymbol{x^{\mathrm{H}}}, c^{\mathrm{E}^\top}\boldsymbol{x^{\mathrm{E}}} > \\ \text{s.t.} & A^{\mathrm{H}}\boldsymbol{x^{\mathrm{H}}} + B^{\mathrm{H}}\boldsymbol{z} \geq b^{\mathrm{H}} \\ & A^{\mathrm{E}}\boldsymbol{x^{\mathrm{E}}} + B^{\mathrm{E}}\boldsymbol{x^{\mathrm{H}}} \geq b^{\mathrm{E}} \end{array} \right\} \tag{11d}$$

where $\boldsymbol{z}$, $\boldsymbol{x^{\mathrm{H}}}$ and $\boldsymbol{x^{\mathrm{E}}}$ represent the vectors of primal variables of the bid selection mechanism, heat and electricity market clearings, respectively, and $\boldsymbol{x^{\mathrm{E}}}$ is the vector of dual variables of the electricity market clearing. The expression of the vectors $\left(c^{\mathrm{bid}}, c^{\mathrm{H}}, b^{\mathrm{H}}, c^{\mathrm{E}}, b^{\mathrm{bid}}, b^{\mathrm{H}}, b^{\mathrm{E}}\right)$ and matrices $\left(A^{\mathrm{bid}}, B^{\mathrm{bid}}, A^{\mathrm{H}}, B^{\mathrm{H}}, A^{\mathrm{E}}, B^{\mathrm{E}}\right)$ of parameters can be derived from the detailed formulations of the upper- (10) lower-level problems (7)-(8).

Proposition 2. *The bid selection mechanism in* (11) *can be asymptotically approximated by the following single-level MILP:*

$$
\begin{aligned}
\min_{\substack{\boldsymbol{z}\in\{0,1\}^N,\boldsymbol{x^H}\geq\boldsymbol{0}\\ \boldsymbol{x^E}\geq\boldsymbol{0},\boldsymbol{y^H},\boldsymbol{y^E}}} \quad & \gamma c^{bid\top}\boldsymbol{z} + \gamma c^{H\top}\boldsymbol{x^H} + (1-\gamma)\, c^{E\top}\boldsymbol{x^E} && \text{(12a)}\\
\textit{s.t.} \quad & \boldsymbol{z} \in \mathcal{Z}^{UC} && \text{(12b)}\\
& A^{bid}\boldsymbol{z} + \frac{1}{(1-\gamma)} B^{bid}\boldsymbol{y^E} \geq b^{bid} && \text{(12c)}\\
& A^{H}\boldsymbol{x^H} + B^{H}\boldsymbol{z} \geq b^{H} && \text{(12d)}\\
& A^{E}\boldsymbol{x^E} + B^{E}\boldsymbol{x^H} \geq b^{E} && \text{(12e)}\\
& \boldsymbol{y^H}^{\top} A^{H} + \boldsymbol{y^E}^{\top} B^{E} \leq \gamma c^{H\top} && \text{(12f)}\\
& \boldsymbol{y^E}^{\top} A^{E} \leq (1-\gamma)\, c^{E\top} && \text{(12g)}\\
& \boldsymbol{y^H}^{\top}\left(b^{H} - B^{H}\boldsymbol{z}\right) + \boldsymbol{y^E}^{\top} b^{E} \\
& \qquad \geq \gamma c^{H\top}\boldsymbol{x^H} + (1-\gamma)\, c^{E\top}\boldsymbol{x^E}. && \text{(12h)}
\end{aligned}
$$

When the penalty factor γ tends to 1, the solutions of (12) *converge to the solutions of* (11).

Note that the bilinear terms in (12h) can be linearized using an exact McCormick relaxation [32]. The proof of Proposition 2 is provided in the Appendix.

