Fairness and Efficiency of Renewable Energy Source Allocation Mechanisms in Micro-Grids

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***Abstract*—This work studies the decentralized energy source selection problem for smart-grid consumers with heterogeneous energy profiles and risk attitudes, which compete for a limited amount of renewable energy sources. We model this problem as a non-cooperative game and tackle the challenging issue of the fair and efficient allocation of a limited resource among users with equal claim to it using a proportional allocation policy. We derive closed-form expressions of the renewable energy demand and social cost at NE under varying game parameters, and design a decentralized algorithm to reach these NE states. A numerical evaluation provides insights on the efficiency of this decentralized scheme compared to a centralized one. Finally, we compare our decentralized scheme in terms of fairness with a decentralized scheme that uses the naive equal sharing policy.**

I. Introduction, Background & Contributions

The large scale penetration of distributed, stochastic and non-dispatchable Renewable Energy Sources (RESs) has triggered the need for energy management solutions in distribution systems [1], [2]. In the meantime, growing environmental and societal awareness, coupled with advances in metering and control technologies have allowed for a more active involvement of end-users in managing their energy consumption []. In this context, smart-grids, which locally coordinate distributed energy resources (DERs) production and consumption, have become a viable and efficient solution to facilitate the integration of renewables into distribution grids and reduce energy procurement costs for consumers [3].

Several works in the literature study demand response programs (DRPs) as an essential mechanism of smart-grids for exploiting demand side flexibility, reducing energy procurement costs, and providing services to the grid. A thorough literature review of the benefits and challenges of various DRPs in smart-grids can be found in [4]–[7]. Specifically, this paper focuses on indirect DRPs, which, in contrast with direct load control, incentivize self-interested consumers to independently and in an uncoordinated manner adapt their consumption in response to price signals. While indirect DRPs may not lead to a globally-optimal energy dispatch, they present benefits, namely (i) scalability and (ii) privacy awareness, which render them desirable compared to traditional direct load control approaches [8], [9].

As consumers participating in indirect DRPs are self-interested and their decisions are independent and uncoordinated, game theory is the most suitable tool to design and analyze these DRPs [10], [11]. In particular, a focus on the literature has been placed on designing efficient price signals for deferrable loads using game theoretic tools. In particular, the game-theoretical frameworks developed in the literature provide models of consumers' decisions in response to price signals. The dynamic pricing scheme introduced in [12] incentivizes consumers to adapt their load profile so that they are conveniently supplied by the providers. This work highlights the importance of information (particularly of load profiles) sharing in order to reduce cost overheads. Similarly, the decentralized DRP developed in [13] uses non-cooperative game theory to design appropriate dynamic prices to control the grid load at peak hours. In [14], prices are derived by formulating the electricity provider's cost minimization problem, which considers consumers' device-specific scheduling flexibility and the provider's cost of purchasing electricity from an electricity generator.

The aforementioned works rely on the exchange of information between a centralized entity and the consumers to design efficient pricing schemes. Further works have focused on developing distributed control algorithms to achieve a Nash equilibrium (NE) in a decentralized manner in indirect DRPs while preserving privacy and independence of the decision of the consumers. In [15], a multi-period DRP in presence of storage is performed via Model Predictive Control (MPC) over a Nash non-cooperative game. Under perfect information, the unique NE strategies are shown to be Pareto optimal. The use of forecasts breaks the condition of perfect information but the NE are re-computed using MPC with updated forecasts so as to be closer to Pareto optimality. Similarly, the work in [9] thoroughly analyzes multi-period decentralized DRPs with hourly prices via a game theoretic viewpoint and proposes MPC algorithms to reach a NE while respecting users' privacy.

Furthermore, while the aforementioned works show the existence of NE for various decentralized DRPs, they fail to derive closed-form solutions of this NE and to formally analyse the loss of efficiency resulting from the self-interested

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behavior of consumers compared to direct load control approaches. The authors in [16] have shown that the NE in a decentralized DRPs for an infinite population of consumers and electric vehicles with identical technical characteristics and preferences is efficient. However, these assumptions are impractical and quite restrictive. In contrast, the authors in [17] compare the centralized or decentralized ownership and control of RESs and storage using Stackelberg game models. The DRPs studied trade-off consumers' cost and convenience and empirically show that a centralized approach brings more benefits for both consumers and utilities. Yet, no bounds on the loss of efficiency is provided. Further works have focused on quantifying the loss of efficiency in decentralized DRPs, using the so-called Price of Anarchy (PoA) metric. The authors in [18] compute the PoA for a non-cooperative game and show that the efficiency loss is mainly due to the decentralization of information. Similarly, the authors in [19] study a decentralized DRP in which the central coordinator aims at setting the optimal price signals for deferrable loads solely based on renewable energy production forecasts. A game-theoretic analysis of this mechanism shows the existence of NE both for price-taker and price-anticipator consumers, and derives a bound on the PoA. Yet, the aforementioned papers, again do not account for competition across multiple energy sources and heterogeneous consumer profiles, namely energy demand needs and attitude towards risks.

The aforementioned works focus on the design and analysis of decentralized DRPs in which consumers compete for a single and unlimited resource across multiple time steps. This paper differentiates itself from these works by focusing on competition across multiple energy sources. This framework is similar to the multi-energy smart-grids considered in [20], [21]. More precisely, we consider the decentralized energy procurement problem of consumers in smart-grids with multiple energy sources, namely low-priced but limited-capacity renewable energy sources, as well as medium- to high-priced fossil-fuel-based power generation. Additionally, this paper covers a broad range of deferrable loads by considering consumers with heterogeneous preferences, namely their energy demand and attitude towards risk.

