Appendix A: Marginal heat production costs of CHPs and HPs

As the heat-to-power ratio of HPs is considered fixed and given by their COP, the marginal heat production cost of HPs can be expressed as a *linear function* of the electricity market prices, such that

$$\dot{\Gamma}_{jt}^{\mathrm{H}} = \frac{\partial \dot{\Gamma}_{jt}^{\mathrm{H}}}{\partial \boldsymbol{Q}_{jt}} = \frac{\boldsymbol{\lambda}^{\mathbf{E}}_{\boldsymbol{zt}}}{\mathrm{COP}_j}, \forall z \in \mathcal{Z}^{\mathrm{E}}, j \in \mathcal{I}_z^{\mathrm{HP}}, t \in \mathcal{T}, \tag{8}$$

Additionally, as the heat-to-power ratio of CHPs is variable, their heat production cost must be computed at the optimal heat-to-power ratio, for a given heat production and expected electricity market prices. Typically, the heat production of CHPs is defined as their total production cost minus revenues from electricity sales, such that

$$\Gamma_{jt}^{\mathrm{H}} = c_j \left(\rho_j^{\mathrm{E}} \boldsymbol{P_{jt}} + \rho_j^{\mathrm{H}} \boldsymbol{Q_{jt}}\right) - \boldsymbol{P_{jt}} \boldsymbol{\lambda}^{\mathbf{E}}_{\boldsymbol{zt}}, \forall z \in \mathcal{Z}^{\mathrm{E}}, j \in \mathcal{I}_z^{\mathrm{HP}}, t \in \mathcal{T}. \tag{9}$$

where $c_j \left(\rho^{\mathrm{E}} \boldsymbol{P_{jt}} + \rho^{\mathrm{H}} \boldsymbol{Q_{jt}}\right)$ represents the total production cost of CHPs as a linear function of their fuel consumption. Note that this linear cost assumption can be relaxed to assume convex piece-wise linear costs, without loss of generality. For low electricity market prices the marginal heat production cost of CHPs represents the incremental heat production cost at the minimum heat-to-power ratio $\left(c_j \left(\rho_j^{\mathrm{H}} + r_j \rho_j^{\mathrm{E}}\right) - r_j \boldsymbol{\lambda}^{\mathbf{E}}_{\boldsymbol{zt}}\right)$, and for high electricity market prices it represents the opportunity loss of producing an extra unit of heat at the maximum heat-to-power ratio $\left(\boldsymbol{\lambda}^{\mathbf{E}}_{\boldsymbol{zt}} \frac{\rho_j^{\mathrm{H}}}{\rho_j^{\mathrm{E}}}\right)$. Therefore, the marginal heat production cost of CHPs can be expressed as a *convex piece-wise linear function* of the electricity prices, such that

$$\dot{\Gamma}_{jt}^{\mathrm{H}} = \max\{\boldsymbol{\lambda}^{\mathbf{E}}_{\boldsymbol{zt}} \frac{\rho_j^{\mathrm{H}}}{\rho_j^{\mathrm{E}}}, c_j \left(\rho_j^{\mathrm{H}} + r_j \rho_j^{\mathrm{E}}\right) - r_j \boldsymbol{\lambda}^{\mathbf{E}}_{\boldsymbol{zt}}\}, \forall z \in \mathcal{Z}^{\mathrm{E}}, j \in \mathcal{I}_z^{\mathrm{CHP}}, t \in \mathcal{T}. \tag{10}$$

