Heat and Electricity Market Coordination: A Scalable Complementarity Approach - Online Appendix

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Abstract

In this appendix, we present the reformulation of the electricity-aware heat market-clearing model introduced in Section 5.2 of the supporting paper into a single-level optimization problem. We then describe the steps of the proposed augmented regularized Benders algorithm introduced in Section 5.4. Finally, we provide the input data for the case study in Section 6 of the supporting paper.

Keywords: OR in energy; Integrated energy system; Stochastic programming; Hierarchical optimization; Regularized Benders decomposition

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1. Nomenclature

Sets and Indexes

$\mathcal{T}$ Set of time periods in day-ahead market, i.e. 24 hours of the following day

$\mathcal{X}$ Set of day-ahead scenarios

$\mathcal{I}^{\mathbf{H}}$ Set of heat market participants

$\mathcal{I}^{\mathbf{HO}}$ Set of heat-only units

$\mathcal{I}^{\mathbf{HS}}$ Set of heat storage tanks

$\mathcal{I}^{\mathbf{CHP}}$ Set of CHPs

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$\mathcal{I}^{\mathbf{HP}}$ Set of HPs

$\mathcal{I}^{\mathbf{E}}$ Set of thermal power plants and wind producers

$\Omega^{\mathbf{DA}}$ Set of day-ahead heat dispatch variables

$\Omega_{\nu}^{\mathbf{LL}}$ Set of primal optimization variables of day-ahead electricity market for underlying scenario ν

$\Xi_{\nu}^{\mathbf{LL}}$ Set of dual optimization variables of day-ahead electricity market for underlying scenario ν

$\Omega_{\nu}^{\mathbf{R}}$ Set of heat redispatch variables for underlying scenario ν

$\Omega_{\nu}^{\mathbf{Int}}$ Set of optimization variables of integrated heat and electricity market for underlying scenario ν

$\Omega^{\mathbf{UL}}$ Set of optimization variables of the upper-level problem in the electricity-aware heat market-clearing

Input Parameters

π_{ν} Probability of scenario ν

$\overline{Q}_j$ Maximum heat output of CHPs, HPs, heat-only, and heat storage tanks (MW)

$\rho_j^{\mathbf{H}}$,$\rho_j^{\mathbf{E}}$ Heat and electricity fuel efficiency of CHP

r_j Heat to electricity ratio of CHP

$\overline{F}_j$ Maximum fuel consumption of CHP (MW)

$\mathbf{COP}_j$ Coefficient of performance of heat pump

$\overline{S}_j$ Maximum heat stored in heat storage tanks (MWh)

$\underline{S}_j$ Minimum heat stored in heat storage tanks (MWh)

$S_j^{\mathbf{init}}$ Initial heat stored in heat storage tanks (MWh)

ρ_j^{-},ρ_j^{+} Heat storage tanks charging and discharging efficiencies

l_j Heat storage losses (MWh)

$L_{t\nu}^{\mathbf{E}}$ Electricity load scenario in day-ahead market (MW)

$L_t^{\mathbf{H}}$ Heat load in day-ahead market (MW)

$\overline{P}_{jt\nu}$ Maximum power output of conventional generators and wind producers (MW)

$\underline{\alpha}$ Minimum price offer in the day-ahead electricity market, i.e., -500EUR/MWh in the Nordpool electricity market

$\overline{\alpha}$ Maximum price offer in the day-ahead electricity market, i.e., 3000EUR/MWh in the Nordpool electricity market

α_j Marginal cost parameter of CHPs and heat-only units (EUR/MWh)

$\tilde{\alpha}^{\mathbf{E}}_{jt\nu}$ Marginal price offer of conventional generators and wind producers in day-ahead electricity market (EUR/MWh)

$\alpha^{\mathbf{E}}_j$ Electricity marginal cost of CHPs (EUR/MWh)

$\alpha^{\uparrow}_j$,$\alpha^{\downarrow}_j$ Up and down redispatch costs of CHPs, HPs, and heat-only units (EUR/MWh)

$\hat{\lambda}^{\mathbf{E}}_t$ Forecast electricity price (EUR/MWh)

Decision variables

Q_{jt} Day-ahead heat dispatch of CHPs, HPs, and heat-only units (MWh)

Q^{-}_{jt},Q^{+}_{jt} Charging and discharging of heat storage tanks in day-ahead market (MWh)

S_{jt} Heat stored in heat storage tanks in day-ahead market (MWh)

$Q^{\uparrow}_{jt\nu}$,$Q^{\downarrow}_{jt\nu}$ Upward and downward heat production adjustment of CHPs and heat-only units (MWh)

$Q_{jt\nu}$ Heat production of CHPs and heat-only units after redispatch (MWh)

$Q^{+}_{jt\nu}$,$Q^{-}_{jt\nu}$ Charging and discharging of heat storage tanks after redispatch (MWh)

$S_{jt\nu}$ Heat stored in heat storage tanks after redispatch (MWh)

$P_{jt\nu}$ Day-ahead electricity dispatch of conventional generators, wind producers, and CHPs (MWh)

$P^0_{jt\nu}$ Electricity production of CHPs below $\underline{P}_j(Q_{jt})$ (MWh)

$P^+_{jt\nu}$ Electricity production of CHPs over $\underline{P}_j(Q_{jt})$ (MWh)

$L^{\mathbf{HP}}_{jt\nu}$ Electricity consumption of HP (MWh)

$\underline{\mu}_{jt\nu}$,$\overline{\mu}_{jt\nu}$,$\underline{\mu}^0_{jt\nu}$,$\overline{\mu}^0_{jt\nu}$ Dual variables of the lower-level problems

$\lambda^{\mathbf{E}}_{t\nu}$ Day-ahead electricity prices

$\lambda^{\mathbf{H}}_{t\nu}$ Day-ahead heat prices

Functions

$\Gamma_j(.)$ Total production cost of CHPs, depending on electricity prices, heat and electricity production (EUR)

$\Gamma^{\mathbf{H}}_j(.)$ Expected heat cost of CHPs and HPs, depending on electricity prices, heat and electricity production (EUR)

$\alpha_j^{\mathbf{H}}(.)$ Expected heat marginal cost of CHPs and HPs, depending on electricity prices (EUR/MWh)

$\overline{P}_j(.)$ Maximum power output of CHPs depending on heat output (MW)

$\underline{P}_j(.)$ Minimum power output of CHPs depending on heat output (MW)

$\underline{L}_j^{\mathbf{HP}}(.)$ Minimum power consumption of HPs depending on heat output (MW)

Benders algorithm indexes, variables and parameters

θ Iterations of Benders algorithm, in $\{1, ..., \theta^{\max}\}$

$\Omega^{\mathbf{MP}}$ Set of optimization variables of master problem

$\Omega_\nu^{\mathbf{SUB}}$ Set of optimization variables of subproblem SUB_ν for underlying scenario ν

