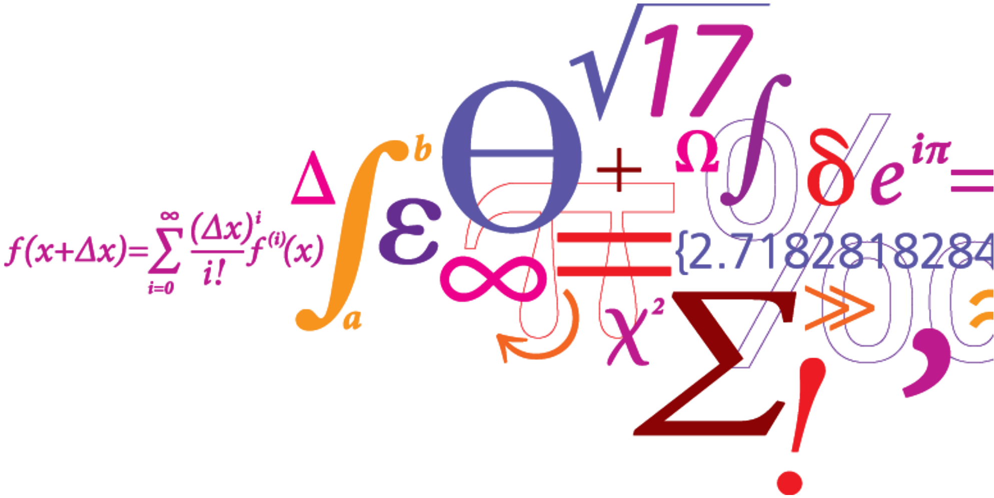
Large-Scale Optimization Problem in Energy Systems: Applications of Decomposition Techniques

Lecture 6: Applications of Bilevel Programming to Power Systems and Electricity Markets



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January 9th, 2018

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Applications to Power Systems

Yesterday we talked about various applications of bilevel programming to power systems:

1. Vulnerability assessment (leader can be the system operator or attacker)
2. Transmission planning (leader is the TSO)
3. Strategic investment (leader is a strategic producer)
4. Strategic offering (leader is a strategic producer)
5. Coupling of energy markets (district heating and electricity)

Etc…

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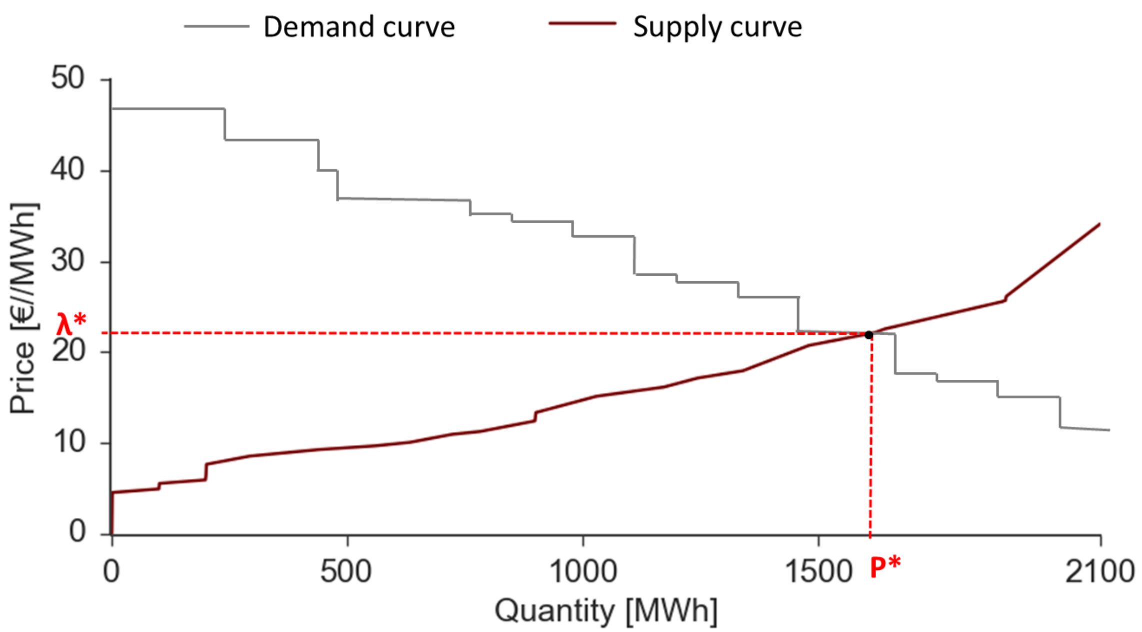
Etc…

Today we will focus on one application: Strategic offering in the day-ahead market

Strategic Offering

- Perfect competition: no producer can exercise market power
- Nash Equilibrium



Strategic Offering

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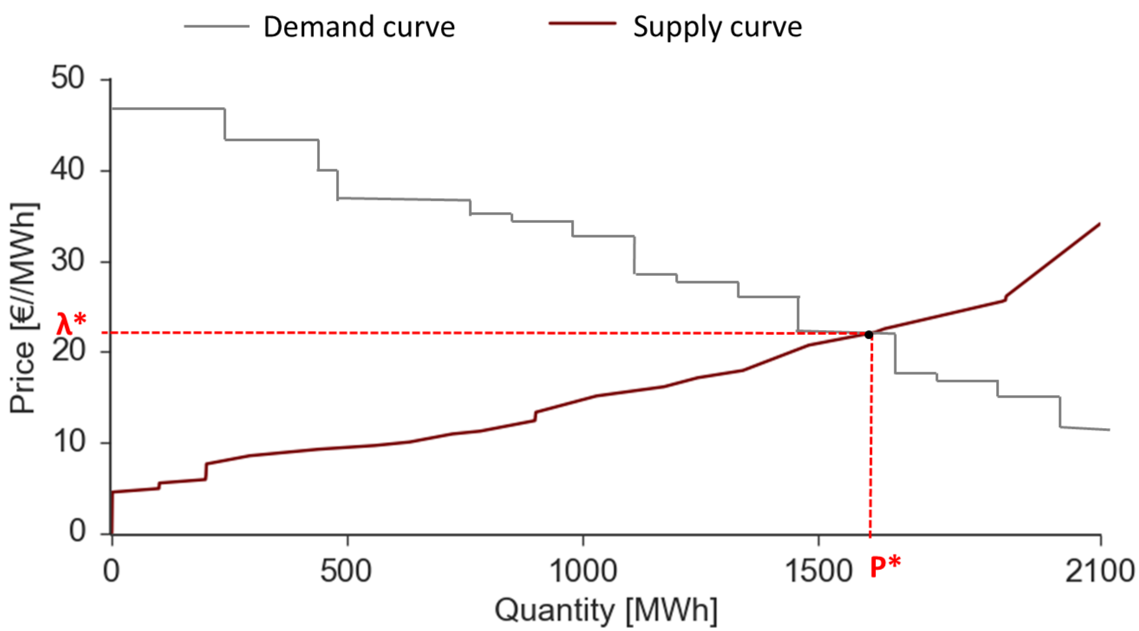

- No participant can deviate unilateraly from the equilibrium to increase its own profit

Strategic Offering

- A large producer participating in the day-ahead market
- Can exercise ”market-power”: modify market equilibrium to increase its profit