Appendix B: Proofs of Propositions

B.1. Proof of Lemma 1

Let us consider a heat market participant $j \in \mathcal{I}_z^{\mathrm{H}}$, whose marginal heat production cost can be expressed as a *convex* piece-wise linear function of the electricity market prices, such that

$$\dot{\Gamma}_{jt}^{\mathrm{H}} = \max_{k=1,\ldots,K} \left\{a_{jkt} \boldsymbol{\lambda}^{\mathbf{E}}_{\boldsymbol{zt}} + b_{jkt}\right\}, \forall j \in \mathcal{I}_z^{\mathrm{H}}, t \in \mathcal{T}, \tag{11}$$

where index $k \in \{1, ..., K\}$ represents the segments of the piece-wise linear function. In addition, $a_{jkt} \in \mathbb{R}$ and $b_{jkt} \in \mathbb{R}$ are parameters. The expression of the segments and parameters in this definition of convex piece-wise linear functions can be computed by the market participants based on their marginal production costs. For instance, using this definition, the heat marginal production cost of CHPs in (10), can be formulated as a piece-wise linear function with two segments, such that

$$\dot{\Gamma}_{jt}^{\mathrm{H}} \left(\boldsymbol{\lambda}^{\mathbf{E}}_{\boldsymbol{zt}}\right) = \max_{k \in \{1,2\}} \left\{a_{jkt} \boldsymbol{\lambda}^{\mathbf{E}}_{\boldsymbol{zt}} + b_{jkt}\right\}, \forall z \in \mathcal{Z}^{\mathrm{E}}, j \in \mathcal{I}_z^{\mathrm{CHP}}, t \in \mathcal{T}, \tag{12}$$

where $a_{j1t} = \frac{\rho_j^{\mathrm{H}}}{\rho_j^{\mathrm{E}}}$, $b_{j1t} = 0$, $a_{j2t} = -r_j$ and $b_{j2t} = c_j \left(\rho_j^{\mathrm{H}} + r_j \rho_j^{\mathrm{E}}\right)$. Note that marginal heat production costs that are affine (or constant) functions of the electricity prices, such as HPs in (8) (or heat-only units) are a straightforward special case of convex piece-wise linear functions with one segment ($k = 1$). Furthermore, costs that are convex quadratic functions of electricity prices can be approximated using a convex piece-wise linear function fitting, for any given number of segments.

As a result, the bid-validity constraints (1) rewrites as

$$\left(c^{\mathrm{H}}_{jbt} - M_j\right) \boldsymbol{u^{\mathrm{bid}}_{jbt}} \geq a_{jkt}\boldsymbol{\lambda^{\mathrm{E}}_{zt}} + b_{jkt} - M_j,\ \forall j \in \mathcal{I}^{\mathrm{H}}_z, k \in \{1,...,K\}, t \in \mathcal{T}, b \in \mathcal{B}^{\mathrm{H}}. \tag{13}$$

This linear formulation of the bid-validity constraints (1) can be expressed in a matrix form as

$$A^{\mathrm{bid}}_j \boldsymbol{u^{\mathrm{bid}}_j} + B^{\mathrm{bid}}_j \boldsymbol{\lambda^{\mathrm{E}}_z} \geq b^{\mathrm{bid}}_j, \quad \forall j \in \mathcal{I}^{\mathrm{H}}_z, \tag{14}$$

where $\boldsymbol{u^{\mathrm{bid}}_j} = \left[\boldsymbol{u^{\mathrm{bid}}_{jbt}}, \forall t \in \mathcal{T}, b \in \mathcal{B}^{\mathrm{H}}\right]$ denotes the vector of selected bids for market participant j. Furthermore, $\boldsymbol{\lambda^{\mathrm{E}}_z} = [\boldsymbol{\lambda^{\mathrm{E}}_{zt}}, \forall t \in \mathcal{T}]$ presents the vector of electricity market prices in zone z, whereas matrices A^{bid}_j, B^{bid}_j and vector b^{bid}_j denote fixed techno-economic parameters of market participant j.

