Appendix A: Computation of electricity-aware heat bids for CHPs and HPs

Let us first consider a heat market participants whose marginal heat productions cost can be expressed as affine functions of the day-ahead electricity prices, i.e., such that

$$\dot{\Gamma}^{\mathrm{H}}_{jt} = a_{jt}\boldsymbol{\lambda^{\mathrm{E}}_{zt}} + b_{jt}, \forall j \in \mathcal{I}^{\mathrm{H}}_{z}, t \in \mathcal{T}, \tag{17}$$

with $a_{jt} \in \mathbb{R}, b_{jt} \in \mathbb{R}$ the fixed affine parameters. As a result, the cost-recovery conditions (10) for each electricity-aware heat bid b with price c^{H}_{jbt} rewrite as

$$\left(c^{\mathrm{H}}_{jbt} - M_j\right) \boldsymbol{u^{\mathrm{bid}}_{jbt}} \geq a_{jt}\boldsymbol{\lambda^{\mathrm{E}}_{zt}} + b_{jt} - M_j, \forall j \in \mathcal{I}^{\mathrm{H}}_{z}, t \in \mathcal{T}, b \in \mathcal{B}^{\mathrm{H}}. \tag{18}$$

Furthermore, we assume that in each electricity market zone $z \in \mathcal{Z}^{\mathrm{E}}$, the set of day-ahead electricity prices is bounded, i.e., $\underline{\lambda}^{\mathrm{E}}_{z} \leq \boldsymbol{\lambda^{\mathrm{E}}_{zt}} \leq \overline{\lambda}^{\mathrm{E}}_{z}$. This assumption is without loss of generality, as the bidding prices in electricity market are typically bounded, and bounds on day-ahead electricity prices can be derived from these bounds and the value of lost loads. Therefore, the bounds on electricity prices for each electricity-aware heat bid b, is computed as

$$\underline{\lambda}^{\mathrm{E}}_{jbt} = \begin{cases} \dfrac{c^{\mathrm{H}}_{jbt} - b_{jt}}{a_{jt}}, \text{ if } \; a_{jt} < 0 \\ \underline{\lambda}^{\mathrm{E}}_{z}, \text{ else} \end{cases}, \forall j \in \mathcal{I}^{\mathrm{H}}_{z}, t \in \mathcal{T}, b \in \mathcal{B}^{\mathrm{H}} \tag{19a}$$

$$\overline{\lambda}^{\mathrm{E}}_{jbt} = \begin{cases} \dfrac{c^{\mathrm{H}}_{jbt} - b_{jt}}{a_{jt}}, \text{ if } \; a_{jt} > 0 \\ \overline{\lambda}^{\mathrm{E}}_{z}, \text{ else} \end{cases}, \forall j \in \mathcal{I}^{\mathrm{H}}_{z}, t \in \mathcal{T}, b \in \mathcal{B}^{\mathrm{H}}. \tag{19b}$$

This general expression of the electricity price bounds in electricity-aware bids can directly be applied to HPs, , using the expression of their affine marginal heat production costs provided in (5).

Let us now generalize to heat market participants whose marginal heat production cost can be expressed as a convex piece-wise linear functions of the day-ahead electricity prices, i.e., such that

$$\dot{\Gamma}^{\mathrm{H}}_{jt} = \max_{k=1,\dots,K} a_{jkt}\{\boldsymbol{\lambda^{\mathrm{E}}_{zt}} + b_{jkt}\}, \forall j \in \mathcal{I}^{\mathrm{H}}_{z}, t \in \mathcal{T}, \tag{20}$$

with $k \in \{1, ..., K\}$ the number of affine pieces, and $a_{jkt}, b_{jkt} \in \mathbb{R}$ their fixed affine parameters. In this expression, $\dot{\Gamma}^{\mathrm{H}}_{jt}$ is defined as the (convex) upper envelope of the lines forming the affine pieces. As a result, the cost-recovery conditions (10) for each electricity-aware heat bid b with price c^{H}_{jbt} rewrite as

$$\left(c^{\mathrm{H}}_{jbt} - M_j\right) \boldsymbol{u^{\mathrm{bid}}_{jbt}} \geq a_{jkt}\boldsymbol{\lambda^{\mathrm{E}}_{zt}} + b_{jkt} - M_j, \forall j \in \mathcal{I}^{\mathrm{H}}_{z}, k \in \{1, ..., K\}, t \in \mathcal{T}, b \in \mathcal{B}^{\mathrm{H}}. \tag{21}$$