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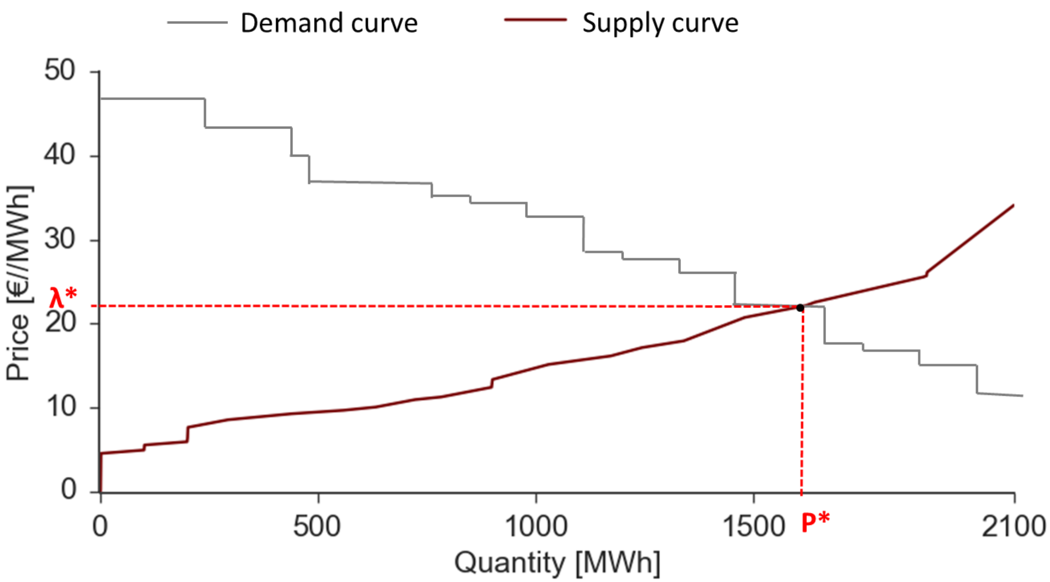

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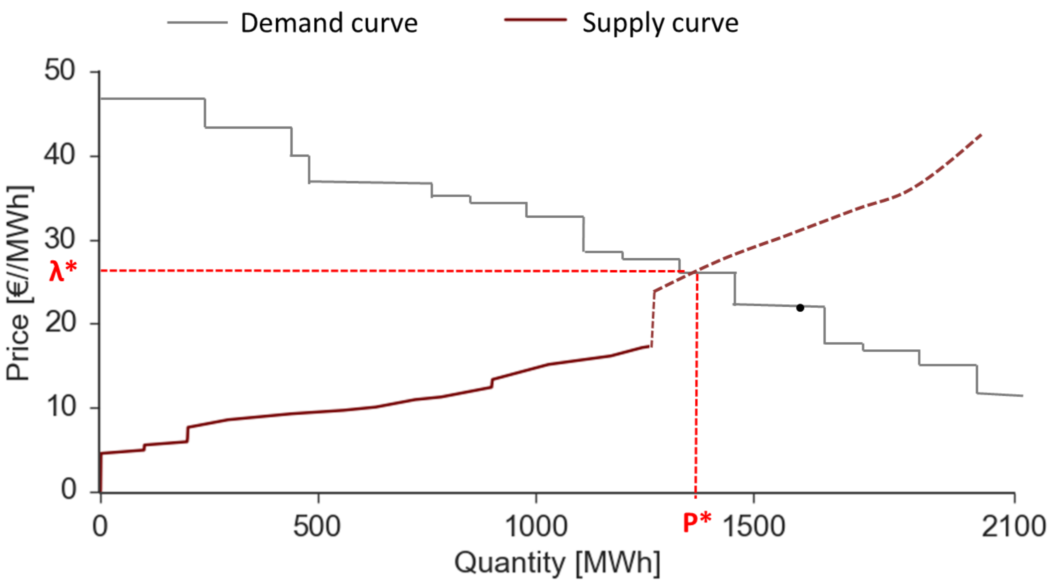

Strategic Offering

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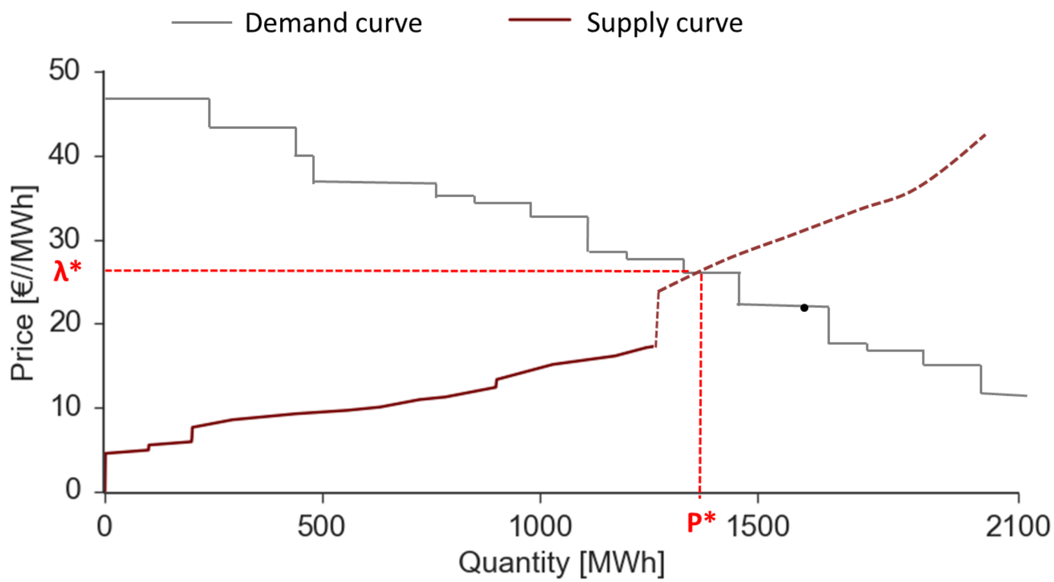
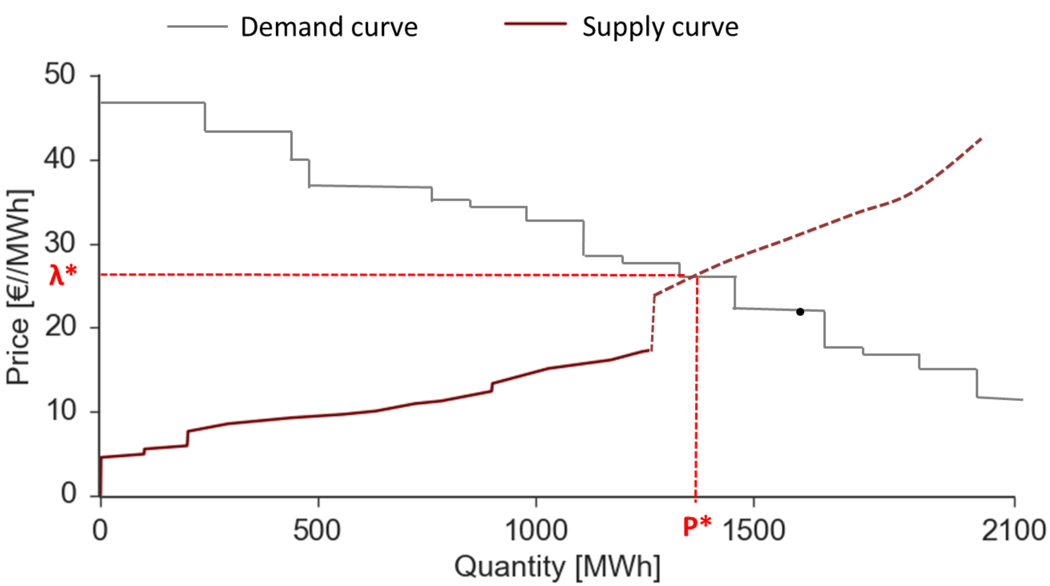


Question: How can the producer model the impact of its decision on market outcomes (prices, quantities)?

Strategic Offering

- A large producer participating in the day-ahead market
- Can exercise ”market-power”: modify market equilibrium to increase its profit



Question: How can the producer model the impact of its decision on market outcomes (prices, quantities)?