V. Numerical analyses

This section illustrates the benefits of the proposed *electricity-aware* bid selection mechanism in terms of renewable energy penetration, cost-effectiveness, and profitability of CHPs and HPs, through two case studies. In both case studies, the penalty factor γ in is fixed to 0.99. A sensitivity analysis reveals that solutions are stable around this value, and therefore, are assumed to have converged. Details on these case studies setup and all relevant data are provided in the online appendices [33], [34].

A. Case study 1: modified 24-bus IEEE Reliability Test System

We first analyse the performance of the proposed electricity-aware bid selection mechanism on a modified version of the 24-bus IEEE Reliability Test System connected to two 3-node district heating networks, over 2 months of operation.

1) Case study setup: As illustrated in Figure 2, this integrated energy system, comprises 12 thermal power plants (G), 6 wind farms (W), 2 extraction CHPs, 2 HPs, and 2 waste incinerator heat-only units (HO). Data for power generation, costs, loads and transmission for the 24-bus IEEE Reliability Test System is derived from [35]. Data for heat generation, loads and transmission for the district heating networks are derived from [9], [11], [36] and representative of the greater Copenhagen area. This small test case represents an energy system with a particularly high penetration of wind production, as well as a large share of CHPs and HPs, which can provide operational flexibility at the interface between heat and electricity systems.

The proposed electricity-aware bid selection mechanism (EA) is compared to: (a) the current sequential and decoupled market mechanism (Dec) described in Section II; and (b) the fully integrated heat and electricity unit commitment and market mechanism (Int), which jointly and simultaneously optimizes the heat and electricity systems. In order to ensure a fair comparison between the electricity-aware and decoupled mechanisms, the hourly heat bids of CHPs and HPs are computed each day as their heat marginal costs using day-ahead electricity price forecasts, as discussed in Section II. These forecast prices are derived by jointly clearing the heat and electricity markets.

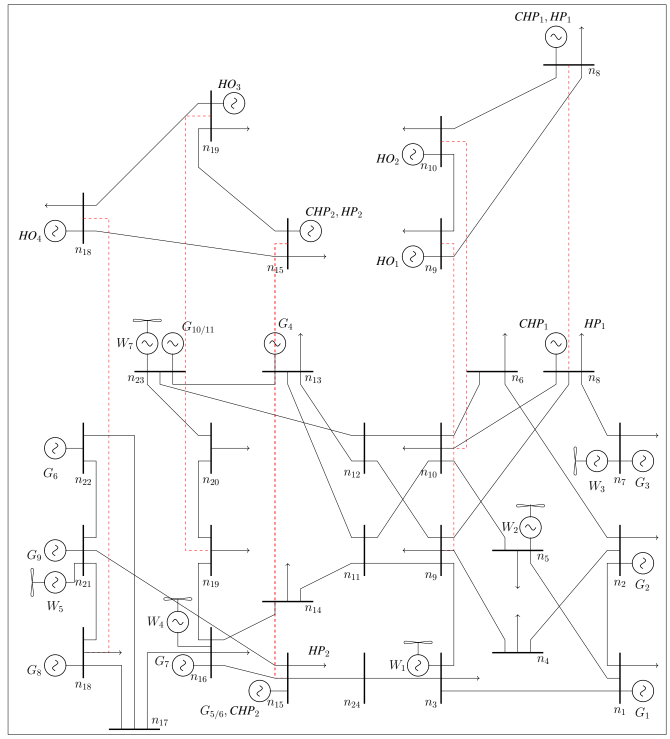

Fig. 2. Case study 1: Modified IEEE 24-node electricity system with 6 wind farms (the bottom system) connected to two isolated 3-node district heating systems (the two systems on the top of the figure)

2) Results: This first analysis focuses on quantifying the *value of coordination* achieved by the proposed electricity-aware bid selection mechanism. As summarized in Table I, the integrated market mechanism reduces the overall system cost by 11.3% compared to the decoupled mechanism. The absolute cost difference, i.e., $1,330$€, between the decoupled and integrated market mechanisms represents the value of coordination between heat and electricity markets [11]. However, as previously discussed, this integrated mechanism is not a realistic alternative to a sequential heat and electricity market framework. Indeed, it can be observed that the heat system operating cost is drastically increased with the integrated mechanism compared to the decoupled and hierarchical mechanisms, which supports the findings by [13], [14]. Meanwhile, the proposed electricity-aware mechanism achieves 77.6% of this so-called value of coordination, while maintaining the sequential and independent structure of the markets.