Therefore, the bounds on electricity prices for each electricity-aware heat bid b, is computed as

$$\underline{\lambda}^{\mathrm{E}}_{jbt} = \begin{cases} \max\left\{\dfrac{c^{\mathrm{H}}_{jbt} - b_{jkt}}{a_{jkt}}, \forall k \in \{1, ..., K\} \mid a_{jkt} < 0\right\}, \text{ if } \exists \; a_{jkt} < 0 \\ \underline{\lambda}^{\mathrm{E}}_{z}, \text{ else} \end{cases}, \forall j \in \mathcal{I}^{\mathrm{H}}_{z}, t \in \mathcal{T}, b \in \mathcal{B}^{\mathrm{H}} \tag{22a}$$

$$\overline{\lambda}^{\mathrm{E}}_{jbt} = \begin{cases} \min\left\{\dfrac{c^{\mathrm{H}}_{jbt} - b_{jkt}}{a_{jkt}}, \forall k \in \{1, ..., K\} \mid a_{jkt} > 0\right\}, \text{ if } \exists \; a_{jkt} > 0 \\ \overline{\lambda}^{\mathrm{E}}_{z}, \text{ else} \end{cases}, \forall j \in \mathcal{I}^{\mathrm{H}}_{z}, t \in \mathcal{T}, b \in \mathcal{B}^{\mathrm{H}}. \tag{22b}$$

This general expression of the electricity price bounds in electricity-aware bids can directly be applied to CHPs, using the expression of their convex piece-wise linear marginal heat production costs provided in (6).

Appendix B: Proofs of Propositions

B.1. Proof of Proposition 1

For a given value of the upper-level variables z^*, the equilibrium formulation of the sequential heat and electricity market-clearing problems can be expressed in a compact form as

$$\boldsymbol{x}^{\mathbf{H}} \in \text{ primal solution of } \min_{\boldsymbol{x}^{\mathbf{H}} \geq \mathbf{0}} \; c^{\mathrm{H}^\top} \boldsymbol{x}^{\mathbf{H}} \tag{23a}$$

$$\text{s.t. } A^{\mathrm{H}} \boldsymbol{x}^{\mathbf{H}} + B^{\mathrm{H}} \boldsymbol{z} \geq b^{\mathrm{H}} \tag{23b}$$

$$\boldsymbol{y}^{\mathbf{E}} \in \text{ dual solution of } \min_{\boldsymbol{x}^{\mathbf{E}} \geq \mathbf{0}} \; c^{\mathrm{E}^\top} \boldsymbol{x}^{\mathbf{E}} \tag{23c}$$

$$\text{s.t. } A^{\mathrm{E}} \boldsymbol{x}^{\mathbf{E}} + B^{\mathrm{E}} \boldsymbol{x}^{\mathbf{H}} \geq b^{\mathrm{E}}, \tag{23d}$$

where (23a)-(23b) represent the heat market-clearing problem, and (23c)-(23d) represent the electricity market-clearing problem. Therefore $\{x^{\mathrm{H}^*}, y^{\mathrm{E}^*}\}$ are optimal solutions to (23) if and only if x^{H^*} is an optimal solution to the heat market-clearing problem (23a)-(23b), and y^{E^*} is an optimal solution to the electricity market-clearing problem (23c)-(23d) with the variables $\boldsymbol{x}^{\mathbf{H}}$ fixed to the values x^{H^*}.

Similarly, for a given value of the upper-level variables z^*, $\{x^{\mathrm{H}^*}, y^{\mathrm{E}^*}\}$ are solutions to the proposed bilevel formulation of the sequential heat and electricity market-clearing problems (15d)-(15g), if and only if x^{H^*} is an optimal solution to the heat market-clearing problem (15d)-(15e), and y^{E^*} is an optimal solution to the electricity market-clearing problem (15f)-(15g) with the variables $\boldsymbol{x}^{\mathbf{H}}$ fixed to the values x^{H^*}.

Therefore, any solutions to the equilibrium problem (23) is equivalent to the bilevel formulation (15d)-(15g), and these two formulations are equivalent.