➢ Needs to model the competition in electricity market endogenously, as a constraint of its optimization problem

Bilevel Formulation

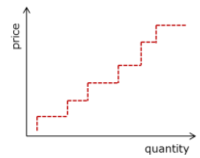
Upper-level: strategic producer

Maximize profit

Subject to:

- Non-negativity of offers

Offering curve



- LMPs
- Quantities

Lower-level: market clearing

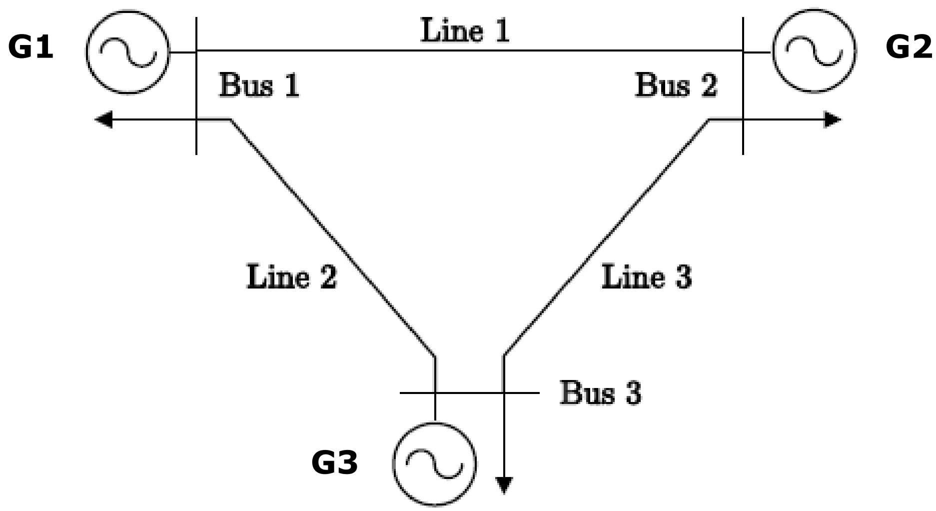
Maximize social welfare

Subject to:

- Balance equations at nodes
- Transmission constraints
- Production bounds

Illustrative Example 1

- Three bus system
- Three generators and loads



Illustrative Example 1

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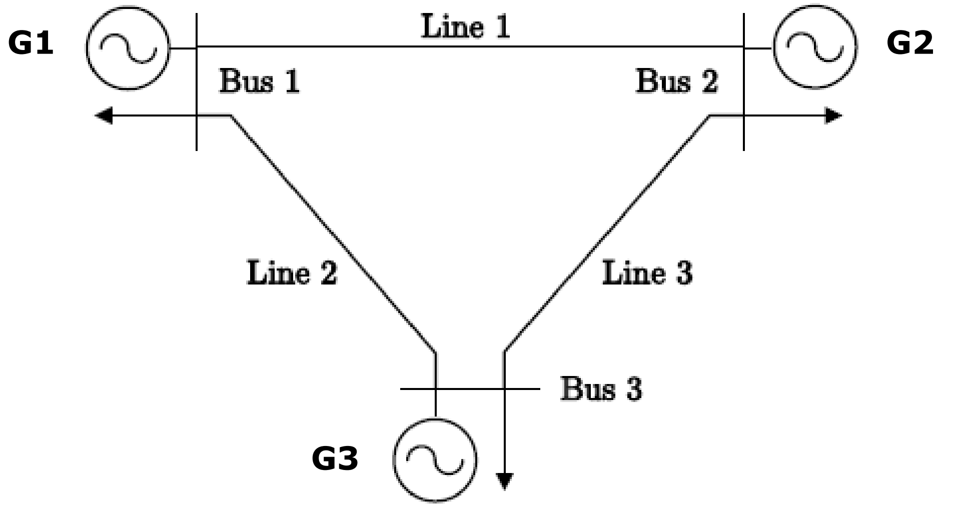

Exercise: Formulate the strategic offering problem of generator G1

Illustrative Example 1

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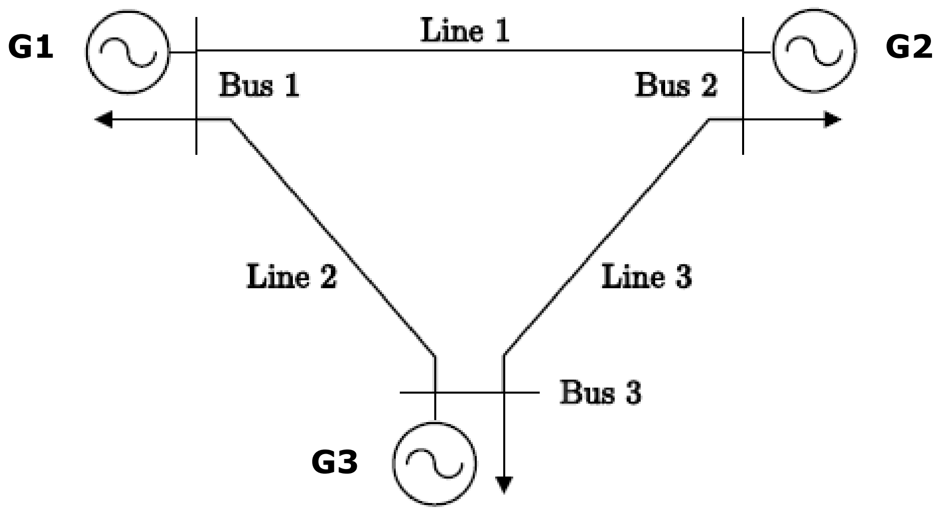
- Three generators and loads



Exercise: Formulate the strategic offering problem of generator G1

Assumptions:

- Single block offer (single price, maximum production)
- Inelastic demand

Bilevel Formulation: Upper-Level Problem

$$\min_{\Theta^{UL} \cup \Theta^{LL} \cup \{\lambda, \mu, \rho\}} \quad c_1 g_1 - \lambda_1 g_1$$
$$\text{s.t.} \quad \begin{cases} \hat{c}_1 \geq 0 \\ \text{Lower-level} \end{cases}$$

Parameters:

c_1 : marginal cost of the strategic producer

Decision variables (upper-level):

$\hat{c}_1$: price offer of the strategic producer

Bilevel Formulation: Upper-Level Problem

$$\min_{\Theta^{UL}\cup\Theta^{LL}\cup\{\lambda,\mu,\rho\}} \quad \boxed{c_1 g_1 - \lambda_1 g_1}$$

$$\text{s.t.} \quad \begin{cases} \hat{c}_1 \geq 0 \\ \text{Lower-level} \end{cases}$$

Lower-level decision variables appear in the objective function:

λ_1 : LMP at node 1 (dual)

g_1 : power dispatch (primal)

Parameters:

c_1 : marginal cost of the strategic producer

Decision variables (upper-level):

$\hat{c}_1$: price offer of the strategic producer

Bilevel Formulation: Lower-Level Problem

$$
\begin{aligned}
\min_{\Theta^{LL}} \quad & \hat{c}_1 g_1 + c_2 g_2 + c_3 g_3 \\
\text{s.t.} \quad & g_1 + B_{l_1}(\theta_2 - \theta_1) + B_{l_2}(\theta_3 - \theta_1) = d_1 && : \lambda_1 \\
& g_2 + B_{l_1}(\theta_1 - \theta_2) + B_{l_3}(\theta_3 - \theta_2) = d_2 && : \lambda_2 \\
& g_3 + B_{l_2}(\theta_1 - \theta_3) + B_{l_3}(\theta_2 - \theta_3) = d_3 && : \lambda_3 \\
& \theta_3 = 0 && : \gamma \\
& 0 \le g_1 \le g_1^{max} && : \mu_1^{min}, \mu_1^{max} \\
& 0 \le g_2 \le g_2^{max} && : \mu_2^{min}, \mu_2^{max} \\
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\end{aligned}
$$