ϵ Positive tolerance parameter

$\eta_{jt\nu}^{Q,(\theta)}$ Sensitivity of subproblem SUB_ν with respect to complicating variable Q_{jt} at iteration θ

$\eta_{jt\nu}^{S,(\theta)}$ Sensitivity of subproblem SUB_ν with respect to complicating variable S_{jt} at iteration θ

$\eta_{jt\nu}^{Q^+,(\theta)}$ Sensitivity of subproblem SUB_ν with respect to complicating variable Q_{jt}^+ at iteration θ

$\eta_{jt\nu}^{Q^-,(\theta)}$ Sensitivity of subproblem SUB_ν with respect to complicating variable Q_{jt}^- at iteration θ

$z_\nu^{\mathbf{SUB},(\theta)}$ Objective value of SUB_ν at iteration θ

$z^{\mathbf{MP},(\theta)}$ Objective value of master problem at iteration θ

$\mathbf{UB}^{(\theta)}$ Value of upper bound at iteration θ

$\mathbf{LB}^{(\theta)}$ Value of lower bound at iteration θ

$X^{(\mathbf{ref})}$ Reference point

$\tau^{(\mathbf{ref})}$ Penalization parameter

m Regularization parameter $(0 < m < 1)$

X Vector of complicating variables

β Decision variable of master problem

2. Sequential heat market-clearing formulation

In this section, we present a sequential market framework for heat and electricity dispatch, inspired by the Danish energy system, and we illustrate the basic features of this market framework using the illustrative example introduced in Section 5.1. CHPs and HPs first participate in the day-ahead heat market (step 1), by computing their heat marginal cost based on expected day-ahead electricity prices. Once the heat system has been dispatched, CHPs can participate in the day-ahead electricity market (step 2), by computing their minimum and maximum electricity production based on their heat dispatch. Finally, after the day-ahead electricity market clearing CHPs' heat and electricity outputs might not be in their FOR. Hence, a heat redispatch (step 3) might be performed.

The heat market operator seeks to minimize the day-ahead heat production cost based on the reported expected heat marginal costs $\alpha_j^{\mathrm{H}}\left(\hat{\lambda}_t^{\mathrm{E}}\right)$ of CHPs and HPs over the set of optimization variables $\Omega^{\mathrm{DA}} = \left\{Q_{jt}, S_{jt}, Q_{jt}^{+}, Q_{jt}^{-}\right\}$, including the day-ahead heat dispatch Q_{jt} of CHPs, HPs, and heat-only units, the state of charge S_{jt}, charge Q_{jt}^{-}, and discharge Q_{jt}^{+} variables of the centrally-operated heat storage tanks. The objective of this optimization problem is to minimize the day-ahead heat production cost (1a), subject to hourly heat balance (1b), heat production bounds (1c), and operating constraints of heat-storage tanks (1d)-(1g). The day-ahead heat market optimization problem can be formulated as follows:

$$\min_{\Omega^{\mathrm{DA}}} \sum_{t\in\mathcal{T}} \left[\sum_{j\in\mathcal{I}^{\mathrm{HO}}} \alpha_j Q_{jt} + \sum_{j\in\mathcal{I}^{\mathrm{HP}}\cup\mathcal{I}^{\mathrm{CHP}}} \alpha_j^{\mathrm{H}}\left(\hat{\lambda}_t^{\mathrm{E}}\right) Q_{jt} \right] \tag{1a}$$

$$\text{s.t.} \quad L_t^{\mathrm{H}} = \sum_{j\in\mathcal{I}^{\mathrm{HP}}\cup\mathcal{I}^{\mathrm{CHP}}\cup\mathcal{I}^{\mathrm{HO}}} Q_{jt} + \sum_{j\in\mathcal{I}^{HS}} \left(Q_{jt}^{+} - Q_{jt}^{-}\right), \ \forall t\in\mathcal{T} \tag{1b}$$

$$0 \le Q_{jt} \le \overline{Q}_j, \ \forall j \in \mathcal{I}^{\mathrm{HO}} \cup \mathcal{I}^{\mathrm{HP}} \cup \mathcal{I}^{\mathrm{CHP}}, t\in\mathcal{T} \tag{1c}$$

$$S_{jt} = S_{j(t-1)} + \rho_j^{-} Q_{jt}^{-} - \rho_j^{+} Q_{jt}^{+} - l_j, \ \forall j\in\mathcal{I}^{HS}, t\in\mathcal{T} \tag{1d}$$

$$0 \le Q_{jt}^{-} \le \overline{Q}_j, \ 0 \le Q_{jt}^{+} \le \overline{Q}_j, \ \forall j\in\mathcal{I}^{HS}, t\in\mathcal{T} \tag{1e}$$

$$\underline{S}_j \le S_{jt} \le \overline{S}_j, \ \forall j\in\mathcal{I}^{HS}, t\in\mathcal{T} \tag{1f}$$

$$S_{j(t=|\mathcal{T}|)} \ge S_j^{\mathrm{init}}, \ \forall j\in\mathcal{I}^{HS}. \tag{1g}$$

Note that, day-ahead heat prices should not be directly derived as the dual variables of the heat balance equations (1b). Indeed, heat marginal costs and heat prices depend on the realization of electricity prices. In practice, heat prices are calculated ex-post and fixed in bilateral contracts. The centrally operated heat storage units can be heat producers ($Q_{jt}^{+} > 0$) when discharging, or consumers ($Q_{jt}^{-} > 0$) when charging. Although (1d) allows simultaneous charging and discharging, this type of operation is not optimal when prices are positive due to the charging ($0 < \rho_j^{-} < 1$) and discharging ($\rho_j^{+} > 1$) loss coefficients. Additionally, a constraint on the charging state of the storage in the last time period is imposed in (1g), so that it is not discharged completely over the optimization period. Alternative approaches could be used to model the final state of charge, in the form of hard and soft constraints, or modification of the objective function.

3. Integrated heat and electricity market-clearing formulation

An optimal approach to couple heat and electricity systems and to fully exploit the flexibility of CHPs and HPs in the day-ahead is to co-dispatch heat and electricity sources (Chen et al., 2015). In this section, we present a fully integrated heat and electricity market framework, and we discuss its benefits using the illustrative example introduced in Section 5.1.