B. Case study 2: Danish energy system

We then analyse the performance of the proposed electricity-aware bid selection mechanism on a realistic case study representing the Danish heat and electricity systems, over 1 year of operation.

TABLE I
CASE STUDY 1: TOTAL, HEAT AND ELECTRICITY SYSTEM COSTS, IN 10^3€, AND VALUE OF COORDINATION ACHIEVED, IN % OF TOTAL VALUE OF COORDINATION FOR THE DECOUPLED (DEC), ELECTRICITY-AWARE (EA), AND INTEGRATED (INT) MARKET FRAMEWORKS.

	Dec	**EA**	**Int**
Total cost	11.75	10.72	10.42
Heat cost	2.66	2.96	10.23
Electricity cost	9.09	7.76	0.19

1) Case study setup: As illustrated in Figure 3, the Danish electricity system is divided into two market zones, $DK1$ and $DK2$, connected via an interconnection. Generation costs and parameters, ATCs, as well as electricity loads, wind and solar power generation for 1 year are derived from the website of the Danish electricity system operator, Energinet.dk [37] as well as [21].

Additionally, we consider three disconnected district heating networks, which geographically cover, respectively, the greater Copenhagen area, Aarhus area, and the multicity TVIS area (Fredericia, Middelfart, Kolding and Vejle). These district heating networks comprise 11 CHPs, 6 incinerators (IS), i.e., CHPs with fixed heat-electricity ratio, 6 HPs, 20 HO units and peak boilers, and 3 heat storage tanks (HS). Heat load profiles for 1 year, generation costs, and technical parameters for these district heating networks are derived from [11], [38].

Fig. 3. Case study 2: Danish electricity market zones ($DK1$ and $DK2$) and three district heating networks (Aarhus, Copenhagen, TVIS). Red zones represent areas with high density of district heating consumption. Red arrow represents the interconnection between the two electricity market zones.

2) Results: This second analysis provides further insights on the operation of the heat and electricity systems under the proposed mechanism in a realistic energy system, and the impact on the financial losses of different market participants. The simulation of the sequential operation of the heat and electricity systems over 1 year with both mechanisms shows that the proposed electricity-aware bid selection mechanism is able to efficiently anticipate the impact of the commitment decisions of CHPs and HPs on the electricity market. As summarized in Table II, the proposed model achieves lower heat, electricity and overall system costs compared to the decoupled approach. This is achieved by switching off certain CHPs during extended periods of low day-ahead electricity prices and when their bids are invalid. For instance, we compare the heat dispatch of CHP5 and CHP6, which are located in the same electricity market zone, but differ in their techno-economic characteristics. As illustrated in Figure 4(a) the bids of CHP6 are rejected by the electricity-aware bid selection mechanism because they are invalid, and CHP5 is used to partially cover this heat production, despite offering more expensive bids. While, for a given hour, this rejected bid may be replaced with more expensive bids, over multiple days, this electricity-aware approach yields a more efficient and less expensive commitment and dispatch in the heat system.

Additionally, as the decoupled mechanism fails to anticipate the impact of day-ahead electricity prices on the heat production costs of CHPs and HPs, in the hours where invalid bids are selected, these units suffer large financial losses, as summarized in Table III for each unit. As illustrated in Figure 4(b) for CHP6, in the days where the electricity-aware bid selection mechanism rejects invalid bids, the decoupled unit commitment incurs large financial losses by selecting these invalid bids. These losses are more significant during the winter time due to the higher production level of CHP6 during these days.