Parameters:

c_i : marginal costs of producers

$\hat{c}_1$: price offer of strategic producer

B_i : Susceptance of transmisison lines

${g^{max}}_i$ $({f^{max}}_l)$: production (transmission) bounds

d_i : demand at each bus

Decision variables (lower-level):

g_i : power dispatch

θ_i : Voltage angle at each bus

Dual variables…

Bilevel Formulation: Lower-Level Problem

$$
\begin{aligned}
\min_{\Theta^{LL}} \quad & \hat{c}_1 g_1 + c_2 g_2 + c_3 g_3 \\
\text{s.t.} \quad & \boxed{\begin{aligned} & g_1 + B_{l_1}(\theta_2 - \theta_1) + B_{l_2}(\theta_3 - \theta_1) = d_1 \\ & g_2 + B_{l_1}(\theta_1 - \theta_2) + B_{l_3}(\theta_3 - \theta_2) = d_2 \\ & g_3 + B_{l_2}(\theta_1 - \theta_3) + B_{l_3}(\theta_2 - \theta_3) = d_3 \end{aligned}} \quad \begin{aligned} & : \lambda_1 \\ & : \lambda_2 \\ & : \lambda_3 \end{aligned} \\
& \theta_3 = 0 && : \gamma \\
& 0 \leq g_1 \leq g_1^{max} && : \mu_1^{min}, \mu_1^{max} \\
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\end{aligned}
$$

Power balance at each node

Parameters:

c_i : marginal costs of producers

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$g^{max}{}_i$ $(f^{max}{}_l)$: production (transmission) bounds

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Dual variables…

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Power balance at each node

Production bounds

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Power balance at each node

Production bounds

Transmission bounds

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Decision variables (lower-level):

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θ_i : Voltage angle at each bus

Dual variables…

MPEC Formulation: KKT Conditions

Stationarity conditions

$$\hat{c}_1 - \lambda_1 + \mu_1^{max} - \mu_1^{min} = 0$$
$$c_2 - \lambda_2 + \mu_2^{max} - \mu_2^{min} = 0$$
$$c_3 - \lambda_3 + \mu_3^{max} - \mu_3^{min} = 0$$
$$B_{l1}(\lambda_1 - \lambda_2 + \rho_{l1}^{max} - \rho_{l1}^{min}) + B_{l2}(\lambda_1 - \lambda_3 - \rho_{l2}^{max} + \rho_{l2}^{min}) = 0$$
$$B_{l1}(\lambda_2 - \lambda_1 - \rho_{l1}^{max} + \rho_{l1}^{min}) + B_{l3}(\lambda_2 - \lambda_3 + \rho_{l3}^{max} - \rho_{l3}^{min}) = 0$$
$$B_{l2}(\lambda_3 - \lambda_1 + \rho_{l2}^{max} - \rho_{l2}^{min}) + B_{l3}(\lambda_3 - \lambda_2 - \rho_{l3}^{max} + \rho_{l3}^{min}) + \gamma = 0$$

Primal feasibility

$$g_1 + B_{l_1}(\theta_2 - \theta_1) + B_{l_2}(\theta_3 - \theta_1) = d_1$$
$$g_2 + B_{l_1}(\theta_1 - \theta_2) + B_{l_3}(\theta_3 - \theta_2) = d_2$$
$$g_3 + B_{l_2}(\theta_1 - \theta_3) + B_{l_3}(\theta_2 - \theta_3) = d_3$$
$$\theta_3 = 0$$

MPEC Formulation: KKT Conditions

$$0 \leq \mu_1^{max} \perp (g_1 - g_1^{max}) \leq 0$$
$$0 \leq \mu_2^{max} \perp (g_2 - g_2^{max}) \leq 0$$
$$0 \leq \mu_3^{max} \perp (g_3 - g_3^{max}) \leq 0$$
$$0 \leq \mu_1^{min} \perp (-g_1) \leq 0$$
$$0 \leq \mu_2^{min} \perp (-g_2) \leq 0$$
$$0 \leq \mu_3^{min} \perp (-g_3) \leq 0$$
$$0 \leq \rho_{l_1}^{max} \perp (B_{l_1}(\theta_1 - \theta_2) - f_{l_1}^{max}) \leq 0$$
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- Complementarity conditions
- Primal feasibility
- Dual feasibility

MPEC Formulation: KKT Conditions

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- Complementarity conditions
- Primal feasibility
- Dual feasibility

Tip: We can linearize the complementarity conditions using binary variables (Fortuny-Amat)

Objective Function Linearization

$$\min_{\Theta^{UL} \cup \Theta^{LL} \cup \{\lambda, \mu, \rho\}} \quad \boxed{c_1 g_1 - \lambda_1 g_1}$$

$$\text{s.t.} \quad \begin{cases} \hat{c}_1 \geq 0 \\ \text{Lower-level} \end{cases}$$

Lower-level decision variables appear in the objective function:

λ_1 : LMP at node 1 (dual)

g_1 : power dispatch (primal)

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Lower-level decision variables appear in the objective function:

λ_1 : LMP at node 1 (dual)

g_1 : power dispatch (primal)

The objective function is bilinear and non-convex!!!

Tip: we can linearize this product of a dual and primal variable using strong duality and KKT conditions (from the lower-level problem)

Objective Function Linearization

$$\boxed{\hat{c}_1 = \lambda_1 - \mu_1^{max} + \mu_1^{min}}$$

(Stationarity condition)

[1] GABRIEL, Steven A., CONEJO, Antonio J., FULLER, J. David, *et al. Complementarity modeling in energy markets*. Springer Science & Business Media, 2012.

Objective Function Linearization

$$\boxed{\hat{c}_1 = \lambda_1 - \mu_1^{max} + \mu_1^{min}}$$

(Stationarity condition)

Multiply by g1

$$\boxed{\hat{c}_1 g_1 = \lambda_1 g_1 - \mu_1^{max} g_1 + \mu_1^{min} g_1}$$

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Objective Function Linearization

$$\boxed{\hat{c}_1 = \lambda_1 - \mu_1^{max} + \mu_1^{min}}$$

(Stationarity condition)

Multiply by g1

$$\boxed{\hat{c}_1 g_1 = \lambda_1 g_1 - \mu_1^{max} g_1 + \mu_1^{min} g_1}$$

Use the complementarity conditions:

$$\mu_1^{max} g_1 = \mu_1^{max} g_1^{max}$$

$$\mu_1^{min} g_1 = 0$$

$$\boxed{\hat{c}_1 g_1 = \lambda_1 g_1 - \mu_1^{max} g_1^{max}}$$

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Objective Function Linearization

$$\hat{c}_1 g_1 = -\sum_{i=2}^{3} c_i g_i - \sum_{i=1}^{3} g_i^{max} \mu_i^{max} + \sum_{i=1}^{3} d_i \lambda_i - \sum_{i=1}^{3} f_{l_i}^{max} \rho_{l_i}^{max} - \sum_{i=1}^{3} f_{l_i}^{max} \rho_{l_i}^{min}$$

(Strong duality)