B.2. Proof of Proposition 2

Firstly, by strong duality of the linear lower-level problem (15f)-(15g), the middle- and lower-level problems (15d)-(15g) are equivalent to

$$\boldsymbol{x}^{\mathbf{H}}, \boldsymbol{y}^{\mathbf{E}} \in \underset{\boldsymbol{x}^{\mathbf{H}}, \boldsymbol{y}^{\mathbf{E}} \geq \mathbf{0}}{\arg\min} \; c^{\mathrm{H}^\top} \boldsymbol{x}^{\mathbf{H}} \tag{24a}$$

$$\text{s.t. } A^{\mathrm{H}} \boldsymbol{x}^{\mathbf{H}} + B^{\mathrm{H}} z^* \geq b^{\mathrm{H}} \tag{24b}$$

$$\boldsymbol{y}^{\mathbf{E}} \in \underset{\boldsymbol{x}^{\mathbf{E}}, \boldsymbol{y}^{\mathbf{E}} \geq \mathbf{0}}{\arg\min} \; c^{\mathrm{E}^\top} \boldsymbol{x}^{\mathbf{E}} \tag{24c}$$

$$\text{s.t. } A^{\mathrm{E}} \boldsymbol{x}^{\mathbf{E}} + B^{\mathrm{E}} \boldsymbol{x}^{\mathbf{H}} \geq b^{\mathrm{E}} \tag{24d}$$

$$\boldsymbol{y}^{\mathbf{E}^\top} A^{\mathrm{E}} \leq c^{\mathrm{E}^\top} \tag{24e}$$

$$\boldsymbol{y}^{\mathbf{E}^\top} \left(B^{\mathrm{E}} - B^{\mathrm{E}} \boldsymbol{x}^{\mathbf{H}}\right) \geq c^{\mathrm{E}^\top} \boldsymbol{x}^{\mathbf{E}}. \tag{24f}$$

Constraint (24e) represents the dual constraints of the lower-level problem (15f)-(15g), i.e., dual feasibility. Constraint (24f) enforces equality of the primal and dual objective values of the lower-level problem (15f)-(15g) at optimality (strong duality).

Furthermore, objective function (24a) and constraints (24b) of the middle-level problem do not depend on the lower-level variables $\boldsymbol{x}^{\mathrm{E}}$ and $\boldsymbol{y}^{\mathrm{E}}$. Therefore, the solutions of middle-level optimization problem are not affected by the solutions of the lower-level problem. Problem (24) can thus be solved in two steps: (i) solve the heat market-clearing problem (24a)-(24b) and obtain the optimal solutions x^{H^*}, (ii) solve the electricity market-clearing problem (24c)-(24f) with the variable $\boldsymbol{x}^{\mathrm{H}}$ fixed to x^{H^*} and obtain the optimal solutions y^{E^*}. Therefore, the middle-level problem (24) can be reformulated using a lexicographic function as

$$\boldsymbol{x}^{\mathbf{H}}, \boldsymbol{y}^{\mathbf{E}} \in \underset{\boldsymbol{x}^{\mathbf{H}}, \boldsymbol{x}^{\mathbf{E}}, \boldsymbol{y}^{\mathbf{E}} \geq \mathbf{0}}{\arg\min} < \quad c^{\mathrm{H}^\top} \boldsymbol{x}^{\mathbf{H}}, \quad c^{\mathrm{E}^\top} \boldsymbol{x}^{\mathbf{E}} > \tag{25a}$$

$$\text{s.t.} \quad A^{\mathrm{H}} \boldsymbol{x}^{\mathbf{H}} + B^{\mathrm{H}} \boldsymbol{z} \geq b^{\mathrm{H}} \tag{25b}$$

$$A^{\mathrm{E}} \boldsymbol{x}^{\mathbf{E}} + B^{\mathrm{E}} \boldsymbol{x}^{\mathbf{H}} \geq b^{\mathrm{E}} \tag{25c}$$

$$\boldsymbol{y}^{\mathbf{E}^\top} A^{\mathrm{E}} \leq c^{\mathrm{E}^\top} \tag{25d}$$

$$\boldsymbol{y}^{\mathbf{E}^\top} \left(B^{\mathrm{E}} - B^{\mathrm{E}} \boldsymbol{x}^{\mathbf{H}} \right) \geq c^{\mathrm{E}^\top} \boldsymbol{x}^{\mathbf{E}}. \tag{25e}$$