In the absence of a centralized dispatch,, consumers independently decide which resource to compete for. Consumers who compete for low-priced RESs during the day to serve their flexible loads incur a risk. Indeed, if the aggregate demand for RESs exceeds its available capacity, the available RES capacity is allocated among consumers based on a given *allocation policy*, and the excess demand is covered by high-priced peak-load generation. Alternatively, consumers may choose to engage their loads during night and pay a risk-free medium price. In essence, each consumer faces the dilemma of competing or not for a limited inexpensive resource: if they compete and are successful they incur a low resource cost; if they compete and fail, they incur a high cost; if they decide not to compete, they incur a medium cost. This general resource selection problem formulation with the ternary cost structure can model a wealth of resource selection cases, such as in the case of parking resources [22].

Without adequately-designed incentives, the intuitive tendency of self-interested consumers to opt for the low-priced but scarce RESs would lead to high demand for RESs, congestion, and lowered social-welfare. While the prices of these respective resources are fixed, an adequate allocation policy should aim at *efficiently* incentivizing consumers to adapt their flexible demand to avoid congestion. Additionally, to facilitate social-acceptance and consumers engagement an adequate allocation policy should satisfy a notion of *fairness*. However, the fair and efficient allocation of a limited resource among multiple players who have equal right to the resource but different levels of demand is challenging. As highlighted in [23], due to the subjective nature of *fairness*, various well-established notions of fairness have been introduced in the literature and no allocation policy is universally accepted as "the most fair". Indeed, various allocation policies, which satisfy one notion of fairness or another, may result in different levels of efficiency and stability. The work in [24] showed that proportional fairness, may provide higher efficiency and a lower "cost of fairness" than other axiomatically justified notions of fairness, by advantaging "strong players", i.e. consumers with high demand. However, the proportional allocation (PA) policy may also lack stability, as "weak players", i.e. consumers with low demand, may continuously change behavior/strategy to improve the acquired service. On the other hand, *equal sharing (ES)*, which is another well-established allocation policy that satisfies a certain notion of fairness, is known to provide greater stability than the PA since it allows small players to be fully satisfied and prevents strong players from obtaining more resources than other players, by allocating equal shares of resources. Yet, equal sharing may result in highly inefficient and wasteful utilization of energy resources. In the context of DRPs and the allocation of renewable resources, this is a major limitation to ES.

Given the described research gaps, the contributions of this paper are threefold:

- Firslty, we present a novel game-theoretic formulation of a decentralized DRP in which consumers in a smart-grid compete for multiple limited-capacity energy resources. We model a wide range of deferrable loads by considering consumers with heterogeneous preferences, namely energy demand and risk attitudes. This decentralized DRP provides a novel application for the PA policy with multiple energy sources. We theoretically analyze this decentralized DRP and we derive closed-form expressions for its stable operational points (NE). To the best of our knowledge, this paper is the first to propose, thoroughly analyze and evaluate a game-theoretic framework for the distributed, uncoordinated competition of consumers across different energy sources.
- Secondly, we formulate the centralized coordinated energy source allocation mechanism that minimizes the social-cost for all consumers; the results serve as benchmark for assessing the efficiency of the game-theoretic mechanism. We quantify the efficiency of the distributed, uncoordinated energy selection using the PoA metric with respect to the benchmark centralized solution. The efficiency of the proposed DRP is compared to another

well-know decentralized allocation policy, namely the ES policy.
- Finally, we provide a distributed, uncoordinated, iterative algorithm so that the users' decisions will reside at a Nash Equilibrium. Our algorithmic solution does not require from consumers to reveal privacy-sensitive information such as their individual constraints on deciding their demand level.

The rest of the paper is organized as follows. In Section II, we introduce the game-theoretic modeling approach. In Section III, we study the NE mixed strategies under different parameter values and the corresponding expected aggregate demand and expected aggregate cost. In Section IV, we investigate the solution via a centralized mechanism. Section V provides a distributed, uncoordinated algorithm with which the players can choose NE mixed strategies. In Section VI, we perform numerical evaluations and comparisons with emphasis on the Price of Anarchy metric. Finally, Section **??** concludes the paper.

II. Uncoordinated energy source selection game

A. Game set up

We consider a smart-grid with N consumers which have access to multiple energy sources in order to serve their daily flexible loads. During the day, they have access to (i) a limited RES capacity $\mathcal{ER} > 0$ (in energy units *e.g.*, kWh) at a low-cost price c_{RES} per unit of energy, and (ii) an unlimited peak-load production at a high-cost price $c_{nonRES,d} = \gamma \cdot c_{RES}$, with $\gamma > 1$. During the night, they have access to an unlimited base-load production, with a medium-cost $c_{nonRES,n} = \beta \cdot c_{RES}$, with $\gamma > \beta > 1$.

At the beginning of a given day, each self-interested consumer independently decides during which time period, i.e. day or night, they will supply their daily flexible load. Once engaged during a time period, their loads cannot be interrupted or shifted to another time period. Such loads can represent a variety of shiftable appliances that can be scheduled to run during a specific time period, such as water heaters, batteries, EVS, washing machines, etc.