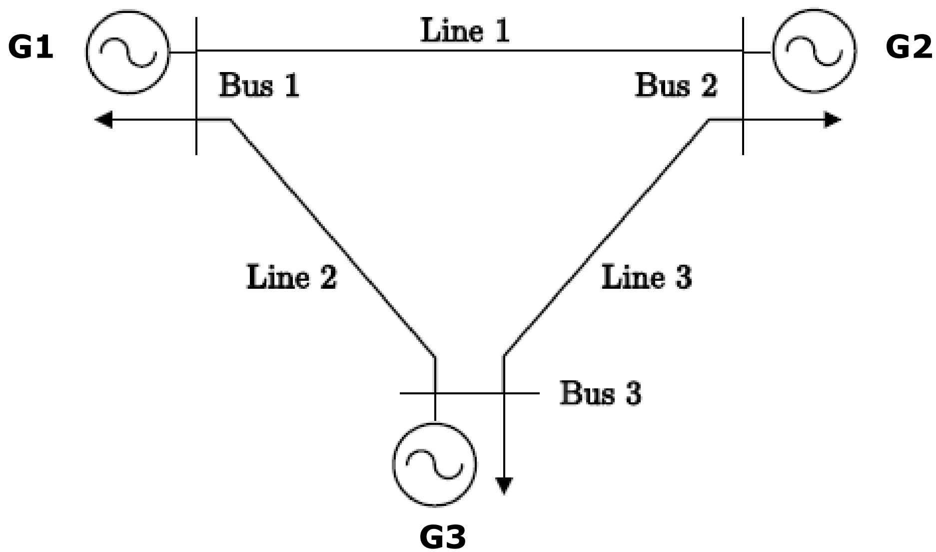
Replace in previous expression

$$\lambda_1 g_1 = -\sum_{i=2}^{3} c_i g_i - \sum_{i=2}^{3} g_i^{max} \mu_i^{max} + \sum_{i=1}^{3} d_i \lambda_i - \sum_{i=1}^{3} f_{l_i}^{max} \rho_{l_i}^{max} - \sum_{i=1}^{3} f_{l_i}^{max} \rho_{l_i}^{min}$$

[1] GABRIEL, Steven A., CONEJO, Antonio J., FULLER, J. David, *et al. Complementarity modeling in energy markets*. Springer Science & Business Media, 2012.

Numerical example

Question: If producers are not strategic, what is the market outcome?

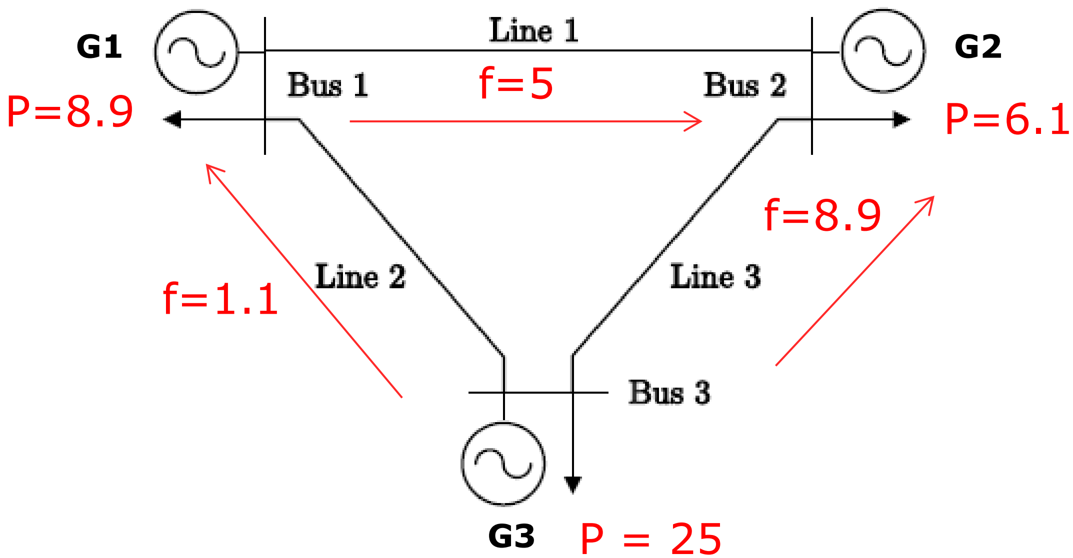


Bus #	Capacity (MW)	Production cost ($/MWh)	Demand (MW)
1	20	16	5
2	10	19	20
3	25	15	15

Line #	From	To	Susceptance (S)	Capacity (MW)
1	1	2	100	5
2	1	3	125	10
3	2	3	150	10

Numerical example

Question: If producers are not strategic, what is the market outcome?



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Line #	From	To	Susceptance (S)	Capacity (MW)
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3	2	3	150	10

Numerical Example

Exercise for this afternoon:

- Formulate the strategic offering problem of generator G1
- Modify the GAMS code provided for the market clearing problem, and solve the strategic offering problem
- How does the market clearing outcomes differ in both models?
- How is the merit order affected?
- How si the profit of each player affected?
- How is the social welfare affected?

GAMS Code

```
  sets
  i   generators /i1*i3/
  d   inelastic loads /d1*d3/
  n   buses /n1*n3/
  l   lines /l1*l3/
;

  parameters

  M0 / 10000 /

  P_max(i)   installed capacity /
  i1  20
  i2  10
  i3  25/

  c(i)   marginal cost /
  i1  16
  i2  19
  i3  15/

  Load(d) Load level /
  d1 5
  d2 20
  d3 15/

  B(l)   Transmission lines susceptance /
  l1 100
  l2 125
  l3 150/
```

```
  parameters

  Fmax(l)   Transmission lines capacity /
  l1 5
  l2 10
  l3 10/
  ;


  Free variables
  cost              Total expected system cost
  P(i)              DA dispatch of generators
  theta(n)          Voltage angles
  lambda(n)         LMPs
  gamma             node reference N3 dual variable
;

  Positive variables
  offer           price offer
  mu_max(i)       max production dual variables
  mu_min(i)       min production dual variables
  rho_max(l)      max flow dual variables
  rho_min(l)      min flow dual variables
;

  Integer variables

  u_mu_max(i)      max production binary variables
  u_mu_min(i)      min production binary variables
  u_rho_max(l)     max flow binary variables
  u_rho_min(l)     min flow binary variables
;
```

GAMS Code

```
 equations
 costfn
 offer_max
 node_balance_1,node_balance_2,node_balance_3
 Prod_max,Prod_min
 flow_max_1,flow_max_2,flow_max_3,flow_min_1,flow_min_2,flow_min_3
 slack_bus
 stat_g1,stat_g2,stat_g3,stat_theta1,stat_theta2,stat_theta3
 comp_gmax_1,comp_gmax_2
 comp_gmin_1,comp_gmin_2
 comp_fmax_1,comp_fmax_21,comp_fmax_22,comp_fmax_23
 comp_fmin_1,comp_fmin_21,comp_fmin_22,comp_fmin_23
;

  costfn.. cost =e= c('i1')*P('i1')
                       - (-c('i2')*P('i2')-c('i3')*P('i3')-P_max('i2')*mu_max('i2')-P_max('i3')*mu_max('i3')
                               +(Load('d1')*lambda('n1')+Load('d2')*lambda('n2')+Load('d3')*lambda('n3'))
                               -sum(l,Fmax(l)*rho_max(l))-sum(l,Fmax(l)*rho_min(l)));
* UL constraint
  offer_max.. offer =l= 50;

* DA constraints
  node_balance_1.. P('i1') + B('l1')*(theta('n2')-theta('n1')) + B('l2')*(theta('n3')-theta('n1')) =e= Load('d1');
  node_balance_2.. P('i2') + B('l1')*(theta('n1')-theta('n2')) + B('l3')*(theta('n3')-theta('n2')) =e= Load('d2');
  node_balance_3.. P('i3') + B('l2')*(theta('n1')-theta('n3')) + B('l3')*(theta('n2')-theta('n3')) =e= Load('d3');
  slack_bus.. theta('n3')=e=0;
  Prod_max(i).. P(i)-P_max(i)=l=0;
  Prod_min(i).. -P(i)=l=0;
  flow_max_1.. B('l1')*(theta('n1')-theta('n2'))-Fmax('l1')=l=0;
  flow_min_1.. -B('l1')*(theta('n1')-theta('n2'))-Fmax('l1')=l=0;
  flow_max_2.. B('l2')*(theta('n3')-theta('n1'))-Fmax('l2')=l=0;
  flow_min_2.. -B('l2')*(theta('n3')-theta('n1'))-Fmax('l2')=l=0;
  flow_max_3.. B('l3')*(theta('n2')-theta('n3'))-Fmax('l3')=l=0;
  flow_min_3.. -B('l3')*(theta('n2')-theta('n3'))-Fmax('l3')=l=0;
```