B.2. Proof of Proposition 1

Let us consider a heat market participant $j \in \mathcal{I}^{\mathrm{H}}_z$, whose marginal heat production cost can be expressed as a convex piece-wise linear function of the electricity market prices, such that

$$\dot{\Gamma}^{\mathrm{H}}_{jt} = \max_{k=1,\dots,K} a_{jkt}\{\boldsymbol{\lambda^{\mathrm{E}}_{zt}} + b_{jkt}\}, \forall t \in \mathcal{T}, \tag{15}$$

with $a_{jkt} \in \mathbb{R}$, and $b_{jkt} \in \mathbb{R}$ fixed parameters. As a result, the bid-validity constraints (1) rewrite as

$$\left(c^{\mathrm{H}}_{jbt} - M_j\right) \boldsymbol{u^{\mathrm{bid}}_{jbt}} \geq a_{jkt}\boldsymbol{\lambda^{\mathrm{E}}_{zt}} + b_{jkt} - M_j, \forall j \in \mathcal{I}^{\mathrm{H}}_z, k \in \{1,...,K\}, t \in \mathcal{T}, b \in \mathcal{B}^{\mathrm{H}}. \tag{16}$$

Furthermore, we assume that in each electricity market zone $z \in \mathcal{Z}^{\mathrm{E}}$, the set of electricity market prices is bounded, i.e., $\underline{\lambda}^{\mathrm{E}}_z \leq \boldsymbol{\lambda^{\mathrm{E}}_{zt}} \leq \overline{\lambda}^{\mathrm{E}}_z$. This assumption is without loss of generality, as the bidding prices in electricity market are typically bounded, and bounds on electricity market prices can be derived from these bounds and the value of lost loads.

We define the minimum $\underline{\lambda}^{\mathrm{E}}_{jbt}$ and maximum $\overline{\lambda}^{\mathrm{E}}_{jbt}$ bounds on electricity market prices for each bid b, such that

$$\underline{\lambda}^{\mathrm{E}}_{jbt} = \begin{cases} \max\left\{\dfrac{c^{\mathrm{H}}_{jbt} - b_{jkt}}{a_{jkt}}, \forall k \in \{1,...,K\} \mid a_{jkt} < 0\right\}, \text{ if } \exists\ a_{jkt} < 0 \\ \underline{\lambda}^{\mathrm{E}}_z, \text{ else} \end{cases}, \forall j \in \mathcal{I}^{\mathrm{H}}_z, t \in \mathcal{T}, b \in \mathcal{B}^{\mathrm{H}} \tag{17a}$$

$$\overline{\lambda}^{\mathrm{E}}_{jbt} = \begin{cases} \min\left\{\dfrac{c^{\mathrm{H}}_{jbt} - b_{jkt}}{a_{jkt}}, \forall k \in \{1,...,K\} \mid a_{jkt} > 0\right\}, \text{ if } \exists\ a_{jkt} > 0 \\ \overline{\lambda}^{\mathrm{E}}_z, \text{ else} \end{cases}, \forall j \in \mathcal{I}^{\mathrm{H}}_z, t \in \mathcal{T}, b \in \mathcal{B}^{\mathrm{H}}. \tag{17b}$$

Therefore, bid-validity constraints (16) guarantee that a *necessary condition* for bid b to be selected, i.e., $\boldsymbol{u^{\mathrm{bid}}_{jbt}} = 1$, is that the electricity market price in zone z and time period t is bounded by $\underline{\lambda}^{\mathrm{E}}_{jbt} \leq \boldsymbol{\lambda^{\mathrm{E}}_{zt}} \leq \overline{\lambda}^{\mathrm{E}}_{jbt}$.