TABLE II
CASE STUDY 2: HEAT, ELECTRICITY AND OVERALL ENERGY SYSTEMS COMMITMENT AND DISPATCH COSTS, IN 10^6€, AND RENEWABLE ENERGY UTILIZATION IN THE ELECTRICITY SYSTEM, IN % OF PRODUCTION, AS A PERCENTAGE OF THE AVAILABLE PRODUCTION, FOR DECOUPLED (DEC) AND ELECTRICITY-AWARE (EA) MARKET FRAMEWORKS.

	Dec	**EA**
Total cost*	$1,967$	$1,881$ (-4.4%)
Heat cost	$1,360$	$1,287$ (-5.4%)
Electricity cost	608	594 (-2.2%)
Renewable utilization	95.9	96.3 $(+0.4\%)$

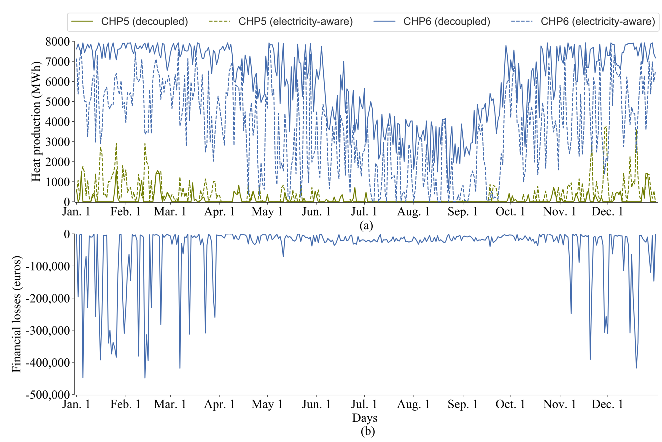

Fig. 4. Case study 2: (a) Daily heat dispatch (in MWh) of CHPs with decoupled and electricity-aware mechanisms over 1 year, and (b) corresponding financial losses (in €) with the decoupled mechanism.

VI. CONCLUSION

This paper proposes a novel electricity-aware bid format and bid selection mechanism, which are modeled using a trilevel optimization problem. This mechanism improves the coordination between heat and and electricity markets by allowing CHPs and HPs to offer electricity-aware heat bids which are conditioned on day-ahead electricity prices, while

TABLE III
CASE STUDY 2: NUMBER OF HOURS FOR WHICH INVALID BIDS ARE SELECTED BY THE DECOUPLED MECHANISM, AND THE RESULTING FINANCIAL LOSSES FOR EACH UNIT.

	Hours	Losses (10^3 €)		Hours	Losses (10^3 €)
IS1	7472	$-6,620$	**CHP1**	3989	$-20,364$
IS2	7220	$-6,717$	**CHP2**	2638	$-3,081$
IS3	0	0	**CHP3**	3192	$-10,668$
IS4	3195	$-8,092$	**CHP4**	4784	$-6,683$
IS5	6899	$-11,918$	**CHP5**	20	-148
IS6	4135	$-6,739$	**CHP6**	3078	$-17,203$
HP1	1920	-787	**CHP7**	2256	$-7,534$
HP2	2669	$-2,086$	**CHP8**	1503	$-5,140$
HP3	2669	-348	**CHP9**	3212	$-7,531$
HP4	594	0	**CHP10**	3161	$-7,395$
HP5	2146	-379	**CHP11**	1457	$-5,839$
HP6	1959	-83			

respecting the current sequential heat and electricity market clearing procedure. These electricity-aware bids are modeled using linear bid-validity conditions. As a result, a tractable MILP reformulation for the proposed trilevel optimization problem is developed. Finally, the value of improving the coordination between heat and electricity systems is illustrated in two case studies. The first illustrative case study shows that the proposed electricity-aware mechanism can achieve 77.6% of the *value of coordination* achieved by the fully integrated mechanism, while maintaining a sequential heat and electricity market framework. The second case study, based on the realistic electricity and heat systems in Denmark, shows that the proposed mechanism is able to ensure cost recovery for CHPs and HPs in the heat market, while reducing the operating cost of the overall energy system by 4.5% and wind curtailment by 0.4% compared to a decoupled mechanism. These benefits are achieved by anticipating the impact of CHPs and HPs on electricity markets. This work allows us to harness and remunerate the flexibility of CHPs and HPs at the interface between heat and electricity systems, and to achieve an efficient and cost-effective operation of the overall energy system.