If a consumer engages its load during the day, they compete with other consumers to use the low-cost RESs but incur the risk to be allocated the high-cost peak-load production during the day. Indeed, if too many consumers compete for RESs and their aggregate demand exceeds the available RESs capacity, the part of their load that cannot be covered by RESs must be covered by the peak-load production during the day. Due to this risk, we consider that risk-averse consumers who chose to engage their daily flexible load during the day can decide to engage only part of it. The remainder of their daily flexible load is then curtailed for that day, and transferred for the following day that the game is played. This behavior represents a variety of appliances that do not need to run every day, or energy storages that do not need to be fully charged at the end of the day. For instance, an EV owner computes their minimum (inflexible) daily energy load, representing the energy needed to cover their transportation needs for the day, as well as their flexible energy load, representing the additional energy needed to fully charge their EV. Then, if they decide to compete for RESs to charge their EV, they may be willing to engage their inflexible load plus only a part of their flexible energy load so as to avoid paying for the high-cost peak-load production for an unnecessary demand if RESs generation is not adequate. At the beginning of the following day that they play this game, they will update their daily inflexible and flexible energy loads and their risk attitude based on their new state-of-charge and their transportation needs.

In the following, we provide the mathematical definition of this uncoordinated *Energy Source Selection Game (ESSG)* for one single day.

Definition 1. *An* Energy Source Selection Game *is a tuple* $\Gamma = (\mathcal{N}, \mathcal{ER}, \{A_i\}_{i\in\mathcal{N}}, \{\vartheta_i\}_{i\in\mathcal{N}}, \mathbf{r}, \{\upsilon_{A_i,\vartheta_i}\}_{i\in\mathcal{N}})$*, where:*
- $\mathcal{N} = \{1, ..., N\}$*, is the set of energy consumers.*
- $\mathcal{ER} > 0$ *is the limited RES capacity in energy units.*
- A_i *is the action of player* i *taking values in the set of potential pure strategies* $\mathcal{A} = \{RES, nonRES\}$ *(which is the same for all consumers).* $\mathcal{A}$ *consists of the choices to engage their loads during the day to compete for RES (*RES*) or engage their loads during the night (*$nonRES$*).*
- $\vartheta_i \in \Theta = \{0, 1, ..., M-1\}$ *(*$M \leq N$*) is the type of consumer* $i \in \mathcal{N}$*, consisting of an energy load profile* U_{ϑ_i} *and a risk-aversion degree* $\mu_{\vartheta_i} \leq 1$*. The risk-aversion degree represents the aversion of consumers to risk their entire load during the day if playing* RES*. In particular, if a consumer* $i \in \mathcal{N}$ *plays* $A_i = nonRES$*, it engages its entire load profile* U_{ϑ_i} *at night. If a consumer plays* $A_i = RES$*, it engages a part of its load equal to* $\mu_{\vartheta_i} U_{\vartheta_i}$ *during the day, while the remaining of its load* $(1-\mu_{\vartheta_i})U_{\vartheta_i}$ *is curtailed or transferred for the following day that the game is played. Therefore,* $\mu_{\vartheta_i} = 1$ *represents a risk-seeking consumer* i *who risks its entire load when playing* RES*, and* $\mu_{\vartheta_i} < 1$ *a risk-conservative consumer.*
- $\mathbf{r} = [r_0, ..., r_{M-1}]^T$ *is a probability distribution with* $0 \leq r_\ell \leq 1$ *the probability that a consumer is of type* $\ell \in \Theta$*.*
- $\upsilon_{A_i,\vartheta_i}(.) : \mathbb{R}^M \to \mathbb{R}$ *is the cost function of a player* i *with type* ϑ_i *and action* A_i*.*

The ESSG is played once a day and the type of each consumer (energy load profile and risk aversion degree) may differ between consecutive games but remains fixed during a particular game. When taking decisions (on the variables $A_i, \forall i \in \mathcal{N}$) the consumers have information on (i) the price parameters (c_{RES}, β, γ), (ii) the available RESs capacity, $\mathcal{ER}$, (iii) the maximum possible expected aggregate demand for RESs (i.e., if everyone competes for RESs during the day), $D^{Total} = N \sum_{\ell\in\Theta} r_\ell \mu_\ell U_\ell$.

In this paper, we will study the game equilibria under mixed strategies. Let $\mathbf{p_\ell} = [p_{RES,\ell}, p_{nonRES,\ell}]^T$ be the *mixed strategy* of consumers of type $\ell \in \Theta$. This is a probability distribution assigning to each action in the set $\mathcal{A}$ a likelihood of being selected. A consumer of type ℓ with a mixed strategy $\mathbf{p_\ell}$ plays the game by randomly selecting an action in $\mathcal{A}$, with $p_{RES,\ell}$ being the probability of playing RES, and $p_{nonRES,\ell}$ the probability of playing $nonRES$. Note that a *pure strategy* is a special case of a mixed strategy where one action has a probability equal to 1 (and the remaining have 0). We denote with $\mathbf{p} = [\mathbf{p_0}^T; \mathbf{p_1}^T; ...; \mathbf{p_{M-1}}^T]$ the vector of mixed strategies for all consumer types.