B.3. Proof of Proposition 2

For a given value of the upper-level variables z^*, the equilibrium formulation of the sequential heat and electricity market-clearing problems can be expressed in a compact form as

$$\boldsymbol{x^{\mathrm{H}}} \in \text{ primal solution of } \min_{\boldsymbol{x^{\mathrm{H}}} \geq \boldsymbol{0}}\ c^{\mathrm{H}\top} \boldsymbol{x^{\mathrm{H}}} \tag{18a}$$

$$\text{s.t. } A^{\mathrm{H}}\boldsymbol{x^{\mathrm{H}}} + B^{\mathrm{H}}\boldsymbol{z} \geq b^{\mathrm{H}} \tag{18b}$$

$$\boldsymbol{y^{\mathrm{E}}} \in \text{ dual solution of } \min_{\boldsymbol{x^{\mathrm{E}}} \geq \boldsymbol{0}}\ c^{\mathrm{E}\top} \boldsymbol{x^{\mathrm{E}}} \tag{18c}$$

$$\text{s.t. } A^{\mathrm{E}}\boldsymbol{x^{\mathrm{E}}} + B^{\mathrm{E}}\boldsymbol{x^{\mathrm{H}}} \geq b^{\mathrm{E}}, \tag{18d}$$

where (18a)-(18b) represent the heat market-clearing problem, and (18c)-(18d) represent the electricity market-clearing problem. Therefore $\{x^{\mathrm{H}^*}, y^{\mathrm{E}^*}\}$ are optimal solutions to (18) if and only if x^{H^*} is an optimal solution to the heat market-clearing problem (18a)-(18b), and y^{E^*} is an optimal solution to the electricity market-clearing problem (18c)-(18d) with the variables $\boldsymbol{x}^{\mathbf{H}}$ fixed to the values x^{H^*}.

Similarly, for a given value of the upper-level variables z^*, $\{x^{\mathrm{H}^*}, y^{\mathrm{E}^*}\}$ are solutions to the proposed bilevel formulation of the sequential heat and electricity market-clearing problems (6d)-(6g), if and only if x^{H^*} is an optimal solution to the heat market-clearing problem (6d)-(6e), and y^{E^*} is an optimal solution to the electricity market-clearing problem (6f)-(6g) with the variables $\boldsymbol{x}^{\mathbf{H}}$ fixed to the values x^{H^*}.

Therefore, any solutions to the equilibrium problem (18) is equivalent to the bilevel formulation (6d)-(6g), and these two formulations are equivalent.

B.4. Proof of Proposition 3

Firstly, by strong duality of the linear lower-level problem (6f)-(6g), the middle- and lower-level problems (6d)-(6g) are equivalent to

$$\boldsymbol{x}^{\mathbf{H}}, \boldsymbol{y}^{\mathbf{E}} \in \underset{\boldsymbol{x}^{\mathbf{H}}, \boldsymbol{y}^{\mathbf{E}} \geq \mathbf{0}}{\arg\min} \quad c^{\mathrm{H}^\top} \boldsymbol{x}^{\mathbf{H}} \tag{19a}$$

$$\text{s.t.} \quad A^{\mathrm{H}} \boldsymbol{x}^{\mathbf{H}} + \quad B^{\mathrm{H}} z^* \geq b^{\mathrm{H}} \tag{19b}$$

$$\boldsymbol{y}^{\mathbf{E}} \in \underset{\boldsymbol{x}^{\mathbf{E}}, \boldsymbol{y}^{\mathbf{E}} \geq \mathbf{0}}{\arg\min} \; c^{\mathrm{E}^\top} \boldsymbol{x}^{\mathbf{E}} \tag{19c}$$

$$\text{s.t.} \quad A^{\mathrm{E}} \boldsymbol{x}^{\mathbf{E}} + B^{\mathrm{E}} \boldsymbol{x}^{\mathbf{H}} \geq b^{\mathrm{E}} \tag{19d}$$

$$\boldsymbol{y}^{\mathbf{E}^\top} A^{\mathrm{E}} \leq c^{\mathrm{E}^\top} \tag{19e}$$

$$\boldsymbol{y}^{\mathbf{E}^\top} \left(B^{\mathrm{E}} - B^{\mathrm{E}} \boldsymbol{x}^{\mathbf{H}} \right) \geq c^{\mathrm{E}^\top} \boldsymbol{x}^{\mathbf{E}}. \tag{19f}$$

Constraint (19e) represents the dual constraints of the lower-level problem (6f)-(6g), i.e., dual feasibility. Constraint (19f) enforces equality of the primal and dual objective values of the lower-level problem (6f)-(6g) at optimality (strong duality).