This study opens up various opportunities for future work. Firstly, the proposed bid selection mechanism coordinates the participation of CHPs and HPs across multiple isolated heat networks and market zones in a centralized way. This requires a central heat system operator to collect information from independent heat market operators, which may be challenging in practice. As a potential alternative, decomposition techniques relying on consensus-based distributed algorithms could be investigated to facilitate the application of the proposed approach in the current energy system. Recent advances in the literature have introduced performance guarantees on the application of such algorithms to large-scale MILPs [39]–[41]. Furthermore, this work does not take into account additional energy products, such as natural gas, and how their day-ahead prices may impact the validity of heat bids. As the day-ahead gas market is cleared after the heat and electricity markets, this limitation may be accounted for by developing an *electricity- and gas-aware* heat bid selection mechanism, modeled as trilevel optimization problem [18].

Secondly, as previously discussed, the proposed bid selection mechanism does not guarantee efficiency and incentive compatibility, due to the unchanged design of the sequential heat and electricity markets. In order to quantify the loss of efficiency in heat and electricity markets resulting from the exercise of market power, further agent-based analysis should be conducted. Providing a rigorous analysis of potential strategic behaviors, exercise of market power, and opportunity costs across both heat and electricity markets would provide useful insights for further market designs. Furthermore, market power may be reduced by designing new market-clearing mechanisms for the day-ahead heat and electricity markets [42], [43], which would require major regulatory and organizational changes.

Thirdly, the proposed model assumes perfect information on the bids of the market participants and wind power availability in the electricity market clearing. However, such information may not be communicated by the independent market operator, due to privacy concerns. Therefore, imperfect information on the parameters of the lower-level problem may be assumed using a scenario-based stochastic programming framework, or a robust counterpart of the middle- and lower-level problems. In addition, in order to mitigate the inefficiencies related to the lack of information exchange, a privacy-preserving extension of the proposed model can be developed, based on the Privacy-Preserving Stackelberg Mechanism (PPSM) introduced in [44]. The PPSM allows the follower in a Stackelberg game, e.g., the electricity market operator, to share differentially-private information, e.g. bids, with the leader, e.g. the heat system operator, while ensuring near-optimal coordination of the sectors.

Finally, while this work focuses on the coordination of different energy markets in the day-ahead stage, accounting for uncertainty of energy delivery in real time is essential to ensure reliability of the overall energy system. Following the work on policy-based reserves proposed by [45] and [46], the proposed bid selection mechanism could be extended to co-optimize energy and operating reserves.

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Appendix A
Proof of Proposition 1

The equilibrium problem representing the sequential heat and electricity market clearing in (7)-(8) can be expressed in a compact manner as:

$$\boldsymbol{x^{\mathrm{H}}} \in \text{ sol. of } \left\{ \begin{array}{ll} \min\limits_{\boldsymbol{x^{\mathrm{H}}} \geq \mathbf{0}} & c^{\mathrm{H}^\top} \boldsymbol{x^{\mathrm{H}}} \\ \text{s.t.} & A^{\mathrm{H}} \boldsymbol{x^{\mathrm{H}}} + B^{\mathrm{H}} \boldsymbol{z} \geq b^{\mathrm{H}} \end{array} \right\} \tag{13a}$$

$$\boldsymbol{x^{\mathrm{E}}} \in \text{ sol. of } \left\{ \begin{array}{ll} \min\limits_{\boldsymbol{x^{\mathrm{E}}} \geq \mathbf{0}} & c^{\mathrm{E}^\top} \boldsymbol{x^{\mathrm{E}}} \\ \text{s.t.} & A^{\mathrm{E}} \boldsymbol{x^{\mathrm{E}}} + B^{\mathrm{E}} \boldsymbol{x^{\mathrm{H}}} \geq b^{\mathrm{E}} \end{array} \right\} \tag{13b}$$

Similarly, the proposed lexicographic optimization problem (9) is formulated in a compact manner as (11d).