In the remainder of the paper, we set the energy load profile of consumer i of type ϑ_i as $U_{\vartheta_i} = \epsilon_{\vartheta_i} E_{\vartheta_i}$ and the risk aversion degree as $\mu_{\theta_i} = \frac{1}{\epsilon_{\vartheta_i}}$, with $\epsilon_{\vartheta_i} \geq 1$. Then, $E_{\vartheta_i} = \mu_{\theta_i} U_{\vartheta_i}$ represents the load that a consumer i is willing to risk during the day if they play RES, which is called as the day-time energy demand. And $U_{\vartheta_i} = \epsilon_{\vartheta_i} E_{\vartheta_i}$ represents the entire load of the consumer that it engages during the night if they play $nonRES$. Also, $\epsilon_{\vartheta_i} = 1$ represents a risk-seeking consumer i, and $\epsilon_{\vartheta_i} > 1$ a risk-conservative consumer. Finally, we assume without loss of generality that $E_0 \leq E_1 \leq ... \leq E_{M-1}$.

B. *Energy allocation policy*

Based on the above game definition, the expected aggregate demand for RESs during the day, can be expressed as

$$D(\mathbf{p}) = N \sum_{\ell \in \Theta} r_\ell \; p_{RES,\ell} \; E_\ell. \tag{1}$$

If this expected aggregate demand exceeds the available RESs capacity $\mathcal{ER}$, and since the individual consumer loads cannot be interrupted or shifted, the excess demand must be covered by the peak-load production during the day.

This paper examines the proportional allocation (PA) policy, which satisfies the well-accepted notion of *proportional fairness*. Under PA, the share of RESs received by a consumer i of type $\vartheta_i \in \Theta$ that plays RES, $rse^{PA}_{\vartheta_i}(\mathbf{p})$, is proportional to its demand, and depends on the strategies of all consumers, such that

$$rse^{PA}_{\vartheta_i}(\mathbf{p}) \quad = \quad \frac{E_{\vartheta_i}}{\max(\mathcal{ER}, D(\mathbf{p}))} \mathcal{ER}. \tag{2}$$

Eq. (2) ensures that, if the available RES capacity $\mathcal{ER}$ is greater than the aggregate expected demand for RES, then each consumer $i \in \mathcal{N}$ competing for RES receives a RES share equal to its energy demand profile E_{ϑ_i}. Otherwise, it receives a share of the available capacity $\mathcal{ER}$ proportional to its energy profile.

C. *Cost function*

The cost function $\upsilon_{RES,\vartheta_i}(.)$ for a consumer i of type ϑ_i that plays RES depends on the strategies of all consumers and the chosen allocation policy, and can be expressed as

$$\upsilon_{RES,\vartheta_i}(\mathbf{p}) = rse^{PA}_{\vartheta_i}(\mathbf{p}) \cdot c_{RES} + (E_{\vartheta_i} - rse^{PA}_{\vartheta_i}(\mathbf{p})) \cdot c_{nonRES,d}. \tag{3}$$

Note that the load that is not enganged, i.e., $(1 - \mu_{\vartheta_i}) U_{\vartheta_i}$ does not incur any cost. Moreover, the cost $\upsilon_{nonRES,\vartheta_i}(.)$ for a consumer i of type ϑ_i that chooses $nonRES$ is

$$\upsilon_{nonRES,\vartheta_i} = \epsilon_{\vartheta_i} \cdot E_{\vartheta_i} \cdot c_{nonRES,n}, \tag{4}$$

and solely depends on that consumer's strategy.

III. Theoretical Game Analysis

In this section, we investigate the operational states of the distributed, uncoordinated energy source selection for both risk-conservative and risk-seeking consumers. The proofs of the theoretical results presented below that are not shown here, are available in the online appendix [].

First, we recall that the expected costs of each pure strategy in the support of a mixed-strategy NE are equal. Using the costs in (3) and (4), we obtain that the amount of RESs allocated to the type $\ell \in \Theta$ at a NE should satisfy:

$$rse^{PA,NE}_\ell(\mathbf{p^{NE}}) = \frac{\gamma - \epsilon_\ell \beta}{\gamma - 1} E_\ell, \; \forall \ell \in \Theta. \tag{5}$$

Thus, when combining the Energy Source Selection Game with the PA policy, the existence of a NE is under the condition

$$rse^{PA}_\ell(\mathbf{p}^{NE}) = rse^{PA,NE}_\ell(\mathbf{p}^{NE}), \forall \ell \in \Theta. \tag{6}$$

Therefore, for all cases, any existing mixed-strategy NE competing probabilities, $\mathbf{p}^{NE}$, are obtained so as to satisfy the relation (6). In the following we distinguish several cases based on the parameters values, namely the RES capacity, the risk aversion degrees, and the day-time energy demand levels.

A. ***Case*** 1***: The RES capacity, $\mathcal{ER}$, exceeds the maximum total demand for RES*** D^{Total} ***($\mathcal{ER} \geq D^{Total}$).***

As the consumers have knowledge of $\mathcal{ER}$ and D^{Total}, it is straightforward to show that the dominant-strategy for all consumers is to select the strategy RES. As a result, the competing probabilities that lead to equilibrium states are equal to 1 for all consumer types, and the expected aggregate demand for RES at NE is equal to D^{Total}.

In all the remaining cases, we assume that $\mathcal{ER} < D^{Total}$. Three distinct cases are defined with respect to the risk aversion degrees of the consumers and energy prices.

B. ***Case*** 2***: The risk aversion degrees for all consumer types satisfy*** $1 \leq \epsilon_\ell < \gamma/\beta$***,*** $\forall \ell \in \Theta$***.***

We distinguish the following sub-cases with respect to the day-time energy demand levels.