The aim of the integrated day-ahead market operator is to minimize the total day-ahead production cost, by co-dispatching heat and electricity sources. The set of optimization variables $\Omega_\nu^{\mathrm{Int}} = \left\{P_{jt\nu}, Q_{jt\nu}, L_{jt\nu}^{\mathrm{HP}}, S_{jt\nu}, Q_{jt\nu}^{+}, Q_{jt\nu}^{-}\right\}$ includes the day-ahead electricity production $P_{jt\nu}$ of electricity generators and CHPs, the electricity consumption $L_{jt\nu}^{\mathrm{HP}}$ of HPs, the heat production $Q_{jt\nu}$ of CHPs, HPs, and heat-only units, the state of charge $S_{jt\nu}$, charge $Q_{jt\nu}^{-}$, and discharge $Q_{jt\nu}^{+}$ variables of heat storage tanks. CHPs participate in this integrated market by offering their total production cost. Additionally, CHPs and HPs communicate their joint FOR, allowing for the market operator to fully exploit this operational flexibility. The objective of this optimization problem is to minimize the total day-ahead production cost (2a), subject to hourly electricity and heat balance (2b)-(2c), production bounds (2d)-(2e), HPs' and CHPs' FOR constraints (2f)-(2j), and heat storage tanks' operating constraints (2k)-(2n). For a given realization of the uncertainty in the power system $\nu \in \mathcal{X}$, the integrated market can be formulated as follows:

$$\min_{\Omega_\nu^{Int}} \sum_{j\in\mathcal{I}^{\mathrm{CHP}},t\in\mathcal{T}} \alpha_j \left(\rho_j^{\mathrm{E}} P_{jt\nu} + \rho_j^{\mathrm{H}} Q_{jt\nu}\right) + \sum_{j\in\mathcal{I}^{\mathrm{HO}},t\in\mathcal{T}} \alpha_j Q_{jt\nu} + \sum_{j\in\mathcal{I}^{\mathrm{E}},t\in\mathcal{T}} \tilde{\alpha}_{jt\nu}^{\mathrm{E}} P_{jt\nu} \tag{2a}$$

$$\text{s.t.} \quad L_{t\nu}^{\mathrm{E}} + \sum_{j\in\mathcal{I}^{\mathrm{HP}}} L_{jt\nu}^{\mathrm{HP}} = \sum_{j\in\mathcal{I}^{\mathrm{E}}\cup\mathcal{I}^{\mathrm{CHP}}} P_{jt\nu} : \lambda_{t\nu}^{\mathrm{E}},\ \forall t\in\mathcal{T} \tag{2b}$$

$$L_t^{\mathrm{H}} = \sum_{j\in\mathcal{I}^{\mathrm{HP}}\cup\mathcal{I}^{\mathrm{CHP}}\cup\mathcal{I}^{\mathrm{HO}}} Q_{jt\nu} + \sum_{j\in\mathcal{I}^{HS}} \left(Q_{jt\nu}^{+} - Q_{jt\nu}^{-}\right) : \lambda_{t\nu}^{\mathrm{H}},\ \forall t\in\mathcal{T} \tag{2c}$$

$$0 \le P_{jt\nu} \le \overline{P}_j,\ \forall j\in\mathcal{I}^{\mathrm{E}}, t\in\mathcal{T}, \tag{2d}$$

$$0 \le Q_{jt\nu} \le \overline{Q}_j,\ \forall j\in\mathcal{I}^{\mathrm{HO}}, t\in\mathcal{T} \tag{2e}$$

$$P_{jt\nu} \ge r_j Q_{jt\nu},\ \forall j\in\mathcal{I}^{\mathrm{CHP}}, t\in\mathcal{T}, \nu\in\mathcal{X} \tag{2f}$$

$$0 \le Q_{jt\nu} \le \overline{Q}_j,\ \forall j\in\mathcal{I}^{\mathrm{CHP}}, t\in\mathcal{T}, \nu\in\mathcal{X} \tag{2g}$$

$$\rho_j^{\mathrm{H}} Q_{jt\nu} + \rho_j^{\mathrm{E}} P_{jt\nu} \le \overline{F}_j,\ \forall j\in\mathcal{I}^{\mathrm{CHP}}, t\in\mathcal{T}, \nu\in\mathcal{X} \tag{2h}$$

$$Q_{jt\nu} = \mathrm{COP}_j L_{jt\nu}^{\mathrm{HP}},\ \forall j\in\mathcal{I}^{\mathrm{HP}}, t\in\mathcal{T}, \nu\in\mathcal{X} \tag{2i}$$

$$0 \le Q_{jt\nu} \le \overline{Q}_j,\ \forall j\in\mathcal{I}^{\mathrm{HP}}, t\in\mathcal{T}, \nu\in\mathcal{X} \tag{2j}$$

$$S_{jt\nu} = S_{j(t-1)\nu} + \rho_j^{-} Q_{jt\nu}^{-} - \rho_j^{+} Q_{jt\nu}^{+} - l_j,\ \forall j\in\mathcal{I}^{HS}, t\in\mathcal{T} \tag{2k}$$

$$\underline{S}_j \le S_{jt\nu} \le \overline{S}_j,\ \forall j\in\mathcal{I}^{HS}, t\in\mathcal{T} \tag{2l}$$

$$0 \le Q_{jt\nu}^{-} \le \overline{Q}_j,\ 0 \le Q_{jt\nu}^{+} \le \overline{Q}_j,\ \forall j\in\mathcal{I}^{HS}, t\in\mathcal{T} \tag{2m}$$

$$S_{j(t=|\mathcal{T}|)\nu} \ge S_j^{init},\ \forall j\in\mathcal{I}^{HS}. \tag{2n}$$

Electricity and heat day-ahead prices are defined as the dual variables of the electricity and heat balance equations (2b) and (2c).

4. Single-level reformulations of electricity-aware heat market-clearing

The electricity-aware heat market-clearing model introduced in Section 5.2 of the supporting paper cannot be solved directly by traditional solvers. In this section, we introduce various methods to reformulate this hierarchical (bilevel) optimization problem as single-level optimization problem.

4.1. Mathematical Problem with Equilibrium Constraints (MPEC)

The hierarchical optimization problem (17) in the supporting paper can be reformulated as a single-level optimization problem by replacing each linear lower-level optimization problem LL_ν by its equivalent Karush-Khun-Tucker (KKT) conditions:

$$\alpha_{jt\nu} + \overline{\mu}_{t\nu} - \underline{\mu}_{jt\nu} - \lambda^{\mathrm{E}}_{t\nu} = 0,\ \forall j \in \mathcal{I}^{\mathrm{E}}, t \in \mathcal{T} \tag{3a}$$

$$\underline{\alpha} + \overline{\mu}^{0}_{t\nu} - \underline{\mu}^{0}_{jt\nu} - \lambda^{\mathrm{E}}_{t\nu} = 0,\ \forall j \in \mathcal{I}^{\mathrm{CHP}}, t \in \mathcal{T} \tag{3b}$$

$$\alpha_{jt\nu}\rho^{\mathrm{E}}_{j} + \overline{\mu}_{t\nu} - \underline{\mu}_{jt\nu} - \lambda^{\mathrm{E}}_{t\nu} = 0,\ \forall j \in \mathcal{I}^{\mathrm{CHP}}, t \in \mathcal{T} \tag{3c}$$