Furthermore, objective function (19a) and constraints (19b) of the middle-level problem do not depend on the lower-level variables $\boldsymbol{x}^{\mathrm{E}}$ and $\boldsymbol{y}^{\mathrm{E}}$. Therefore, the solutions of middle-level optimization problem are not affected by the solutions of the lower-level problem. Problem (19) can thus be solved in two steps: (i) solve the heat market-clearing problem (19a)-(19b) and obtain the optimal solutions x^{H^*}, (ii) solve the electricity market-clearing problem (19c)-(19f) with the variable $\boldsymbol{x}^{\mathrm{H}}$ fixed to x^{H^*} and obtain the optimal solutions y^{E^*}. Therefore, the middle-level problem (19) can be reformulated using a lexicographic function as

$$\boldsymbol{x}^{\mathbf{H}}, \boldsymbol{y}^{\mathbf{E}} \in \underset{\boldsymbol{x}^{\mathbf{H}}, \boldsymbol{x}^{\mathbf{E}}, \boldsymbol{y}^{\mathbf{E}} \geq \mathbf{0}}{\arg\min} \; < \quad c^{\mathrm{H}^\top} \boldsymbol{x}^{\mathbf{H}}, \quad c^{\mathrm{E}^\top} \boldsymbol{x}^{\mathbf{E}} > \tag{20a}$$

$$\text{s.t.} \quad A^{\mathrm{H}} \boldsymbol{x}^{\mathbf{H}} + B^{\mathrm{H}} \boldsymbol{z} \geq b^{\mathrm{H}} \tag{20b}$$

$$A^{\mathrm{E}} \boldsymbol{x}^{\mathbf{E}} + B^{\mathrm{E}} \boldsymbol{x}^{\mathbf{H}} \geq b^{\mathrm{E}} \tag{20c}$$

$$\boldsymbol{y}^{\mathbf{E}^\top} A^{\mathrm{E}} \leq c^{\mathrm{E}^\top} \tag{20d}$$

$$\boldsymbol{y}^{\mathbf{E}^\top} \left(B^{\mathrm{E}} - B^{\mathrm{E}} \boldsymbol{x}^{\mathbf{H}} \right) \geq c^{\mathrm{E}^\top} \boldsymbol{x}^{\mathbf{E}}. \tag{20e}$$

Any optimal solution $x^{\mathrm{H}^*}, x^{\mathrm{E}^*}, y^{\mathrm{E}^*}$ of problem (20) satisfies the following properties:

$$x^{\mathrm{H}^*} \in \underset{\boldsymbol{x}^{\mathbf{H}}}{\arg\min} \quad c^{\mathrm{H}^\top} \boldsymbol{x}^{\mathbf{H}} \tag{21a}$$
$$\text{s.t.} \quad A^{\mathrm{H}} \boldsymbol{x}^{\mathbf{H}} + B^{\mathrm{H}} \boldsymbol{z} \geq b^{\mathrm{H}}, \tag{21b}$$

and

$$x^{\mathrm{E}^*}, y^{\mathrm{E}^*} \in \underset{\boldsymbol{x}^{\mathbf{E}}, \boldsymbol{y}^{\mathbf{E}} \geq \mathbf{0}}{\arg\min} \quad c^{\mathrm{E}^\top} \boldsymbol{x}^{\mathbf{E}} \tag{22a}$$
$$\text{s.t.} \quad A^{\mathrm{E}} \boldsymbol{x}^{\mathbf{E}} + B^{\mathrm{E}} x^{\mathrm{H}^*} \geq b^{\mathrm{E}} \tag{22b}$$
$$\boldsymbol{y}^{\mathbf{E}^\top} A^{\mathrm{E}} \leq c^{\mathrm{E}^\top} \tag{22c}$$
$$\boldsymbol{y}^{\mathbf{E}^\top} \left(B^{\mathrm{E}} - B^{\mathrm{E}} x^{\mathrm{H}^*} \right) \geq c^{\mathrm{E}^\top} \boldsymbol{x}^{\mathbf{E}}. \tag{22d}$$