1) ***Sub-case*** $2(a)$***: The day-time energy demand levels satisfy*** $E_\ell \leq \mathcal{ER}\frac{(\gamma-1)}{(\gamma-\epsilon_\ell\beta)}$***, for all*** $\ell \in \Theta$*:* In this case, a mixed-strategy NE with the PA policy exists if and only if for every pair of consumers, i, j with day-time energy demand levels E_{ϑ_i}, E_{ϑ_j}, respectively, it holds that

$$\mathcal{ER}\frac{(\gamma-1)}{(\gamma-\epsilon_{\vartheta_i}\beta)} - E_{\vartheta_i} \quad = \quad \mathcal{ER}\frac{(\gamma-1)}{(\gamma-\epsilon_{\vartheta_j}\beta)} - E_{\vartheta_j}. \tag{7}$$

Proof. To derive the condition (7) we first consider a consumer i with type $\vartheta_i \in \Theta$ that plays RES. Then, we can substitute the left-hand side of (6) with (2) where the demand for RESs is expressed as $D(\mathbf{p^{NE}}) = E_{\vartheta_i} + \sum_{\ell \in \Theta} r_\ell \, (N-1) \, E_\ell \, p^{NE}_{RES,\ell}$. And, by also substituting the right-hand side of (6) with (5) for consumer i, we obtain:

$$\mathcal{ER}\frac{(\gamma-1)}{(\gamma-\epsilon_{\vartheta_i}\beta)} - E_{\vartheta_i} = \sum_{\ell \in \Theta} r_\ell \; (N-1) \; E_\ell \; p^{NE}_{RES,\ell}, \tag{8}$$

By analogy, we can re-write (8) for a consumer j with type $\vartheta_j \in \Theta \setminus \{\vartheta_i\}$ that plays RES as:

$$\mathcal{ER}\frac{(\gamma-1)}{(\gamma-\epsilon_{\vartheta_j}\beta)} - E_{\vartheta_j} = \sum_{\ell \in \Theta} r_\ell \; (N-1) \; E_\ell \; p^{NE}_{RES,\ell}. \tag{9}$$

Since the right-hand sides of (8)-(9) are equal, the left-hand sides will be also equal and (7) derives. $\square$

Under condition (7), the competing probabilities p_{RES,ϑ_i}^{NE} for each consumer i of type $\vartheta_i \in \Theta$ that lead to equilibrium states lie in the range:

$$\Bigg[\max\Bigg\{0, \frac{\mathcal{E}\mathcal{R}\frac{(\gamma-1)}{(\gamma-\epsilon_{\vartheta_i}\beta)} - E_{\vartheta_i} - \sum_{\ell\in\Theta\setminus\{\vartheta_i\}} r_\ell (N-1) E_\ell}{r_{\vartheta_i}(N-1)E_{\vartheta_i}}\Bigg\}, \min\Bigg\{1, \frac{\mathcal{E}\mathcal{R}\frac{(\gamma-1)}{(\gamma-\epsilon_{\vartheta_i}\beta)} - E_{\vartheta_i}}{r_{\vartheta_i}(N-1)E_{\vartheta_i}}\Bigg\}\Bigg]. \tag{10}$$

Furthermore, at NE, the expected aggregate demand for RES for any consumer type $\ell \in \Theta$ can be expressed as:

$$D(\mathbf{p}^{NE}) = \min\Big\{D^{Total}, \max\Big\{\Big[\mathcal{E}\mathcal{R}\frac{(\gamma-1)}{(\gamma-\epsilon_\ell\beta)} - E_\ell\Big]\frac{N}{(N-1)}, 0\Big\}\Big\}. \tag{11}$$

Remark 1. *If all consumers are risk-seeking (i.e., $\epsilon_{\vartheta_i} = 1, \forall i \in \mathcal{N}$), a NE can exist only if $E_0 = E_1 = ... = E_{M-1}$.*

Remark 2. *Note that condition (7) can hold, and therefore a NE can exist, only if $\epsilon_0 \le \epsilon_1 \le .. \le \epsilon_{M-1}$. Since by assumption, $E_0 \le E_1 \le ... \le E_{M-1}$, this means that consumers with lower day-time energy demand levels should be less risk-averse than those with higher ones.*

*2) **Sub-case** $2(b)$**: The day-time energy demand levels satisfy** $E_\ell > \mathcal{E}\mathcal{R}\frac{(\gamma-1)}{(\gamma-\epsilon_\ell\beta)}$ **for all** $\ell \in \Theta$:* In this case, the dominant strategy for all consumers is to play $nonRES$.

As result, the competing probabilities that lead to equilibrium states are equal to 0 for all consumer types $\vartheta_i \in \Theta$, and the expected aggregate demand for RES at NE is equal to 0.

*3) **Sub-case** $2(c)$**: There exist two distinct subsets of consumer types,** $\Sigma_1, \Sigma_2 \subset \Theta$**, such that** $\{E_\ell > \mathcal{E}\mathcal{R}\frac{(\gamma-1)}{(\gamma-\epsilon_\ell\beta)}, \forall \ell \in \Sigma_1\}$ **and** $\{E_\ell \le \mathcal{E}\mathcal{R}\frac{(\gamma-1)}{(\gamma-\epsilon_\ell\beta)}, \forall \ell \in \Sigma_2\}$:* For consumers whose types are in the set Σ_1, the dominant strategy is to play $nonRES$. For consumers whose types are in the set Σ_2, the mixed strategy NE is determined under the condition of (7).