GAMS Code

```
 equations
 costfn
 offer_max
 node_balance_1,node_balance_2,node_balance_3
 Prod_max,Prod_min
 flow_max_1,flow_max_2,flow_max_3,flow_min_1,flow_min_2,flow_min_3
 slack_bus
 stat_g1,stat_g2,stat_g3,stat_theta1,stat_theta2,stat_theta3
 comp_gmax_1,comp_gmax_2
 comp_gmin_1,comp_gmin_2
 comp_fmax_1,comp_fmax_21,comp_fmax_22,comp_fmax_23
 comp_fmin_1,comp_fmin_21,comp_fmin_22,comp_fmin_23
;

  costfn.. cost =e= c('i1')*P('i1')
                       - (-c('i2')*P('i2')-c('i3')*P('i3')-P_max('i2')*mu_max('i2')-P_max('i3')*mu_max('i3')
                               +(Load('d1')*lambda('n1')+Load('d2')*lambda('n2')+Load('d3')*lambda('n3'))
                               -sum(l,Fmax(l)*rho_max(l))-sum(l,Fmax(l)*rho_min(l)));
* UL constraint
  offer_max.. offer =l= 50;
```

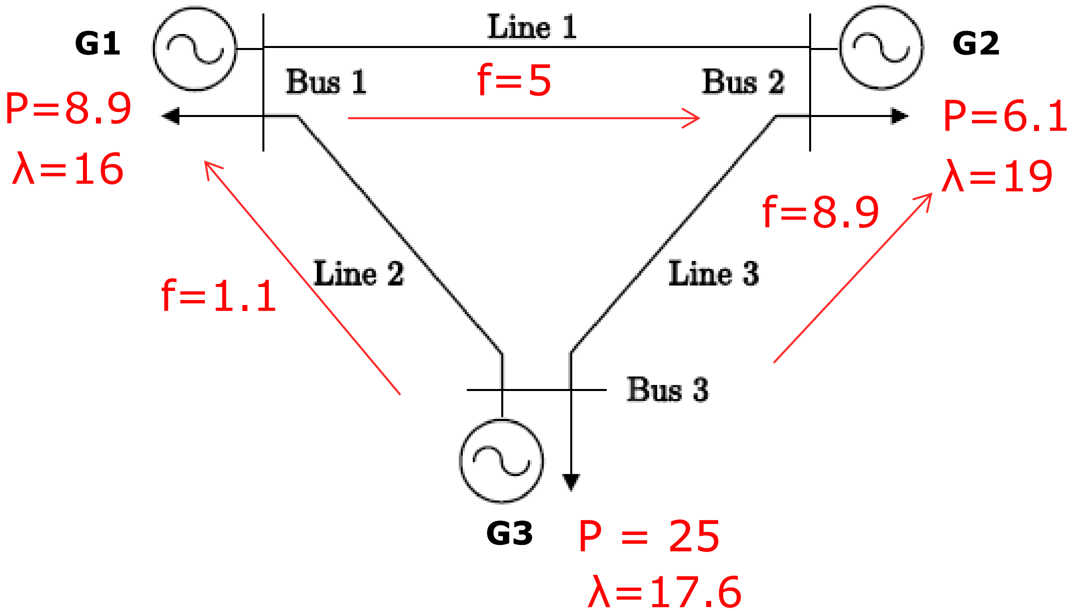
Need to add a bound on the price offer! Why?

```
* DA constraints
  node_balance_1.. P('i1') + B('l1')*(theta('n2')-theta('n1')) + B('l2')*(theta('n3')-theta('n1')) =e= Load('d1');
  node_balance_2.. P('i2') + B('l1')*(theta('n1')-theta('n2')) + B('l3')*(theta('n3')-theta('n2')) =e= Load('d2');
  node_balance_3.. P('i3') + B('l2')*(theta('n1')-theta('n3')) + B('l3')*(theta('n2')-theta('n3')) =e= Load('d3');
  slack_bus.. theta('n3')=e=0;
  Prod_max(i).. P(i)-P_max(i)=l=0;
  Prod_min(i).. -P(i)=l=0;
  flow_max_1.. B('l1')*(theta('n1')-theta('n2'))-Fmax('l1')=l=0;
  flow_min_1.. -B('l1')*(theta('n1')-theta('n2'))-Fmax('l1')=l=0;
  flow_max_2.. B('l2')*(theta('n3')-theta('n1'))-Fmax('l2')=l=0;
  flow_min_2.. -B('l2')*(theta('n3')-theta('n1'))-Fmax('l2')=l=0;
  flow_max_3.. B('l3')*(theta('n2')-theta('n3'))-Fmax('l3')=l=0;
  flow_min_3.. -B('l3')*(theta('n2')-theta('n3'))-Fmax('l3')=l=0;
```

GAMS Code

```
* KKT conditions

  stat_g1..  offer - lambda('n1') + mu_max('i1') - mu_min('i1') =e=0;
  stat_g2..  c('i2') - lambda('n2') + mu_max('i2') - mu_min('i2') =e=0;
  stat_g3..  c('i3') - lambda('n3') + mu_max('i3') - mu_min('i3')  =e=0;

  stat_theta1.. B('l1')*(lambda('n1')-lambda('n2')+rho_max('l1')-rho_min('l1'))
                + B('l2')*(lambda('n1')-lambda('n3')-rho_max('l2')+rho_min('l2')) =e= 0;
  stat_theta2.. B('l1')*(lambda('n2')-lambda('n1')-rho_max('l1')+rho_min('l1'))
                + B('l3')*(lambda('n2')-lambda('n3')+rho_max('l3')-rho_min('l3')) =e= 0;
  stat_theta3.. B('l2')*(lambda('n3')-lambda('n1')+rho_max('l2')-rho_min('l2'))
                + B('l3')*(lambda('n3')-lambda('n2')-rho_max('l3')+rho_min('l3')) + gamma =e= 0;

  comp_gmax_1(i).. mu_max(i) =l= M0*u_mu_max(i);
  comp_gmax_2(i).. P_max(i)-P(i)=l= M0*(1-u_mu_max(i));

  comp_gmin_1(i).. mu_min(i) =l= M0*u_mu_min(i);
  comp_gmin_2(i).. P(i)=l= M0*(1-u_mu_min(i));

  comp_fmax_1(l).. rho_max(l) =l= M0*u_rho_max(l);
  comp_fmax_21.. Fmax('l1') - B('l1')*(theta('n1')-theta('n2')) =l= M0*(1-u_rho_max('l1'));
  comp_fmax_22.. Fmax('l2') - B('l2')*(theta('n3')-theta('n1')) =l= M0*(1-u_rho_max('l2'));
  comp_fmax_23.. Fmax('l3') - B('l3')*(theta('n2')-theta('n3')) =l= M0*(1-u_rho_max('l3'));

  comp_fmin_1(l).. rho_min(l) =l= M0*u_rho_min(l);
  comp_fmin_21.. Fmax('l1') + B('l1')*(theta('n1')-theta('n2')) =l= M0*(1-u_rho_min('l1'));
  comp_fmin_22.. Fmax('l2') + B('l2')*(theta('n3')-theta('n1')) =l= M0*(1-u_rho_min('l2'));
  comp_fmin_23.. Fmax('l3') + B('l3')*(theta('n2')-theta('n3')) =l= M0*(1-u_rho_min('l3'));

  model market /  all / ;
  solve market using mip minimizing cost;

  display
  cost.l, P.l , offer.l , lambda.l;
```

Results: Market Clearing (Perfect Competition)