Any optimal solution $x^{\mathrm{H}^*}, x^{\mathrm{E}^*}, y^{\mathrm{E}^*}$ of problem (25) satisfies the following properties:

$$x^{\mathrm{H}^*} \in \underset{\boldsymbol{x}^{\mathbf{H}}}{\arg\min} \quad c^{\mathrm{H}^\top} \boldsymbol{x}^{\mathbf{H}} \tag{26a}$$

$$\text{s.t.} \quad A^{\mathrm{H}} \boldsymbol{x}^{\mathbf{H}} + B^{\mathrm{H}} \boldsymbol{z} \geq b^{\mathrm{H}}, \tag{26b}$$

and

$$x^{\mathrm{E}^*}, y^{\mathrm{E}^*} \in \underset{\boldsymbol{x}^{\mathbf{E}}, \boldsymbol{y}^{\mathbf{E}} \geq \mathbf{0}}{\arg\min} \quad c^{\mathrm{E}^\top} \boldsymbol{x}^{\mathbf{E}} \tag{27a}$$

$$\text{s.t.} \quad A^{\mathrm{E}} \boldsymbol{x}^{\mathbf{E}} + B^{\mathrm{E}} x^{\mathrm{H}^*} \geq b^{\mathrm{E}} \tag{27b}$$

$$\boldsymbol{y}^{\mathbf{E}^\top} A^{\mathrm{E}} \leq c^{\mathrm{E}^\top} \tag{27c}$$

$$\boldsymbol{y}^{\mathbf{E}^\top} \left(B^{\mathrm{E}} - B^{\mathrm{E}} x^{\mathrm{H}^*} \right) \geq c^{\mathrm{E}^\top} \boldsymbol{x}^{\mathbf{E}}. \tag{27d}$$

In addition, by strong duality of problem (27), any feasible solution $\hat{x}^{\mathrm{E}}$, $\hat{y}^{\mathrm{E}}$ is optimal. Therefore, $\hat{x}^{\mathrm{E}}$ is an optimal solution of the primal formulation of the lower-level problem (15f)-(15g), such that

$$\hat{x}^{\mathrm{E}} \in \underset{\boldsymbol{x}^{\mathbf{E}} \geq \mathbf{0}}{\arg\min} \quad c^{\mathrm{E}^\top} \boldsymbol{x}^{\mathbf{E}} \tag{28a}$$

$$\text{s.t.} \quad A^{\mathrm{E}} \boldsymbol{x}^{\mathbf{E}} + B^{\mathrm{E}} x^{\mathrm{H}^*} \geq b^{\mathrm{E}}, \tag{28b}$$

and $\hat{y}^{\mathrm{E}}$ is an optimal solution of the dual formulation of the lower-level problem (15f)-(15g), such that

$$\hat{y}^{\mathrm{E}} \in \underset{\boldsymbol{y}^{\mathbf{E}} \geq \mathbf{0}}{\arg\max} \quad \boldsymbol{y}^{\mathbf{E}^\top} \left(B^{\mathrm{E}} - B^{\mathrm{E}} x^{\mathrm{H}^*} \right) \tag{29a}$$

$$\text{s.t.} \quad \boldsymbol{y}^{\mathbf{E}^\top} A^{\mathrm{E}} \leq c^{\mathrm{E}^\top}. \tag{29b}$$

As problem (28) is a relaxation of problem (27) and the set of solutions $\hat{x}^{\mathrm{E}}$, $\hat{y}^{\mathrm{E}}$ is feasible to problem (27), then problem (25) can be approximated by the following linear program, with $\gamma \in]0,1[$:

$$\min_{\boldsymbol{x}^{\mathbf{H}}, \boldsymbol{x}^{\mathbf{E}} \geq \mathbf{0}} \quad \gamma c^{\mathrm{H}^\top} \boldsymbol{x}^{\mathbf{H}} + (1-\gamma) \, c^{\mathrm{E}^\top} \boldsymbol{x}^{\mathbf{E}} \tag{30a}$$

$$\text{s.t.} \quad A^{\mathrm{H}} \boldsymbol{x}^{\mathbf{H}} + B^{\mathrm{H}} \boldsymbol{z} \geq b^{\mathrm{H}} \tag{30b}$$

$$A^{\mathrm{E}} \boldsymbol{x}^{\mathbf{E}} + B^{\mathrm{E}} \boldsymbol{x}^{\mathbf{H}} \geq b^{\mathrm{E}}, \tag{30c}$$