$$-\overline{\alpha} + \overline{\mu}_{t\nu} - \underline{\mu}_{jt\nu} + \lambda^{\mathrm{E}}_{t\nu} = 0,\ \forall j \in \mathcal{I}^{\mathrm{HP}}, t \in \mathcal{T} \tag{3d}$$

$$L^{\mathrm{E}}_{t\nu} + \sum_{j\in\mathcal{I}^{\mathrm{HP}}} L^{\mathrm{HP}}_{jt\nu} = \sum_{j\in\mathcal{I}^{\mathrm{E}}} P_{jt\nu} + \sum_{j\in\mathcal{I}^{\mathrm{CHP}}} \left(P^{0}_{jt\nu} + P^{+}_{jt\nu}\right)\ :\lambda^{\mathrm{E}}_{t\nu},\ \forall t \in \mathcal{T} \tag{3e}$$

$$0 \leq \underline{\mu}_{jt\nu} \perp P_{jt\nu} \geq 0,\ \forall j \in \mathcal{I}^{\mathrm{E}}, t \in \mathcal{T} \tag{3f}$$

$$0 \leq \overline{\mu}_{jt\nu} \perp \left(\overline{P}_{jt\nu} - P_{jt\nu}\right) \geq 0,\ \forall j \in \mathcal{I}^{\mathrm{E}}, t \in \mathcal{T} \tag{3g}$$

$$0 \leq \underline{\mu}^{0}_{jt\nu} \perp P^{0}_{jt\nu} \geq 0,\ \forall j \in \mathcal{I}^{\mathrm{CHP}}, t \in \mathcal{T} \tag{3h}$$

$$0 \leq \overline{\mu}^{0}_{jt\nu} \perp \left(\underline{P}_{j}\left(Q_{jt}\right) - P^{0}_{jt\nu}\right) \geq 0,\ \forall j \in \mathcal{I}^{\mathrm{CHP}}, t \in \mathcal{T} \tag{3i}$$

$$0 \leq \underline{\mu}_{jt\nu} \perp P^{+}_{jt\nu} \geq 0,\ \forall j \in \mathcal{I}^{\mathrm{CHP}}, t \in \mathcal{T} \tag{3j}$$

$$0 \leq \overline{\mu}_{jt\nu} \perp \left(\overline{P}_{j}\left(Q_{jt}\right) - \underline{P}_{j}\left(Q_{jt}\right) - P^{+}_{jt\nu}\right) \geq 0,\ \forall j \in \mathcal{I}^{\mathrm{CHP}}, t \in \mathcal{T} \tag{3k}$$

$$0 \leq \underline{\mu}_{jt\nu} \perp L^{\mathrm{HP}}_{jt\nu} \geq 0,\ \forall j \in \mathcal{I}^{\mathrm{HP}}, t \in \mathcal{T} \tag{3l}$$

$$0 \leq \overline{\mu}_{jt\nu} \perp \left(\underline{L}^{\mathrm{HP}}_{j}\left(Q_{jt}\right) - L^{\mathrm{HP}}_{jt\nu}\right) \geq 0,\ \forall j \in \mathcal{I}^{\mathrm{HP}}, t \in \mathcal{T}. \tag{3m}$$

Equation (3a)-(3d) represent the stationarity conditions, and (3e)-(3m) represent the primal and dual constraints, and complementarity conditions. By replacing each lower-level problem LL_ν by its equivalent KKT conditions, the hierarchical optimization problem (17) in the supporting paper is recast as Mathematical Problem with Equilibrium Constraints (MPEC). Due to the non-convex complementarity conditions, MPECs are challenging to solve and few solvers (e.g. KNITRO, NLPEC) support them. Another approach is to reformulate the complementarity conditions by introducing SOS variables or auxiliary binary variables, as will be detailed in Section 4.2.

4.2. Mixed Integer Linear Program (MILP)

The MPEC introduced in Appendix 4.1 can be reformulated as a Mixed Integer Linear Problem (MILP). First, the complementarity conditions (3f)-(3m) can be linearized using the well-known Fortuny-Amat linearization (Gabriel et al., 2012; Fortuny-Amat & McCarl, 1981). Additionally, the bilinear terms in the objective function (17a) in the supporting paper can be exactly linearized, using the complementarity conditions (3f)-(3m) and the strong duality theorem (5a) (Gabriel et al., 2012), such that:

$$\sum_{j\in\mathcal{I}^{\mathrm{HP}}} \lambda^{\mathrm{E}}_{t\nu} L^{\mathrm{HP}}_{jt\nu} - \sum_{j\in\mathcal{I}^{\mathrm{CHP}}} \lambda^{\mathrm{E}}_{t\nu} P_{jt\nu} = \sum_{j\in\mathcal{I}^{\mathrm{E}}} \left(\alpha_{jt\nu} P_{jt\nu} + \overline{\mu}_{jt\nu}\overline{P}_{jt\nu}\right) - \lambda^{\mathrm{E}}_{t\nu} L^{\mathrm{E}}_{t\nu},\ \forall t\in\mathcal{T}, \nu\in\mathcal{X}. \tag{4}$$

This MILP problem can be readily solved with traditional solvers. However, the number of auxiliary binary variables used to linearize the complementarity conditions increases proportionally to the number of scenarios considered, i.e. as $2\left|\mathcal{X}\right|\left|\mathcal{T}\right|\left|\mathcal{I}^{\mathrm{CHP}}\cup\mathcal{I}^{\mathrm{HP}}\cup\mathcal{I}^{\mathrm{E}}\right|$. In order to cope with the computational complexity, we will introduce a decomposition-based method below.

4.3. Primal-dual formulation

Additionally, as the lower-level problems LL_ν of the hierarchical optimization problem (17) in the supporting paper are linear in the continuous variables Ω^{LL}_ν, strong duality applies. Therefore, the hierarchical optimization problem can be reformulated as a single-level optimization problem by replacing each lower-level problem LL_ν by the following equivalent primal-dual formulation:

$$\begin{aligned}
&\sum_{j\in\mathcal{I}^{\mathrm{E}},t\in\mathcal{T}} \tilde{\alpha}^{\mathrm{E}}_{jt\nu} P_{jt\nu} + \sum_{j\in\mathcal{I}^{\mathrm{CHP}},t\in\mathcal{T}} \left(\underline{\alpha} P^0_{jt\nu} + \alpha^{\mathrm{E}}_j P^+_{jt\nu}\right) - \overline{\alpha} L^{\mathrm{HP}}_{jt\nu} \\
&= \sum_{t\in\mathcal{T}} \Bigg[- \sum_{j\in\mathcal{I}^{\mathrm{HP}}} \overline{\mu}_{jt\nu} \underline{L}^{\mathrm{HP}}_j\left(Q_{jt}\right) - \sum_{j\in\mathcal{I}^{\mathrm{CHP}}} \overline{\mu}^0_{jt\nu} \underline{P}_j\left(Q_{jt}\right) \\
&\quad - \sum_{j\in\mathcal{I}^{\mathrm{CHP}}} \overline{\mu}_{jt\nu}\left(\overline{P}_j\left(Q_{jt}\right) - \underline{P}_j\left(Q_{jt}\right)\right) - \sum_{j\in\mathcal{I}^{\mathrm{E}}} \overline{\mu}_{jt\nu}\overline{P}_{jt\nu} + \lambda^{\mathrm{E}}_{t\nu} L^{\mathrm{E}}_{t\nu}\Bigg]
\end{aligned} \tag{5a}$$