Additionally, for consumers whose types are in Σ_1, the competing probabilities at NE are equal to 0, whereas, for consumers i of type $\vartheta_i \in \Sigma_2$, the competing probabilities that lead to a NE states lie in the range:

$$\Bigg[\max\Bigg\{0, \frac{\mathcal{E}\mathcal{R}\frac{(\gamma-1)}{(\gamma-\epsilon_{\vartheta_i}\beta)} - E_{\vartheta_i} - \sum_{\ell\in\Sigma_2\setminus\{\vartheta_i\}} r_\ell (N-1) E_\ell}{r_{\vartheta_i}(N-1)E_{\vartheta_i}}\Bigg\}, \min\Bigg\{1, \frac{\mathcal{E}\mathcal{R}\frac{(\gamma-1)}{(\gamma-\epsilon_{\vartheta_i}\beta)} - E_{\vartheta_i}}{r_{\vartheta_i}(N-1)E_{\vartheta_i}}\Bigg\}\Bigg]. \tag{12}$$

Finally, the aggregate demand for RESs in Σ_1 is equal to 0, and the total aggregate demand for RESs can be expressed as

$$D(\mathbf{p}^{NE}) = \min\Big\{D_{\Sigma_2}^{Total}, \max\Big\{\Big[\mathcal{E}\mathcal{R}\frac{(\gamma-1)}{(\gamma-\epsilon_\ell\beta)} - E_\ell\Big]\frac{N}{(N-1)}, 0\Big\}\Big\}, \tag{13}$$

where $D_{\Sigma_2}^{Total} = N\sum_{\tilde{\ell}\in\Sigma_2} r_{\tilde{\ell}} E_{\tilde{\ell}}$ is the maximum demand for RESs of the consumers whose types are in Σ_2.

Remark 3. *We note that Remark 2 now holds for all consumer types in Σ_2.*

*C. **Case** 3**: The risk aversion degrees satisfy** $\epsilon_\ell \ge \gamma/\beta$**,** $\forall \ell \in \Theta$**.***

In this case, the dominant strategy for all consumers is to play RES.

Therefore, the competing probabilities that lead to equilibrium states are equal to 1 for all consumer types, and the aggregate demand for RESs at NE is equal to D^{Total}.

*D. **Case** 4**: There exist two distinct subsets of consumer types,** $\Sigma_1, \Sigma_2 \subset \Theta$**, such that** $\Big\{\epsilon_\ell \ge \gamma/\beta,\ \forall \ell \in \Sigma_1\Big\}$**, and** $\Big\{\epsilon_\ell < \gamma/\beta,\ \forall \ell \in \Sigma_2\Big\}$**.***

By analogy with Case 3, for the consumers whose type is in the set Σ_1, the dominant strategy is to compete for RESs. By analogy with Case 2, for the consumers whose type is in the set Σ_2, three sub-cases $4(a)-(c)$ are defined with respect to their day-time energy demand levels. All the conditions derived in Case 2 can straightforwardly be extended to the consumer types in Σ_2, considering that all consumers in Σ_1 play RES. To do so, we consider that the available RES capacity for consumers in Σ_2 is equal to $\mathcal{E}\mathcal{R}$ minus the aggregate demand for RESs of consumer types in Σ_1. For the sake of concision we do not detail these conditions.

IV. Centralized Energy Source Allocation Mechanism

In this section, we study the centralized mechanism that allocates the different energy sources to the N consumers of the micro-grid.

A. Optimization Problem Formulation

In practice, in the centralized mechanism, all consumers, $i \in \mathcal{N}$ with type $\vartheta_i \in \Theta$ issue energy demand requests E_{ϑ_i} to a central micro-grid operator that optimally processes their requests and allocates resources. The centralized mechanism aims at minimizing the social cost of all consumers by optimally allocating this demand between the night and day.

In practice, this coordinator's energy source allocation policy is represented as an optimization problem which aims at minimizing the social cost $C(\mathbf{p}^{OPT})$ for all the consumers, defined as:

$$\begin{aligned} C(\mathbf{p}^{OPT}) = \min\Big\{&\mathcal{E}\mathcal{R}, N\sum_{\ell\in\Theta} r_\ell p_{RES,\ell}^{OPT} E_\ell\Big\} c_{RES} \\ &+ \max\Big\{0, N\sum_{\ell\in\Theta} r_\ell p_{RES,\ell}^{OPT} E_\ell - \mathcal{E}\mathcal{R}\Big\} c_{nonRES} \\ &+ N\sum_{\ell\in\Theta} r_\ell p_{nonRES,\ell}^{OPT} \varepsilon_\ell E_\ell c_{nonRES,N}, \end{aligned} \tag{14}$$

where, the decision variables of the centralized mechanism are the optimal probability distributions ($p_{RES,\vartheta_i}^{OPT}$, $p_{nonRES,\vartheta_i}^{OPT}$), which represent the optimal probability that consumer i with type ϑ_i engages its load during day, or during the night, respectively, for all consumers $i \in \mathcal{N}$. [1]

The optimal solutions of this centralized mechanism will provide a benchmark against which to evaluate the side-effects

[1]The vector $\mathbf{p}^{\mathbf{OPT}}$ is defined similarly to $\mathbf{p}^{\mathbf{NE}}$.

that stem from the distributed, uncoordinated energy source selection, in terms of social cost. In the following, we provide insights and analytical formulations for the energy allocation, the social cost and the PoA under the PA mechanism.