where $\boldsymbol{y}^{\mathbf{E}}$ is obtained as the dual variable associated with constraint (30c) (Byeon and Van Hentenryck 2020). As a result, problem (15) can be approximated by the following linear bilevel optimization problem:

$$\min_{\substack{\boldsymbol{z}\in\{0,1\}^{\mathrm{N}},\boldsymbol{x}^{\mathbf{H}},\boldsymbol{x}^{\mathbf{E}}\geq\mathbf{0}\\ \boldsymbol{y}^{\mathbf{H}},\boldsymbol{y}^{\mathbf{E}}\geq\mathbf{0}}} \gamma {c^0}^\top \boldsymbol{z} + \gamma {c^{\mathrm{H}}}^\top \boldsymbol{x}^{\mathbf{H}} + (1-\gamma)\, {c^{\mathrm{E}}}^\top \boldsymbol{x}^{\mathbf{E}} \tag{31a}$$

$$\text{s.t.} \quad \boldsymbol{z}\in\mathcal{Z}^{\mathrm{UC}} \tag{31b}$$

$$A^{\mathrm{bid}}\boldsymbol{z} + \frac{1}{(1-\gamma)} B^{\mathrm{bid}}\boldsymbol{y}^{\mathbf{E}} \geq b^{\mathrm{bid}} \tag{31c}$$

$$\boldsymbol{x}^{\mathbf{H}}, \boldsymbol{y}^{\mathbf{E}} \text{ primal and dual sol. of (30).} \tag{31d}$$

Besides, by strong duality of the lower-level problem (31d), problem (31) is equivalent to problem (16).

It remains to show that problem (16) is an asymptotic approximation to problem (15), i.e., as $\gamma\to1$ the solutions to problem (16) become optimal solutions to problem (15). By introducing the auxiliary variables $\tilde{\boldsymbol{y}}^{\mathbf{H}} = \frac{\boldsymbol{y}^{\mathrm{H}}}{\gamma}$, and $\tilde{\boldsymbol{y}}^{\mathbf{E}} = \frac{\boldsymbol{y}^{\mathrm{E}}}{1-\gamma}$, problem (16) is equivalent to

$$\min_{\substack{\boldsymbol{z}\in\{0,1\}^{N},\boldsymbol{x}^{\mathbf{H}}\geq\mathbf{0}\\ \boldsymbol{x}^{\mathbf{E}}\geq\mathbf{0},\boldsymbol{y}^{\mathbf{H}},\boldsymbol{y}^{\mathbf{E}}}} \gamma {c^0}^\top \boldsymbol{z} + \gamma {c^{\mathrm{H}}}^\top \boldsymbol{x}^{\mathbf{H}} + (1-\gamma)\, {c^{\mathrm{E}}}^\top \boldsymbol{x}^{\mathbf{E}} \tag{32a}$$

$$\text{s.t.} \quad \boldsymbol{z}\in\mathcal{Z}^{\mathrm{UC}} \tag{32b}$$

$$A^{\mathrm{bid}}\boldsymbol{z} + B^{\mathrm{bid}}\tilde{\boldsymbol{y}}^{\mathbf{E}} \geq b^{\mathrm{bid}} \tag{32c}$$

$$A^{\mathrm{H}}\boldsymbol{x}^{\mathbf{H}} + B^{\mathrm{H}}\boldsymbol{z} \geq b^{\mathrm{H}} \tag{32d}$$

$$A^{\mathrm{E}}\boldsymbol{x}^{\mathbf{E}} + B^{\mathrm{E}}\boldsymbol{x}^{\mathbf{H}} \geq b^{\mathrm{E}} \tag{32e}$$

$${\tilde{\boldsymbol{y}}^{\mathbf{H}}}^\top A^{\mathrm{H}} + \frac{(1-\gamma)}{\gamma} {\tilde{\boldsymbol{y}}^{\mathbf{E}}}^\top B^{\mathrm{E}} \leq {c^{\mathrm{H}}}^\top \tag{32f}$$

$${\tilde{\boldsymbol{y}}^{\mathbf{E}}}^\top A^{\mathrm{E}} \leq {c^{\mathrm{E}}}^\top \tag{32g}$$

$${\tilde{\boldsymbol{y}}^{\mathbf{H}}}^\top \left(b^{\mathrm{H}} - B^{\mathrm{H}}\boldsymbol{z}\right) - {c^{\mathrm{H}}}^\top \boldsymbol{x}^{\mathbf{H}} \geq \frac{(1-\gamma)}{\gamma} \left({c^{\mathrm{E}}}^\top \boldsymbol{x}^{\mathbf{E}} - {\tilde{\boldsymbol{y}}^{\mathbf{E}}}^\top b^{\mathrm{E}}\right). \tag{32h}$$