$$(3\mathrm{a}) - (3\mathrm{d}) \tag{5b}$$

$$L^{\mathrm{E}}_{t\nu} + \sum_{j\in\mathcal{I}^{\mathrm{HP}}} L^{\mathrm{HP}}_{jt\nu} = \sum_{j\in\mathcal{I}^{\mathrm{E}}\cup\mathcal{I}^{\mathrm{CHP}}} P_{jt\nu}\ :\lambda^{\mathrm{E}}_{t\nu},\ \forall t\in\mathcal{T} \tag{5c}$$

$$0\le P^0_{jt\nu} \le \underline{P}_j\left(Q_{jt}\right)\ :\underline{\mu}_{jt\nu}, \overline{\mu}_{jt\nu},\ \forall j\in\mathcal{I}^{\mathrm{CHP}}, t\in\mathcal{T} \tag{5d}$$

$$0\le P^+_{jt\nu} \le \overline{P}_j\left(Q_{jt}\right) - \underline{P}_j\left(Q_{jt}\right)\ :\underline{\mu}_{jt\nu}, \overline{\mu}_{jt\nu},\ \forall j\in\mathcal{I}^{\mathrm{CHP}}, t\in\mathcal{T} \tag{5e}$$

$$0\le P_{jt\nu} \le \overline{P}_{jt\nu}\ :\underline{\mu}_{jt\nu}, \overline{\mu}_{jt\nu},\ \forall j\in\mathcal{I}^{\mathrm{E}}, t\in\mathcal{T} \tag{5f}$$

$$0\le L^{\mathrm{HP}}_{jt\nu} \le \underline{L}^{\mathrm{HP}}_j\left(Q_{jt}\right)\ :\underline{\mu}_{jt\nu}, \overline{\mu}_{jt\nu},\ \forall j\in\mathcal{I}^{\mathrm{HP}}, t\in\mathcal{T} \tag{5g}$$

$$\underline{\mu}_{jt\nu}, \underline{\mu}^0_{jt\nu}, \overline{\mu}_{jt\nu}, \overline{\mu}^0_{jt\nu} \ge 0,\ \forall j\in\mathcal{I}^{\mathrm{E}}\cup\mathcal{I}^{\mathrm{CHP}}\cup\mathcal{I}^{\mathrm{HP}}, t\in\mathcal{T}. \tag{5h}$$

Equation (5a) represents the strong duality theorem, (5b) stationarity conditions, (5c)-(5g) primal feasibility, and (5h) dual feasibility. The strong duality condition (5a) is non-convex due to the bilinear terms $\overline{\mu}^0_{jt\nu}\underline{P}_j\left(Q_{jt}\right)$, $\overline{\mu}_{jt\nu}\left(\overline{P}_j\left(Q_{jt}\right) - \underline{P}_j\left(Q_{jt}\right)\right)$, and $\overline{\mu}_{jt\nu}\underline{L}^{\mathrm{HP}}_j\left(Q_{jt}\right)$. As a result, this primal-dual formulation is challenging to solve directly using traditional solvers.

5. Motivating example

This section introduces a simple motivating example that illustrates the synergies between heat and electricity systems, and the impact of the three different market frameworks discussed, i.e., sequential, integrated, and electricity-aware.

5.1. Potential synergies between heat and electricity systems

In order to illustrate the potential synergies between heat and electricity systems, we consider a small system representing a reduced version of the Danish energy system. The power system comprises two conventional electricity generators, G_1 and G_2, and a wind producer W_1. An extraction CHP and a HP are at the interface between heat and electricity systems. Additionally, we consider a centralized Heat Storage (HS) tank and a heat-only unit (HO). The technical characteristics of the networks and generation units are detailed in Table 1. Figure 1 depicts the FOR of the extraction CHP, and the equivalent FOR of the CHP coupled with a HP and a heat storage tank, assuming that the storage is charged at 50%. Heat and electricity loads are derived from Madsen (2015), and are seen as representative of the Danish energy system. Wind production for a sample day is derived from Bukhsh (2015). As illustrated in the left-hand side plot of Figure 2, heat and electricity systems in Denmark have different peak hours during the day. Indeed, heat demand peak hours occur during the evening and morning due to lower ambient temperatures, while electricity demand peak hours are during the day. Figure 2 displays the excess wind production during certain

Table 1: Generation units' parameters

		G_1	G_2	W_1	CHP	HP	HS	HO
$\overline{P}$	(MW)	150	200	500	-	-	-	-
$\overline{Q}$	(MW)	-	-	-	300	200	50	500
$\overline{F}$	(MW)	-	-	-	600	-	-	-
$\overline{S}$	(MWh)	-	-	-	-	-	150	-
COP	-	-	-	-	-	3.0	-	-
r	-	-	-	-	0.6	-	-	-
$\rho^{\mathrm{E}}/\rho^{\mathrm{H}}$	-	-	-	-	2.4/0.25	-	-	0/1
ρ^{+}/ρ^{-}	-	-	-	-	-	-	1.1/0.9	-
α	(€ /MWh)	11	33	0	12.5	-	-	30

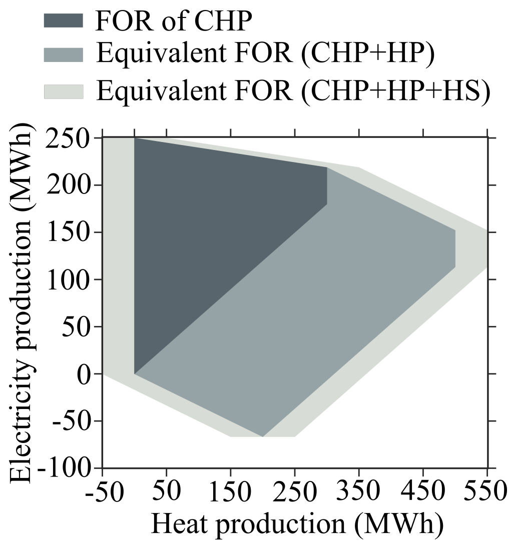

Figure 1: Equivalent FOR of the extraction CHP, coupled with the HP and the HS (inspired by Chen et al. (2015))