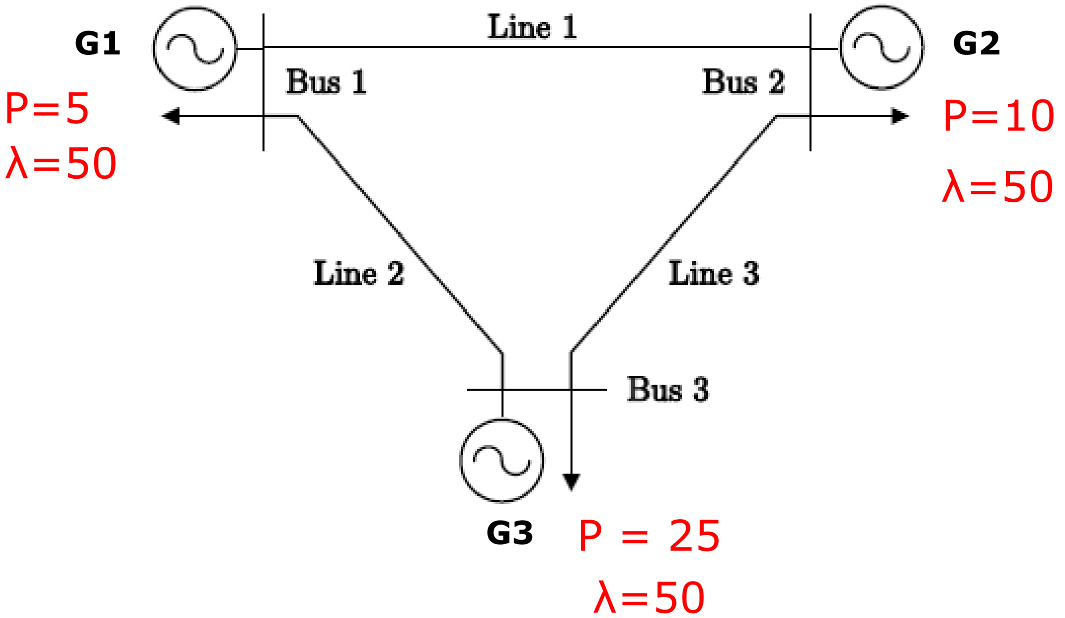
Cost = 633.4 $

Profit (g1) = 8.9$

Bus #	Capacity (MW)	Production cost ($/MWh)	Demand (MW)
1	20	16	5
2	10	19	20
3	25	15	15

Line #	From	To	Susceptance (S)	Capacity (MW)
1	1	2	100	5
2	1	3	125	10
3	2	3	150	10

Results: Strategic Offering



Offer max =50 $/MWh

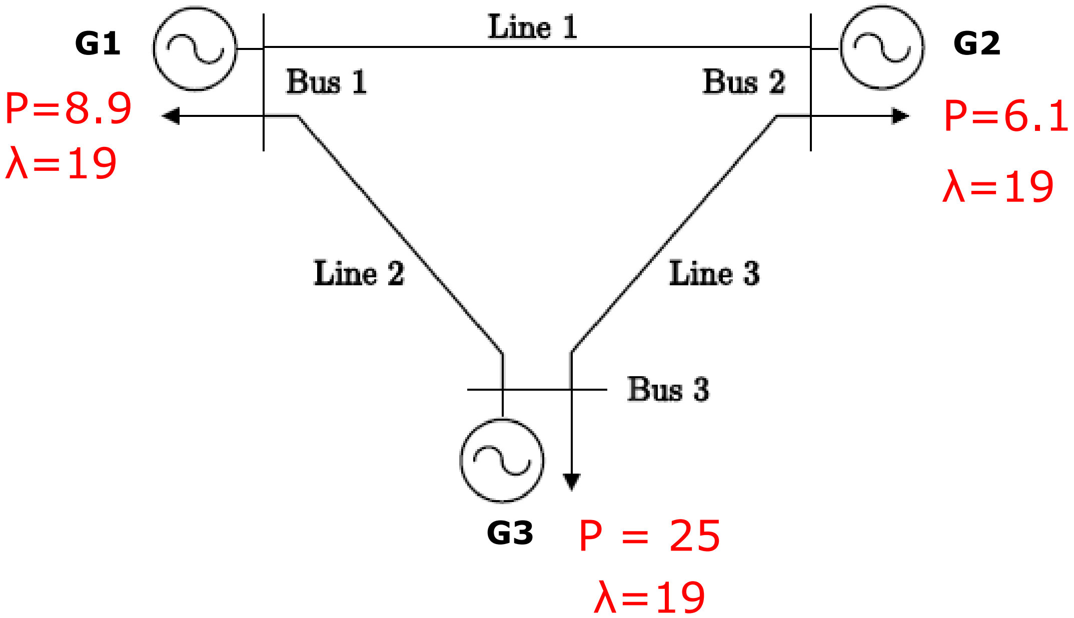
Offer =50 $/MWh

Profit (g1) = 170$

Bus #	Capacity (MW)	Production cost ($/MWh)	Demand (MW)
1	20	16	5
2	10	19	20
3	25	15	15

Line #	From	To	Susceptance (S)	Capacity (MW)
1	1	2	100	5
2	1	3	125	10
3	2	3	150	10

Results: Strategic Offering



Offer max =20 $/MWh

Offer = 19 $/MWh

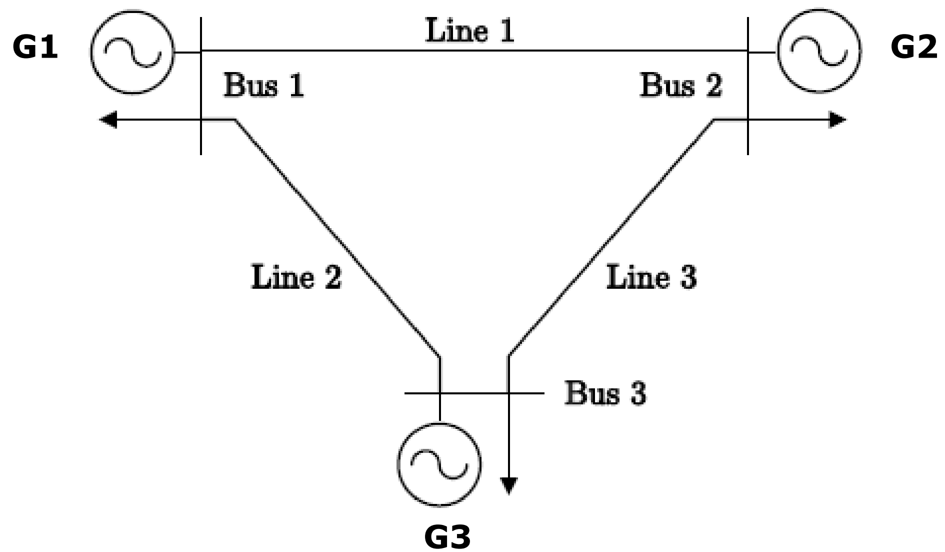
Profit (g1) = 35.6$

Bus #	Capacity (MW)	Production cost ($/MWh)	Demand (MW)
1	20	16	5
2	10	19	20
3	25	15	15

Line #	From	To	Susceptance (S)	Capacity (MW)
1	1	2	100	5
2	1	3	125	10
3	2	3	150	10

Introducing uncertainty…

Question: is it realistic to assume all parameters perfectly know? What are the sources of uncertainty?