This lexicographic optimization problem can be solved in two steps:

1) Find the optimal heat dispatch cost such that:

$$\begin{aligned} \Theta^{\mathrm{H}^*} = \min_{\boldsymbol{x^{\mathrm{H}}}, \boldsymbol{x^{\mathrm{E}}} \geq \mathbf{0}} \quad & c^{\mathrm{H}^\top} \boldsymbol{x^{\mathrm{H}}} && (14a) \\ \text{s.t.} \quad & (7b)-(7g) && (14b) \\ & (8b)-(8i) && (14c) \end{aligned}$$

2) Find an optimal heat and electricity dispatch such that:

$$\begin{aligned} \{x^{\mathrm{H}^*}, x^{\mathrm{E}^*}\} \in \operatorname*{argmin}_{\boldsymbol{x^{\mathrm{H}}}, \boldsymbol{x^{\mathrm{E}}} \geq \mathbf{0}} \quad & c^{\mathrm{E}^\top} \boldsymbol{x^{\mathrm{E}}} && (15a) \\ \text{s.t.} \quad & (7b)-(7g) && (15b) \\ & (8b)-(8i) && (15c) \\ & c^{\mathrm{H}^\top} \boldsymbol{x^{\mathrm{H}}} \leq \Theta^{\mathrm{H}^*} && (15d) \end{aligned}$$

Due to Constraints (15b) and (15d), for any optimal solution $\{x^{\mathrm{H}^*}, x^{\mathrm{E}^*}\}$ to (14)-(15), x^{H^*} is also an optimal solution to (13a), and x^{E^*} is an optimal solution to (13b) with $\boldsymbol{x^{\mathrm{H}}}$ fixed to x^{H^*}.

Appendix B
Proof of Proposition 2

We consider the following approximation of the lexicographic optimization problem (11d), with $\gamma \in]0, 1[$:

$$\begin{aligned} \min_{\boldsymbol{x^{\mathrm{H}}}, \boldsymbol{x^{\mathrm{E}}} \geq \mathbf{0}} \quad & \gamma c^{\mathrm{H}^\top} \boldsymbol{x^{\mathrm{H}}} + (1-\gamma)\, c^{\mathrm{E}^\top} \boldsymbol{x^{\mathrm{E}}} && (16a) \\ \text{s.t.} \quad & A^{\mathrm{H}} \boldsymbol{x^{\mathrm{H}}} + B^{\mathrm{H}} \boldsymbol{z} \geq b^{\mathrm{H}} && (16b) \\ & A^{\mathrm{E}} \boldsymbol{x^{\mathrm{E}}} + B^{\mathrm{E}} \boldsymbol{x^{\mathrm{H}}} \geq b^{\mathrm{E}}, && (16c) \end{aligned}$$

where $\boldsymbol{y^{\mathrm{E}}}$ is obtained as the dual variable associated with constraint (16c) [18]. As a result, problem (10) can be approximated by the following linear bilevel optimization problem:

$$\begin{aligned} \min_{\substack{\boldsymbol{z} \in \{0,1\}^{\mathrm{N}}, \boldsymbol{x^{\mathrm{H}}}, \boldsymbol{x^{\mathrm{E}}} \geq \mathbf{0} \\ \boldsymbol{y^{\mathrm{H}}}, \boldsymbol{y^{\mathrm{E}}} \geq \mathbf{0}}} \quad & \gamma c^{0^\top} \boldsymbol{z} + \gamma c^{\mathrm{H}^\top} \boldsymbol{x^{\mathrm{H}}} + (1-\gamma)\, c^{\mathrm{E}^\top} \boldsymbol{x^{\mathrm{E}}} && (17a) \\ \text{s.t.} \quad & \boldsymbol{z} \in \mathcal{Z}^{\mathrm{UC}} && (17b) \\ & A^{\mathrm{bid}} \boldsymbol{z} + \frac{1}{(1-\gamma)} B^{\mathrm{bid}} \boldsymbol{y^{\mathrm{E}}} \geq b^{\mathrm{bid}} && (17c) \\ & \{\boldsymbol{x^{\mathrm{H}}}, \boldsymbol{y^{\mathrm{E}}}\} \in \text{ primal and dual sol. of (16).} && (17d) \end{aligned}$$

Besides, by strong duality of the lower-level problem (17d), problem (17) is equivalent to (12).

It remains to show that problem (12) is an asymptotic approximation to problem (11), i.e., as $\gamma \to 1$ the solutions to problem (12) become optimal solutions to problem (11). By introducing the auxiliary variables $\tilde{\boldsymbol{y}}^{\mathbf{H}} = \dfrac{\boldsymbol{y}^{\mathrm{H}}}{\gamma}$, and $\tilde{\boldsymbol{y}}^{\mathbf{E}} = \dfrac{\boldsymbol{y}^{\mathrm{E}}}{1-\gamma}$, problem (12) is equivalent to:

$$\begin{aligned} \min_{\substack{\boldsymbol{z} \in \{0,1\}^N, \boldsymbol{x^{\mathrm{H}}} \geq \mathbf{0} \\ \boldsymbol{x^{\mathrm{E}}} \geq \mathbf{0}, \boldsymbol{y^{\mathrm{H}}}, \boldsymbol{y^{\mathrm{E}}}}} \quad & \gamma c^{0^\top} \boldsymbol{z} + \gamma c^{\mathrm{H}^\top} \boldsymbol{x^{\mathrm{H}}} + (1-\gamma)\, c^{\mathrm{E}^\top} \boldsymbol{x^{\mathrm{E}}} && (18a) \\ \text{s.t.} \quad & \boldsymbol{z} \in \mathcal{Z}^{\mathrm{UC}} && (18b) \\ & A^{\mathrm{bid}} \boldsymbol{z} + B^{\mathrm{bid}} \tilde{\boldsymbol{y}}^{\mathbf{E}} \geq b^{\mathrm{bid}} && (18c) \\ & A^{\mathrm{H}} \boldsymbol{x^{\mathrm{H}}} + B^{\mathrm{H}} \boldsymbol{z} \geq b^{\mathrm{H}} && (18d) \\ & A^{\mathrm{E}} \boldsymbol{x^{\mathrm{E}}} + B^{\mathrm{E}} \boldsymbol{x^{\mathrm{H}}} \geq b^{\mathrm{E}} && (18e) \\ & \tilde{\boldsymbol{y}}^{\mathbf{H}^\top} A^{\mathrm{H}} + \frac{(1-\gamma)}{\gamma} \tilde{\boldsymbol{y}}^{\mathbf{E}^\top} B^{\mathrm{E}} \leq c^{\mathrm{H}^\top} && (18f) \\ & \tilde{\boldsymbol{y}}^{\mathbf{E}^\top} A^{\mathrm{E}} \leq c^{\mathrm{E}^\top} && (18g) \\ & \tilde{\boldsymbol{y}}^{\mathbf{H}^\top} \left(b^{\mathrm{H}} - B^{\mathrm{H}} \boldsymbol{z} \right) - c^{\mathrm{H}^\top} \boldsymbol{x^{\mathrm{H}}} \\ & \qquad \geq \frac{(1-\gamma)}{\gamma} \left(c^{\mathrm{E}^\top} \boldsymbol{x^{\mathrm{E}}} - \tilde{\boldsymbol{y}}^{\mathbf{E}^\top} b^{\mathrm{E}} \right). && (18h) \end{aligned}$$