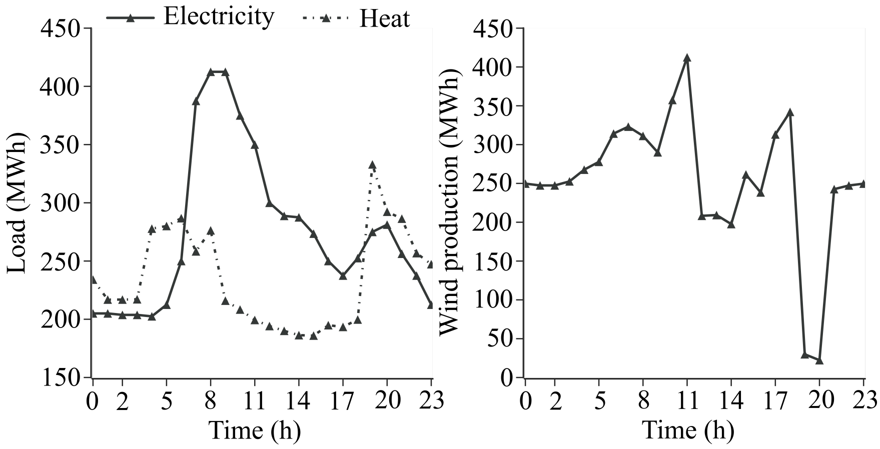

Figure 2: Electricity and heat loads (left plot), and wind production (right plot) in a sample day

hours, especially during the night. The heat system can provide flexibility to the power system in two ways. First, the HP can supply heat with excess wind production. Second, the extraction CHP can supply heat and electricity during the electricity peak hours. Additionally, heat storage can provide inter-temporal flexibility by storing excess heat production from HPs and CHPs. In an integrated market, this flexibility is exploited by co-dispatching heat and electricity sources. However, in a sequential market framework, the interaction between heat and power systems is captured by the way CHPs and HPs compute their heat marginal costs.

In order to compute their heat marginal costs $\alpha_j^{\mathrm{H}}\left(\hat{\lambda}_t^{\mathrm{E}}\right)$, CHPs and HPs need to forecast electricity prices $\hat{\lambda}_t^{\mathrm{E}}$. Table 2 demonstrates how the forecast electricity prices influence heat marginal costs and thereby the merit order in the heat market. The values for the forecast electricity prices are derived from the heat and electricity dispatch of the electricity-aware market framework. During the electricity peak hours, electricity prices are expected to be high. In this case, the CHP has a lower heat marginal cost than the HP. During the night and hours with excess wind production, electricity prices are expected to drop, and the HP has a lower heat marginal cost.

Table 2: Estimated electricity spot prices and heat marginal costs (EUR/MWh)

	$\hat{\lambda}^{\mathrm{E}}$	$\alpha^{\mathrm{H}}_{\mathrm{CHP}}$	$\alpha^{\mathrm{H}}_{\mathrm{HP}}$		$\hat{\lambda}^{\mathrm{E}}$	$\alpha^{\mathrm{H}}_{\mathrm{CHP}}$	$\alpha^{\mathrm{H}}_{\mathrm{HP}}$
$\boldsymbol{t_{00}-t_{03}}$	12.5	13.6	4.2	$\boldsymbol{t_{13}}$	12.5	13.6	4.2
$\boldsymbol{t_{04}-t_{06}}$	0	21.1	0	$\boldsymbol{t_{14}}$	30	3.1	10
$\boldsymbol{t_{07}-t_{08}}$	12.5	13.6	4.2	$\boldsymbol{t_{15}-t_{16}}$	12.5	13.6	4.2
$\boldsymbol{t_{09}}$	30	3.1	10	$\boldsymbol{t_{17}-t_{18}}$	0	21.1	0
$\boldsymbol{t_{10}-t_{11}}$	12.5	13.6	4.2	$\boldsymbol{t_{19}-t_{20}}$	30	3.1	10
$\boldsymbol{t_{12}}$	30	3.1	10	$\boldsymbol{t_{21}-t_{23}}$	12.5	13.6	4.2

Note that, in Table 2, the CHP and the HP use the same value of the forecast electricity prices to derive their heat marginal cost, which is a strong assumption. It is expected that relaxing this assumption would introduce additional inefficiencies in the sequential heat market-clearing. Indeed, small deviations on electricity spot prices can have a remarkable influence on the merit order in the heat market, and thereby on the heat dispatch. In addition, heat marginal costs in

Table 2 do not reflect the fact that the participation of CHPs and HPs in the electricity market may influence electricity prices.

5.2. *Limitations of sequential market framework*

In order to illustrate the challenges raised by the sequential market framework introduced above, we implement it on the illustrative example presented in Section 5.1. Figure 3 shows the resulting heat and electricity dispatch.

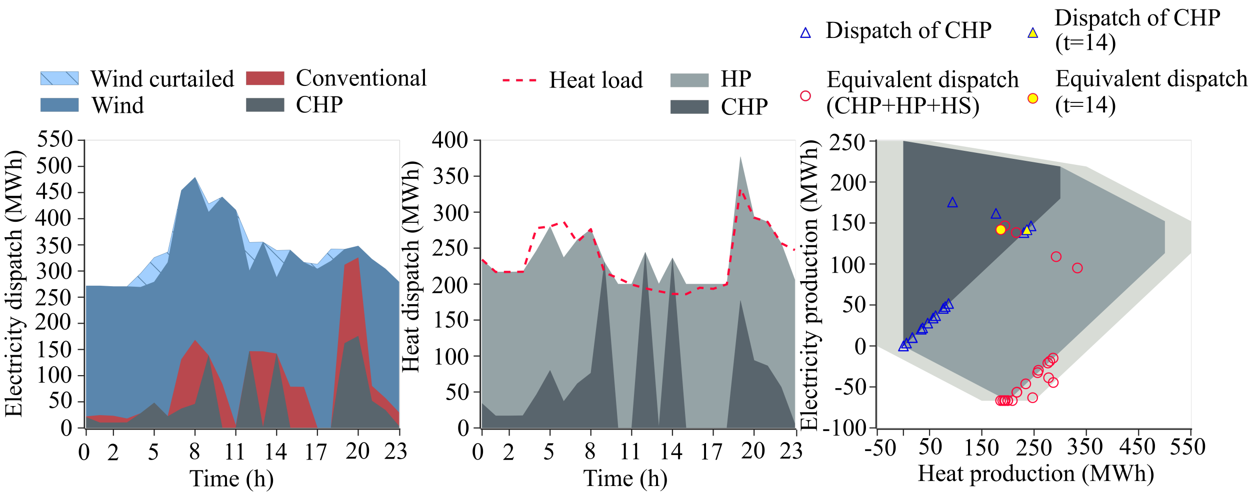

Figure 3: Electricity and heat dispatch in the sequential market framework (left plot: hourly electricity dispatch, middle plot: hourly heat dispatch, right plot: hourly dispatch of CHP, HP, and HS based on their FOR)

Due to the large penetration of wind in the power system, wind curtailment occurs during the night (left-hand side plot). Additionally, wind utilization is decreased during certain hours due to the lack of flexibility of CHPs and HPs when participating in the electricity market. Furthermore, the right-hand side plot of Figure 3 highlights the linkage between heat and electricity outputs of the CHP alone, and the equivalent heat and electricity outputs of the CHP combined with the HP and the heat storage tank. For example, at hour $t = 14$, the heat marginal cost of the CHP is expected to be lower than the one of the HP, and the CHP is dispatched instead of the HP in the heat market. As a result, the HP does not participate in the electricity market and the CHP's minimum electricity production causes wind curtailment. In larger power systems this curtailed wind energy can be exported when the transmission capacity is sufficient, or can be stored. However, in the case of a large scale penetration of renewable energy sources, these solutions may require additional investment costs. A better coordination between heat and power systems can increase the flexibility of the integrated energy system, which eventually helps the power system to integrate higher shares of renewable energy generation.