In addition, by strong duality of problem (22), any feasible solution $\hat{x}^{\mathrm{E}}$, $\hat{y}^{\mathrm{E}}$ is optimal. Therefore, $\hat{x}^{\mathrm{E}}$ is an optimal solution of the primal formulation of the lower-level problem (6f)-(6g), such that

$$\hat{x}^{\mathrm{E}} \in \underset{\boldsymbol{x}^{\mathbf{E}} \geq \mathbf{0}}{\arg\min} \quad c^{\mathrm{E}^\top} \boldsymbol{x}^{\mathbf{E}} \tag{23a}$$
$$\text{s.t.} \quad A^{\mathrm{E}} \boldsymbol{x}^{\mathbf{E}} + B^{\mathrm{E}} x^{\mathrm{H}^*} \geq b^{\mathrm{E}}, \tag{23b}$$

and $\hat{y}^{\mathrm{E}}$ is an optimal solution of the dual formulation of the lower-level problem (6f)-(6g), such that

$$\hat{y}^{\mathrm{E}} \in \underset{\boldsymbol{y}^{\mathbf{E}} \geq \mathbf{0}}{\arg\max} \quad \boldsymbol{y}^{\mathbf{E}^\top} \left(B^{\mathrm{E}} - B^{\mathrm{E}} x^{\mathrm{H}^*} \right) \tag{24a}$$
$$\text{s.t.} \quad \boldsymbol{y}^{\mathbf{E}^\top} A^{\mathrm{E}} \leq c^{\mathrm{E}^\top}. \tag{24b}$$

As problem (23) is a relaxation of problem (22) and the set of solutions $\hat{x}^{\mathrm{E}}$, $\hat{y}^{\mathrm{E}}$ is feasible to problem (22), then problem (20) can be approximated by the following linear program, with $\gamma \in]0,1[$:

$$\min_{\boldsymbol{x}^{\mathbf{H}}, \boldsymbol{x}^{\mathbf{E}} \geq \mathbf{0}} \quad \gamma c^{\mathrm{H}^\top} \boldsymbol{x}^{\mathbf{H}} + (1-\gamma)\, c^{\mathrm{E}^\top} \boldsymbol{x}^{\mathbf{E}} \tag{25a}$$
$$\text{s.t.} \quad A^{\mathrm{H}} \boldsymbol{x}^{\mathbf{H}} + B^{\mathrm{H}} \boldsymbol{z} \geq b^{\mathrm{H}} \tag{25b}$$
$$A^{\mathrm{E}} \boldsymbol{x}^{\mathbf{E}} + B^{\mathrm{E}} \boldsymbol{x}^{\mathbf{H}} \geq b^{\mathrm{E}}, \tag{25c}$$

where $\boldsymbol{y}^{\mathbf{E}}$ is obtained as the dual variable associated with constraint (25c) (Byeon and Van Hentenryck 2020). As a result, problem (6) can be approximated by the following linear bilevel optimization problem:

$$\min_{\substack{\boldsymbol{z} \in \{0,1\}^{\mathrm{N}}, \boldsymbol{x}^{\mathbf{H}}, \boldsymbol{x}^{\mathbf{E}} \geq \mathbf{0} \\ \boldsymbol{y}^{\mathbf{H}}, \boldsymbol{y}^{\mathbf{E}} \geq \mathbf{0}}} \quad \gamma c^{0^\top} \boldsymbol{z} + \gamma c^{\mathrm{H}^\top} \boldsymbol{x}^{\mathbf{H}} + (1-\gamma)\, c^{\mathrm{E}^\top} \boldsymbol{x}^{\mathbf{E}} \tag{26a}$$
$$\text{s.t.} \quad \boldsymbol{z} \in \mathcal{Z}^{\mathrm{UC}} \tag{26b}$$
$$A^{\mathrm{bid}} \boldsymbol{z} + \frac{1}{(1-\gamma)} B^{\mathrm{bid}} \boldsymbol{y}^{\mathbf{E}} \geq b^{\mathrm{bid}} \tag{26c}$$
$$\boldsymbol{x}^{\mathbf{H}}, \boldsymbol{y}^{\mathbf{E}} \text{ primal and dual sol. of (25).} \tag{26d}$$