B. Optimal Centralized Solutions

We derive the properties of the solution provided by the centralized mechanism, $\mathbf{p^{OPT}}$, in four cases, identical to the ones defined in Section III.

*1) **Case** 1**: The RES capacity exceeds the maximum consumer demand**:* In this trivial case, the social cost reduces to:

$$C(\mathbf{p}^{OPT}) = N\sum_{\ell\in\Theta} r_\ell p^{OPT}_{RES,\ell} E_\ell c_{RES} + N\sum_{\ell\in\Theta} r_\ell p^{OPT}_{nonRES,\ell} \varepsilon_\ell E_\ell c_{nonRES,N}, \quad (15)$$

and the optimal solution of the centralized mechanism is $p^{OPT}_{RES,\vartheta_i} = 1$, and $p^{OPT}_{nonRES,\vartheta_i} = 0$, $\forall\vartheta_i \in \Theta$.

In all other cases considered, the RES capacity is lower than the maximum consumer demand.

*2) **Case** 2**: The risk-aversion degrees of all consumer types are lower than** γ/β:* In this case, the optimal probability distributions are shaped so that the total renewable energy capacity is utilized and the remaining demand is shifted to the night zone. Specifically, the social cost reduces to:

$$C(\mathbf{p}^{OPT}) = \mathcal{ER}\cdot c_{RES} + N\left[\sum_{\ell\in\Theta} r_\ell p^{OPT}_{nonRES,\ell} \varepsilon_\ell E_\ell\right] c_{nonRES,N} \quad (16)$$

Therefore, to derive $\mathbf{p^{OPT}}$ the centralized mechanism needs to minimize the night-rate cost, under the constraint that the total energy demand for RES equals the RES capacity:

$$N\sum_{\ell\in\Theta} r_\ell E_\ell p^{OPT}_{RES,\ell} = \mathcal{ER}. \quad (17)$$

This can be done using a software optimization tool such as Mathematica or MATLAB. And we show that the optimal competing probabilities $p^{OPT}_{RES,\vartheta_i}$ for all $\vartheta_i \in \Theta$ lie in the range:

$$\left[max\left\{0, \frac{\left[\mathcal{ER} - \sum_{\ell\in\Theta\setminus\{\vartheta_i\}} r_\ell N E_\ell\right]}{r_{\vartheta_i} N E_{\vartheta_i}}\right\}, min\left\{1, \frac{\mathcal{ER}}{r_{\vartheta_i} N E_{\vartheta_i}}\right\}\right]. \quad (18)$$

*3) **Case** 3**: The risk-aversion degrees of all consumer types are greater than** γ/β:* In this case, the optimal centralized mechanism directs all consumers to play RES, i.e., $p^{OPT}_{RES,\vartheta_i} = 1$ $\forall\vartheta_i \in \Theta$. This result is derived in a similar fashion with the result of Case 3 in Section III.

*4) **Case** 4**: The risk aversion degrees of certain consumer types are lower than** γ/β **(set** Σ_1**), while others are greater than** γ/β **(set** Σ_2**)**:* Then, it is an optimal solution for all consumers whose types are in the set Σ_1 to play RES, i.e., $p^{OPT}_{RES,\vartheta_i} = 1$ $\forall\vartheta_i \in \Sigma_1$. Furthermore, the social cost of the consumers whose types are in Σ_2 can be expressed as:

$$C_{\Sigma_2}(\mathbf{p}^{OPT}) = (\mathcal{ER} - \sum_{\ell\in\Sigma_1} r_\ell N E_\ell)\cdot c_{RES} + N\left[\sum_{\ell\in\Theta} r_\ell p^{OPT}_{nonRES,\ell} \varepsilon_\ell E_\ell\right] c_{nonRES,N} \quad (19)$$

where $\mathcal{ER} - \sum_{\ell\in\Sigma_1} r_\ell N E_\ell$ represents the remaining available RES capacity, i.e., the available RES capacity minus the aggregate demand of consumers whose types are in Σ_1, who all play RES. Therefore, the probability that consumers whose type is in the set Σ_2 play RES is optimized to minimize their night-time cost, while ensuring that they totally utilize the remaining RES capacity, such that:

$$N\sum_{\ell\in\Theta} r_\ell E_\ell p^{OPT}_{RES,\ell} = \mathcal{ER} - \sum_{\ell\in\Sigma_1} r_\ell N E_\ell. \quad (20)$$

This centralized problem can be solved using a software optimization tool. And, we show that the optimal competing probabilities $p^{OPT}_{RES,\vartheta_i}$ for all $\vartheta_i \in \Sigma_2$ lie in the range:

$$\left[\max\left\{0, \frac{\mathcal{ER} - \sum_{\ell\in\Theta\setminus\{\vartheta_i\}} r_\ell N E_\ell}{r_{\vartheta_i} N E_{\vartheta_i}}\right\}, \min\left\{1, \frac{\mathcal{ER} - \sum_{\ell\in\Sigma_1} r_\ell N E_\ell}{r_{\vartheta_i} N E_{\vartheta_i}}\right\}\right]. \quad (21)$$

By analogy with Case 2, this range is derived similarly to the one defined in (18).