Let us denote $P(z^*)$ and $\tilde{P}(z^*)$ the optimal objective value of problems (25) and (32), respectively, with the value of the unit commitment variable $\boldsymbol{z}$ fixed to z^*. Let $\left\{x^{\mathrm{H}^*}, x^{\mathrm{E}^*}, y^{\mathrm{H}^*}, y^{\mathrm{E}^*}\right\}$ be the optimal solutions to $\tilde{P}(z^*)$. As $\gamma\to1$, (32f) and (32h) become

$${\tilde{\boldsymbol{y}}^{\mathbf{H}}}^\top A^{\mathrm{H}} \leq {c^{\mathrm{H}}}^\top \tag{33a}$$

$${\tilde{\boldsymbol{y}}^{\mathbf{H}}}^\top \left(b^{\mathrm{H}} - B^{\mathrm{H}}\boldsymbol{z}\right) - \geq {c^{\mathrm{H}}}^\top \boldsymbol{x}^{\mathbf{H}}. \tag{33b}$$

Constraint (32d) guarantees that x^{H^*} is feasible to problem (26) with $\boldsymbol{z}$ fixed to z^*. Additionally, (33a) guarantees that y^{H^*} becomes feasible to problem (26) with $\boldsymbol{z}$ fixed to z^* when $\gamma\to1$. Moreover, (33b) guarantees that x^{H^*} and y^{H^*}, together, satisfy the strong duality equation of problem (26) with $\boldsymbol{z}$ fixed to z^* when $\gamma\to1$. Therefore, x^{H^*} and y^{H^*} approximate the primal and dual optimal solutions to problem (26) with $\boldsymbol{z}$ fixed to z^* when $\gamma\to1$. This implies that x^{H^*} and y^{H^*} become feasible solutions to $P(z^*)$ when $\gamma\to1$.

Moreover, the combination of (32h) and (32f) $\times x^{\mathrm{H}^*}$ gives

$${\tilde{\boldsymbol{y}}^{\mathbf{H}}}^\top \left(b^{\mathrm{H}} - B^{\mathrm{H}}\boldsymbol{z} - A^{\mathrm{H}}x^{\mathrm{H}^*}\right) \geq \frac{(1-\gamma)}{\gamma} \left({c^{\mathrm{E}}}^\top \boldsymbol{x}^{\mathbf{E}} - {\tilde{\boldsymbol{y}}^{\mathbf{E}}}^\top \left(b^{\mathrm{E}} - B^{\mathrm{E}}x^{\mathrm{H}^*}\right)\right). \tag{34}$$

It follows from (32d) that, for any gamma $\gamma \in [0,1]$

$$\tilde{\boldsymbol{y}}^{\mathbf{E}\top}\left(b^{\mathrm{E}} - B^{\mathrm{E}} x^{\mathrm{H}^*}\right) \geq c^{\mathrm{E}^\top} \boldsymbol{x}^{\mathbf{E}}. \tag{35}$$

Constraints (32e) and (32g) guarantee that x^{E^*} and y^{E^*} are feasible to problem (27) with $\boldsymbol{x}^{\mathrm{H}}$ fixed to x^{H^*}. Additionally, (35) guarantees that x^{E^*} and y^{E^*}, together, satisfy the strong duality equation of problem (27) with $\boldsymbol{x}^{\mathrm{H}}$ fixed to x^{H^*}. Therefore, x^{E^*} and y^{E^*} are the primal and dual optimal solutions to problem (27) with $\boldsymbol{x}^{\mathrm{H}}$ fixed to x^{H^*} for any $\gamma \in [0,1]$.

In summary, x^{H^*} is a feasible solution to $P(z^*)$, which converges towards the optimal solution when $\gamma \to 1$, and y^{E^*} is the optimal solution of the lower-level problem with respect to x^{H^*} for any $\gamma \in [0,1]$. Hence, problem (16) always provides a feasible solution to problem (12), which converges towards the optimal solution when $\gamma \to 1$.