For the value of the unit commitment variable $\boldsymbol{z}$ fixed to z^*, let us denote $\Theta(z^*)$ the optimal objective value to (11), and $\tilde{\Theta}(z^*)$ and $\{x^{\mathrm{H}^*}, x^{\mathrm{E}^*}, y^{\mathrm{H}^*}, y^{\mathrm{E}^*}\}$ the optimal objective and solutions to (18). As $\gamma \to 1$, (18f) and (18h) become

$$\begin{aligned} & \tilde{\boldsymbol{y}}^{\mathbf{H}^\top} A^{\mathrm{H}} \leq c^{\mathrm{H}^\top} && (19a) \\ & \tilde{\boldsymbol{y}}^{\mathbf{H}^\top} \left(b^{\mathrm{H}} - B^{\mathrm{H}} \boldsymbol{z} \right) \geq c^{\mathrm{H}^\top} \boldsymbol{x^{\mathrm{H}}}. && (19b) \end{aligned}$$

Constraint (18d) guarantees that x^{H^*} is feasible to problem (14) with $\boldsymbol{z}$ fixed to z^*. Additionally, (19a) guarantees that y^{H^*} becomes feasible to the dual of problem (14) with $\boldsymbol{z}$ fixed to z^* when $\gamma \to 1$. Moreover, (19b) guarantees that x^{H^*} and y^{H^*}, together, satisfy the strong duality equation of problem (14) with $\boldsymbol{z}$ fixed to z^* when $\gamma \to 1$. Therefore, x^{H^*} and y^{H^*} approximate a primal and dual optimal solution to problem (14) with $\boldsymbol{z}$ fixed to z^* when $\gamma \to 1$. This implies that x^{H^*} and y^{H^*} become feasible solutions to (11) when $\gamma \to 1$.

Moreover, the combination of (18f) $\times x^{\mathrm{H}^*}$ and (18h) gives

$$\begin{aligned} & \tilde{\boldsymbol{y}}^{\mathbf{H}^\top} \left(b^{\mathrm{H}} - B^{\mathrm{H}} \boldsymbol{z} - A^{\mathrm{H}} x^{\mathrm{H}^*} \right) \\ & \qquad \geq \frac{(1-\gamma)}{\gamma} \left(c^{\mathrm{E}^\top} \boldsymbol{x^{\mathrm{E}}} - \tilde{\boldsymbol{y}}^{\mathbf{E}^\top} \left(b^{\mathrm{E}} - B^{\mathrm{E}} x^{\mathrm{H}^*} \right) \right). && (20) \end{aligned}$$

It follows from (20) and (18d) that, for any gamma $\gamma \in]0, 1[$:

$$\tilde{\boldsymbol{y}}^{\mathbf{E}^\top} \left(b^{\mathrm{E}} - B^{\mathrm{E}} x^{\mathrm{H}^*} \right) \geq c^{\mathrm{E}^\top} \boldsymbol{x^{\mathrm{E}}}. \tag{21}$$

Constraints (18e) and (18g) guarantee that x^{E^*} and y^{E^*} are feasible primal and dual solutions to problem (15) with $\boldsymbol{x}^{\mathrm{H}}$ fixed to x^{H^*}. Additionally, (21) guarantees that x^{E^*} and y^{E^*}, together, satisfy the strong duality equation of problem (15). Therefore, x^{E^*} and y^{E^*} are the primal and dual optimal solutions to problem (15) with $\boldsymbol{x}^{\mathrm{H}}$ fixed to x^{H^*} for any $\gamma \in]0, 1[$.

In summary, x^{H^*} is a feasible solution to (14), which converges towards an optimal solution when $\gamma \to 1$, and y^{E^*} is an optimal dual solution of the lower-level problem for any $\gamma \in]0,1[$. Hence, problem (12) always provides a feasible solution to problem (11), which converges towards the optimal solution when $\gamma \to 1$.