5.3. *Benefits of integrated market framework*

We implement this model to the illustrative example described in Section 5.1. Figure 4 displays the heat and electricity dispatch for the entire day.

The integrated market provides an ideal benchmark for the dispatch of heat and electricity systems by optimally exploiting the operational flexibility of the CHP and the HP. For example, at hour $t = 14$, the CHP is not dispatched and heat is covered by the HP in order to integrate wind

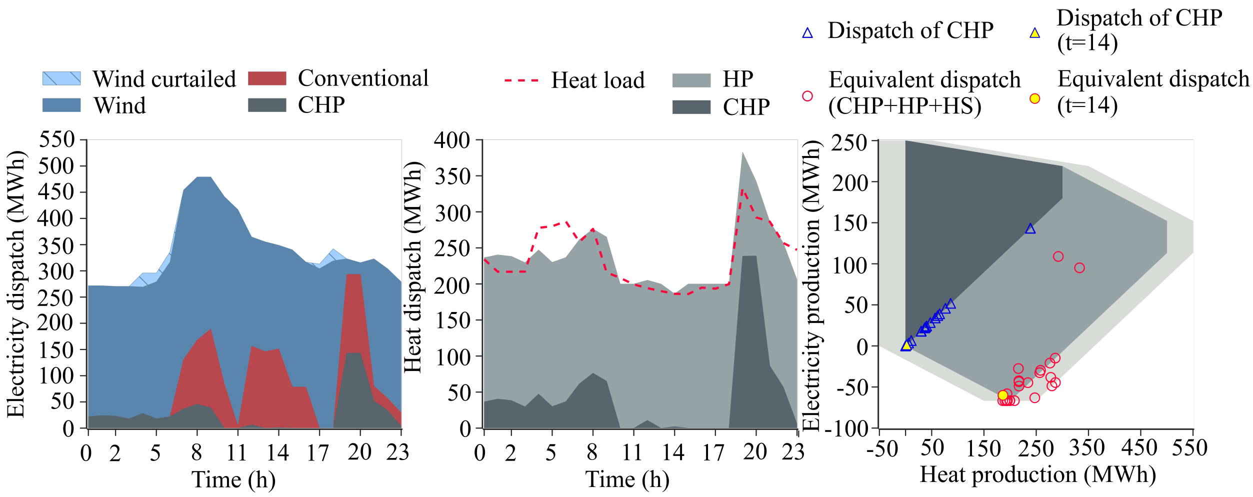

Figure 4: Heat and electricity dispatch in the integrated market framework (left plot: hourly electricity dispatch, middle plot: hourly heat dispatch, right plot: hourly dispatch of CHP, HP, and HS based on their FOR)

production, as highlighted in the right-hand side plot of Figure 4. Subsequently, wind curtailment is reduced compared to the sequential model (left-hand side plot). Additionally, the total energy cost is decreased by 19.5% compared to the sequential market model. Although this market framework goes against current market regulations, it provides a lower bound for the total production cost of the integrated energy system, and a basis for quantifying the social value of coupling heat and electricity systems.

5.4. A trade-off between sequential and integrated market frameworks

We implement the proposed electricity-aware market framework in the illustrative example introduced in Section 5.1. Figure 5 shows the heat and electricity dispatch for 24 hours.

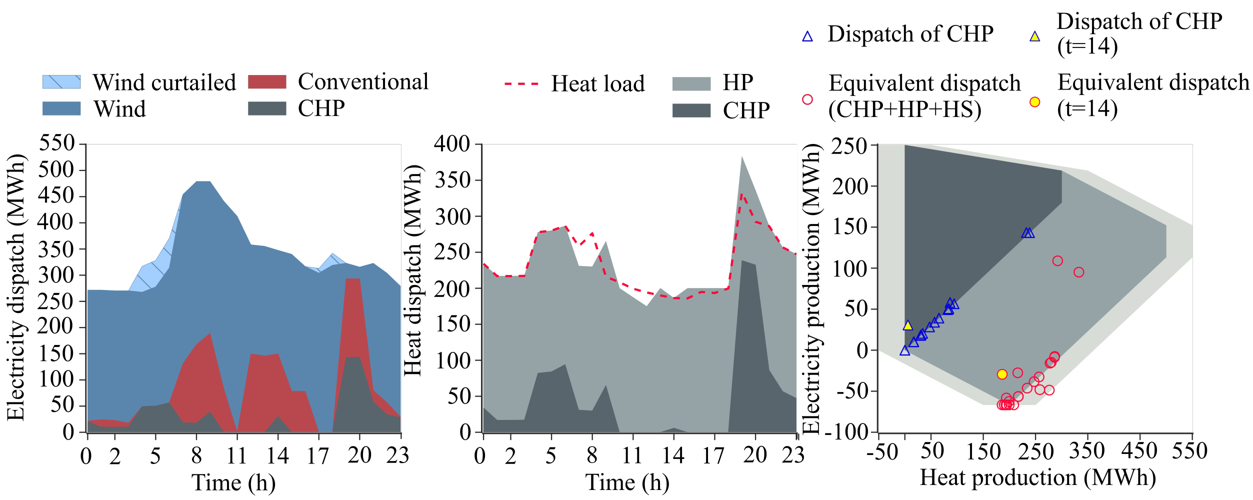

Figure 5: Electricity and heat dispatch in the electricity-aware market framework (left plot: hourly electricity dispatch, middle plot: hourly heat dispatch, right plot: hourly dispatch of CHP, HP, and HS based on their FOR)

The electricity-aware market framework allows the heat market operator to anticipate the impact of CHPs and HPs in the electricity market, and exploit their operational flexibility. For example, at hour $t = 14$, the CHP is not dispatched and heat is covered by the HP in order to reduce wind curtailment. The right-hand side plot of Figure 5 highlights the heat and electricity dispatch at this specific hour of the day. Subsequently, wind curtailment is reduced compared to the sequential model.