Besides, by strong duality of the lower-level problem (26d), problem (26) is equivalent to problem (7).

It remains to show that problem (7) is an asymptotic approximation to problem (6), i.e., as $\gamma \to 1$ the solutions to problem (7) become optimal solutions to problem (6). By introducing the auxiliary variables $\tilde{\boldsymbol{y}}^{\mathbf{H}} = \dfrac{\boldsymbol{y}^{\mathrm{H}}}{\gamma}$, and $\tilde{\boldsymbol{y}}^{\mathbf{E}} = \dfrac{\boldsymbol{y}^{\mathrm{E}}}{1-\gamma}$, problem (7) is equivalent to

$$\min_{\substack{\boldsymbol{z}\in\{0,1\}^N, \boldsymbol{x}^{\mathbf{H}}\geq\mathbf{0} \\ \boldsymbol{x}^{\mathbf{E}}\geq\mathbf{0}, \boldsymbol{y}^{\mathbf{H}}, \boldsymbol{y}^{\mathbf{E}}}} \gamma c^{0^\top}\boldsymbol{z} + \gamma c^{\mathrm{H}^\top}\boldsymbol{x}^{\mathbf{H}} + (1-\gamma)\, c^{\mathrm{E}^\top}\boldsymbol{x}^{\mathbf{E}} \tag{27a}$$

$$\text{s.t.} \quad \boldsymbol{z} \in \mathcal{Z}^{\mathrm{UC}} \tag{27b}$$

$$A^{\mathrm{bid}}\boldsymbol{z} + B^{\mathrm{bid}}\tilde{\boldsymbol{y}}^{\mathbf{E}} \geq b^{\mathrm{bid}} \tag{27c}$$

$$A^{\mathrm{H}}\boldsymbol{x}^{\mathbf{H}} + B^{\mathrm{H}}\boldsymbol{z} \geq b^{\mathrm{H}} \tag{27d}$$

$$A^{\mathrm{E}}\boldsymbol{x}^{\mathbf{E}} + B^{\mathrm{E}}\boldsymbol{x}^{\mathbf{H}} \geq b^{\mathrm{E}} \tag{27e}$$

$$\tilde{\boldsymbol{y}}^{\mathbf{H}^\top} A^{\mathrm{H}} + \frac{(1-\gamma)}{\gamma}\tilde{\boldsymbol{y}}^{\mathbf{E}^\top} B^{\mathrm{E}} \leq c^{\mathrm{H}^\top} \tag{27f}$$

$$\tilde{\boldsymbol{y}}^{\mathbf{E}^\top} A^{\mathrm{E}} \leq c^{\mathrm{E}^\top} \tag{27g}$$

$$\tilde{\boldsymbol{y}}^{\mathbf{H}^\top}\left(b^{\mathrm{H}} - B^{\mathrm{H}}\boldsymbol{z}\right) - c^{\mathrm{H}^\top}\boldsymbol{x}^{\mathbf{H}} \geq \frac{(1-\gamma)}{\gamma}\left(c^{\mathrm{E}^\top}\boldsymbol{x}^{\mathbf{E}} - \tilde{\boldsymbol{y}}^{\mathbf{E}^\top} b^{\mathrm{E}}\right). \tag{27h}$$