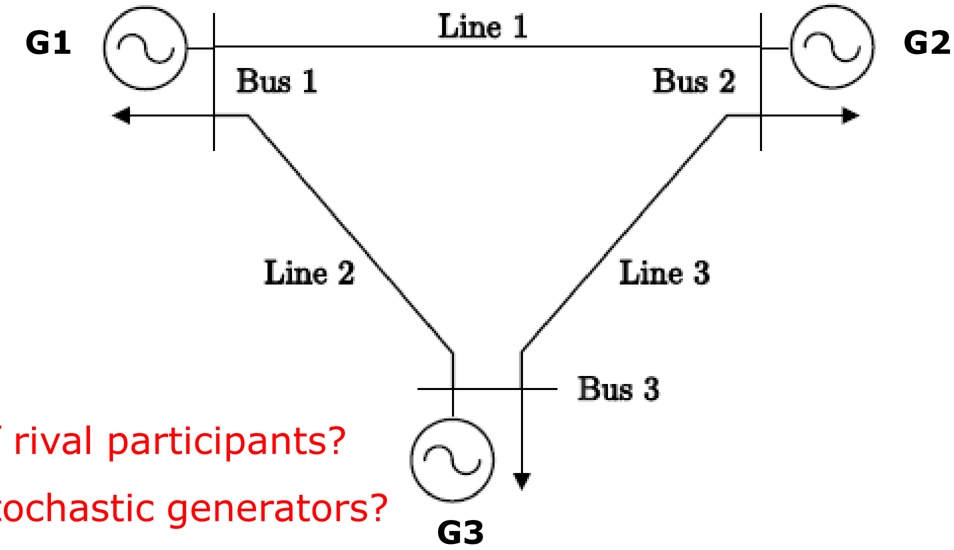


Bus #	Capacity (MW)	Production cost ($/MWh)	Demand (MW)
1	20	16	5
2	10	19	20
3	25	15	15

Line #	From	To	Susceptance (S)	Capacity (MW)
1	1	2	100	5
2	1	3	125	10
3	2	3	150	10

Introducing uncertainty…

Question: is it realistic to assume all parameters perfectly know? What are the sources of uncertainty?



- Supply curve of rival participants?
- Production of stochastic generators?
- Demand?

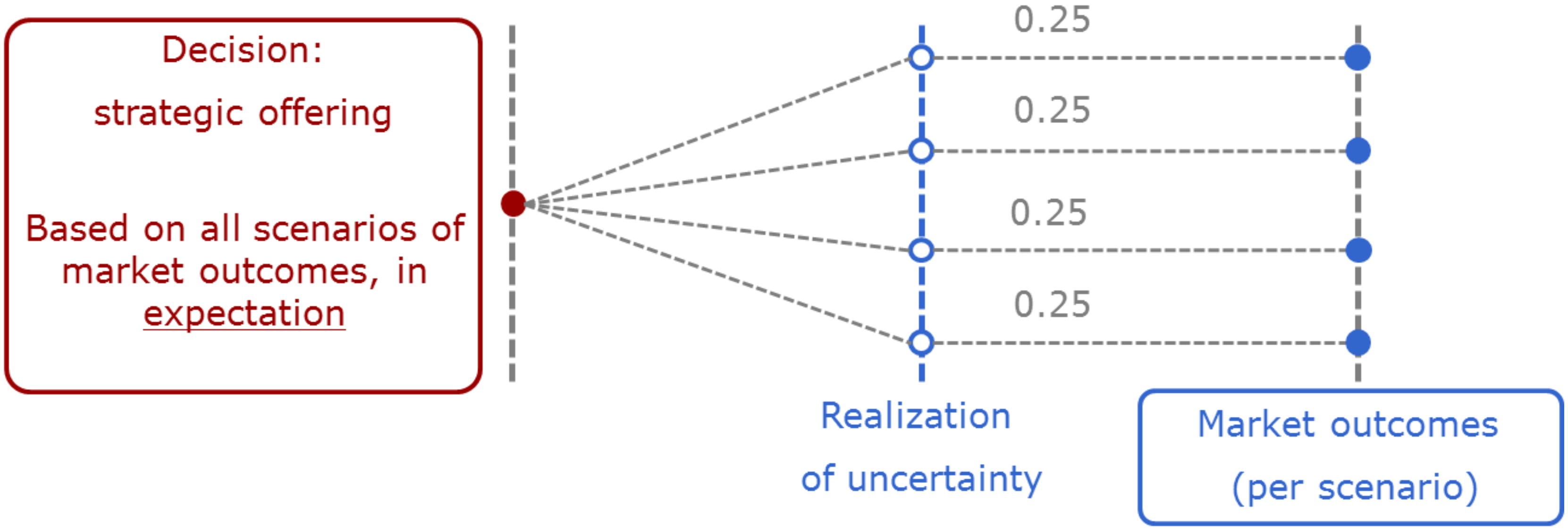
Bus #	Capacity (MW)	Production cost ($/MWh)	Demand (MW)
1	20	16	5
2	10	19	20
3	25	15	15

Line #	From	To	Susceptance (S)	Capacity (MW)
1	1	2	100	5
2	1	3	125	10
3	2	3	150	10

Stochastic Programming (reminder)

Strategic producer:

- considers each supply curve, demand, wind power available, as a potential "scenario" in the day-ahead market,
- evaluate market ouotcomes under every scenario
- determine strategic offering according to all scenarios (in expectation



Stochastic MPEC

Upper-level: strategic producer

Maximize profit

Subject to:

Lower-level: market clearing

Maximize social welfare

Subject to:

- Balance equations at nodes
- Transmission constraints
- Production bounds

Stochastic MPEC

Upper-level: strategic producer

Maximize Expected profit

Subject to:

market clearing (scenario 1)

Maximize social welfare

Subject to:

- Balance equations at nodes
- Transmission constraints
- Production bounds

…

market clearing (scenario N)

Maximize social welfare

Subject to:

- Balance equations at nodes
- Transmission constraints
- Production bounds

Stochastic MPEC

Upper-level: strategic producer

Maximize **Expected** profit

Subject to:

market clearing (scenario 1)

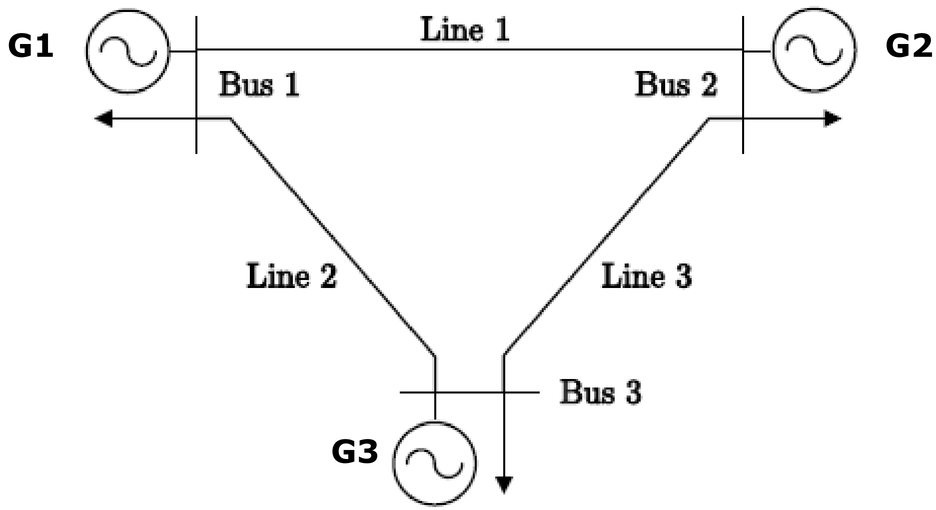
KKT conditions

...

market clearing (scenario N)

KKT conditions

Numerical Example 2



Bus #	Capacity (MW)	Production cost ($/MWh)	Demand (MW)
1	20	16	5
2	10	19	20
3	25	15	15

Line #	From	To	Susceptance (S)	Capacity (MW)
1	1	2	100	5
2	1	3	125	10
3	2	3	150	10

Exercise: Consider now d_3 as uncertain. Consider the 4 scenarios for d_3 (5,10,15,30) and modify your GAMS code to solve a stochastic MPEC