Table 3 compares the total system cost and wind curtailment across the three market frameworks. While the integrated market achieves the lowest cost, the electricity-aware market framework manages to substantially reduce both the wind curtailment and the total system cost compared to those in the sequential market framework.

Table 3: Total system cost and wind curtailment across the three market frameworks

		Sequential	Integrated	Electricity-aware
Total system cost	(EUR)	$50,683$	$41,216$	$41,264$
Wind curtailment	(MWh)	254	95	95

6. Numerical example: data

6.1. Heat and electricity systems

A modified 24-bus IEEE Reliability Test System composes the integrated energy system. It consists of 7 thermal power plants, 6 wind farms, 3 CHPs, and 1 heat-only unit. Data for the power system is derived from the 24-bus IEEE Reliability Test System provided by Ordoudis et al. (2016). Data for the heat system and heat demand are derived from the greater Copenhagen area, and data presented by Madsen (2015) and Zugno et al. (2016). Tables 4-6 summarize the technical parameters and marginal cost parameters of these units. Additionally, a heat-only peak unit,

Table 4: Electricity System – generation units' parameters

		G_1	G_2	G_3	G_4	G_5	G_6	G_7
$\overline{P}$	(MW)	76	76	175	30	200	200	1000
μ_α	(EUR/MWh)	13.32	13.32	20.7	26.11	6.02	5.47	100
σ_α	(EUR/MWh)	1.3	1.3	2.1	2.6	0.6 -	0.5	0

Table 5: Heat System – generation units' parameters

		CHP_1	CHP_2	CHP_3	CHP_4	HP_1	HP_2	HP_3
$\overline{Q}$	(MW)	300	300	300	400	250	250	250
$\overline{F}$	(MW)	600	600	600	600	-	-	-
COP	-	-	-	-	2.8	3.1	2.5	
r	0.5	0.5	0.5	0.5	-	-	-	
ρ^{E}	2.1	2.1	2.1	2.4	-	-	-	
ρ^{H}	0.25	0.21	0.25	0.21	-	-	-	
α	(EUR/MWh)	5	7.5	10	12.5	-	-	-

with a maximum capacity of 8000MW and a marginal cost of 100EUR/MWh is considered.

Table 6: Heat storage tanks parameters

		HS_1	HS_2	HS_3
$\overline{S}$	(MWh)	150	150	150
$\overline{Q}$	(MW)	50	50	50
ρ^+	1.1	1.1	1.1	
ρ^-	0.9	0.9	0.9	
S^{init}	(MW)	100	100	100

6.2. Scenario generation

Scenarios of supply functions are generated from data in 4, assuming a normal distribution of mean μ_{α_j} and standard deviation σ_{α_j} for each participants' bids, such that:

$$\tilde{\alpha}^{\text{E}}_{jt\nu} \sim \mathcal{N}\left(\mu_{\alpha_j}, \sigma_{\alpha_j}\right) \tag{6}$$

Electricity demand scenarios are derived from market data from Nordpool for January 2018, available at Nordpool (2018). Wind power uncertainty is modeled by a set of scenarios with temporal and spatial correlation, which are available at Bukhsh (2015). We first generate 15 independent scenarios per source of uncertainty, namely wind production, electricity loads and supply functions. We then use a scenario reduction technique to merge similar scenarios (Gabriel et al., 2009; Morales et al., 2009). Figure 6 illustrates the scenarios for wind production and electricity demand, before and after the scenario reduction.

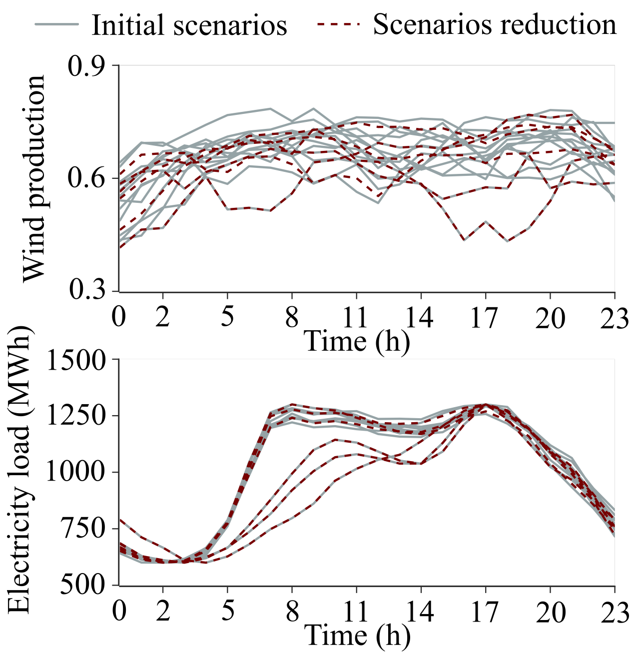

Figure 6: Original 15 scenarios and scenario reduction (6 scenarios) of wind production, as a ratio of total installed wind capacity (top plot) and electricity load (bottom plot)

Finally, we carry out an out-of-sample analysis to provide a more rigorous comparison of the performance of the three market frameworks against unseen scenarios. To this purpose, we generate

10 new scenarios per source of uncertainty, namely wind production, electricity loads and supply functions, from the same distributions that we generated in-sample scenarios. Figure 7 shows the out-of-sample scenarios for wind production and electricity demand.

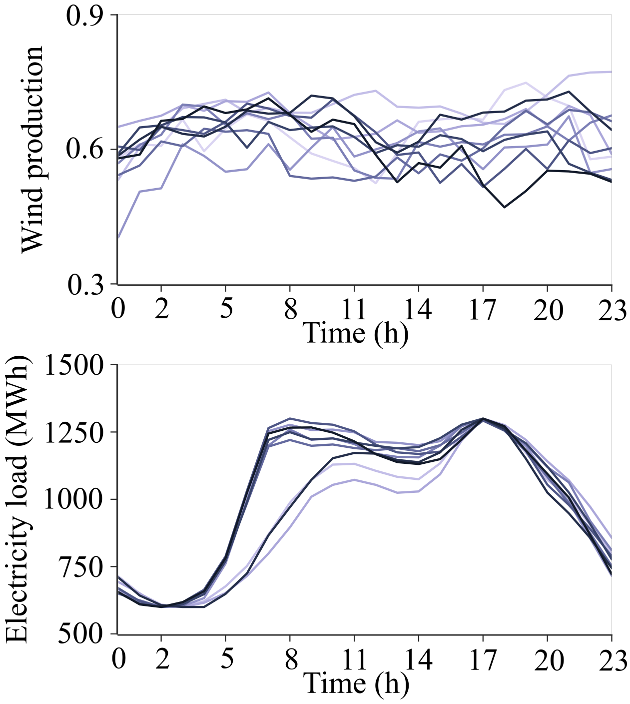

Figure 7: Out-of-sample scenarios of wind production, as a ratio of total installed wind capacity (top plot) and electricity load (bottom plot)

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