Let us denote $P(z^*)$ and $\tilde{P}(z^*)$ the optimal objective value of problems (20) and (27), respectively, with the value of the unit commitment variable $\boldsymbol{z}$ fixed to z^*. Let $\left\{x^{\mathrm{H}^*}, x^{\mathrm{E}^*}, y^{\mathrm{H}^*}, y^{\mathrm{E}^*}\right\}$ be the optimal solutions to $\tilde{P}(z^*)$. As $\gamma \to 1$, (27f) and (27h) become

$$\tilde{\boldsymbol{y}}^{\mathbf{H}^\top} A^{\mathrm{H}} \leq c^{\mathrm{H}^\top} \tag{28a}$$

$$\tilde{\boldsymbol{y}}^{\mathbf{H}^\top}\left(b^{\mathrm{H}} - B^{\mathrm{H}}\boldsymbol{z}\right) - \geq c^{\mathrm{H}^\top}\boldsymbol{x}^{\mathbf{H}}. \tag{28b}$$

Constraint (27d) guarantees that x^{H^*} is feasible to problem (21) with $\boldsymbol{z}$ fixed to z^*. Additionally, (28a) guarantees that y^{H^*} becomes feasible to problem (21) with $\boldsymbol{z}$ fixed to z^* when $\gamma \to 1$. Moreover, (28b) guarantees that x^{H^*} and y^{H^*}, together, satisfy the strong duality equation of problem (21) with $\boldsymbol{z}$ fixed to z^* when $\gamma \to 1$. Therefore, x^{H^*} and y^{H^*} approximate the primal and dual optimal solutions to problem (21) with $\boldsymbol{z}$ fixed to z^* when $\gamma \to 1$. This implies that x^{H^*} and y^{H^*} become feasible solutions to $P(z^*)$ when $\gamma \to 1$.

Moreover, the combination of (27h) and (27f) $\times x^{\mathrm{H}^*}$ gives

$$\tilde{\boldsymbol{y}}^{\mathbf{H}^\top}\left(b^{\mathrm{H}} - B^{\mathrm{H}}\boldsymbol{z} - A^{\mathrm{H}}x^{\mathrm{H}^*}\right) \geq \frac{(1-\gamma)}{\gamma}\left(c^{\mathrm{E}^\top}\boldsymbol{x}^{\mathbf{E}} - \tilde{\boldsymbol{y}}^{\mathbf{E}^\top}\left(b^{\mathrm{E}} - B^{\mathrm{E}}x^{\mathrm{H}^*}\right)\right). \tag{29}$$

It follows from (27d) that, for any gamma $\gamma \in [0,1]$

$$\tilde{\boldsymbol{y}}^{\mathbf{E}^\top}\left(b^{\mathrm{E}} - B^{\mathrm{E}}x^{\mathrm{H}^*}\right) \geq c^{\mathrm{E}^\top}\boldsymbol{x}^{\mathbf{E}}. \tag{30}$$

Constraints (27e) and (27g) guarantee that x^{E^*} and y^{E^*} are feasible to problem (22) with $\boldsymbol{x}^{\mathrm{H}}$ fixed to x^{H^*}. Additionally, (30) guarantees that x^{E^*} and y^{E^*}, together, satisfy the strong duality equation of problem (22) with $\boldsymbol{x}^{\mathrm{H}}$ fixed to x^{H^*}. Therefore, x^{E^*} and y^{E^*} are the primal and dual optimal solutions to problem (22) with $\boldsymbol{x}^{\mathrm{H}}$ fixed to x^{H^*} for any $\gamma \in [0,1]$.

In summary, x^{H^*} is a feasible solution to $P(z^*)$, which converges towards the optimal solution when $\gamma \to 1$, and y^{E^*} is the optimal solution of the lower-level problem with respect to x^{H^*} for any $\gamma \in [0,1]$. Hence, problem (7) always provides a feasible solution to problem (3), which converges towards the optimal solution when $\gamma \to 1$.