C. PoA

The (in)efficiency of equilibrium strategies in the decentralized, uncoordinated mechanism is quantified by the Price of Anarchy (PoA) [25]. The PoA is expressed as the ratio of the worst case social cost among all mixed strategy NE, denoted as $C_w^{PA,NE}$, over the optimal minimum social cost of the centralized mechanism, denoted as $C(\mathbf{p^{OPT^*}})$, such that:

$$PoA^{PA} = \frac{C_w^{PA,NE}}{C(\mathbf{p^{OPT^*}})}. \quad (22)$$

First observe that $C(\mathbf{p^{OPT^*}})$ is uniquely determined for each particular case. Now, in order to obtain $C_w^{PA,NE}$ when there exist multiple possible NE (e.g., in Cases 2 and 4), we can solve a simple optimization problem to maximize the social cost $C^{PA,NE}(\mathbf{p^{NE}})$ with respect to $\mathbf{p^{NE}}$ and subject to the corresponding probability constraints for NE defined in Section III, using a software optimization tool.

Note that $C^{PA,NE}(\mathbf{p^{NE}})$ takes its optimal value (i.e, minimum value) when the night-time cost is minimized. This solution coincides with the optimal centralized solution and thus in this case PoA takes the optimal (unity) value.

V. Distributed Algorithm to Obtain NE

In this section, we design a distributed, uncoordinated algorithm that leads to mixed-strategies NE for the energy source selection game in cases where there exists no dominant-strategy for all consumers (i.e., Cases $2(a)$, $2(c)$, $4(a)$, and $4(c)$). In the presence of dominant strategies (i.e. Cases 1, $2(b)$, 3, and $4(b)$), the decisions of each player are independent on the other players' decisions and can be computed directly.

The proposed distributed iterative algorithm is based on a best response scheme for each consumer, and requires minimum information exchange between the consumers. In particular, there is no central coordinator or direct communication channels between consumers since the exchanged information can be just broadcasted. The steps of the proposed algorithm are summarized in Algorithm 1 below.

The outer iterative loop represents the algorithm's steps, and the inner loop iterates over all consumers who are randomly ordered in a list Σ and at each iteration, they update their strategies. As all consumers with a certain type share the same strategy, in each algorithm's step, although the algorithm iterates over all consumers, it practically computes sequentially the best response of each consumer type $\vartheta_i \in \Theta$ to the strategies that have been already been chosen by the other consumer types at this iteration, and updates their probabilities (p_{RES,ϑ_i}, p_{nonRES,ϑ_i}). So if consumer i's type is already assigned a probability by another consumer of the same type in a previous iteration of the inner loop, the consumer i just retrieves this probability value (line 14), otherwise it computes the best response of its type to the types that have already played (lines 17- 21).

The vector EQT is introduced to represent which consumer types have played at a given algorithm's step ($EQT(\vartheta_i) = 1$ if consumer type $\vartheta_i \in \Theta$ has played), and is re-initialized to $\mathbf{0}_M$ at the beginning of each outer iteration. At a given algorithm's step, after consumer i has played and if no other consumer with the same type has played before (in a previous inner iteration), the vector EQP that contains the NE probability distribution for all consumer types is updated (line 20) and $EQT(\vartheta_i) \leftarrow 1$ (line 19), denoting that this consumer type has played.

As we aim to limit information exchange, the strategies that have already been chosen by the other consumer types are not known. Instead the consumer types update a common variable X_Σ, which encodes that information as an aggregate amount, i.e., it is the total average energy competing for RES up to the current inner iteration. Based on the value of X_Σ in their turn, each consumer type $\vartheta_i \in \Theta$ computes their best response by minimizing their expected cost of energy (line 17), which is given by:

$$cost^{PA}(p_{RES,\vartheta_i}) = p_{RES,\vartheta_i}\big[rse^{PA}_{\vartheta_i}(\mathbf{n_i})c_{RES} + (E_{\vartheta_i} - rse^{PA}_{\vartheta_i}(\mathbf{n_i})) \cdot \gamma \cdot c_{RES}\big] + (1 - p_{RES,\vartheta_i})\epsilon_{\vartheta_i} \cdot E_{\vartheta_i} \cdot \beta \cdot c_{RES}. \tag{23}$$

The information that should be broadcasted among players includes X_Σ, EQP, and EQT. When a consumer plays broadcast this information set to the remaining ones.

One limitation of the best response is that the first consumer type to play can choose as much RES as they want. In order to mitigate this effect, we introduce a *capping system* at each inner iteration, which multiplies the best response with a parameter $cap \in [0, 1]$[2] (line 24), such that the adjusted response is

$$f^{rand}_{cap}(p_{RES,\vartheta_i}) = cap \cdot p_{RES,\vartheta_i}. \tag{24}$$

[2] cap can be constant through the algorithm or drawn from a uniform distribution. This will be discussed in the numerical evaluations.

As a result, after an outer iteration has been completed, the total available RES capacity may not have been allocated. If this happens, the algorithm requires additional outer iterations or steps to reach an equilibrium state. In practice the algorithm continues until one of the two following conditions hold: (i) a NE is reached, which means that the players do not wish to change their actions unilaterally, with respect to the previous iteration (line 25), or (ii) a maximum number of outer iterations (N_{iter}) is reached.

Algorithm 1: Distributed algorithm for uncoordinated NE.

1 **Input** N_{iter}: number of algorithm's steps
2 **Output** EQP: Vector of NE probability distributions for each consumer type;
3 **Initialization:**
4 $(p_{RES,\vartheta_i}, p_{nonRES,\vartheta_i}) \leftarrow (0, 1), \forall \vartheta_i \in \vartheta$;
5 If Case 2(a): $X_\Sigma \leftarrow 0$, $\Sigma \leftarrow$ all consumers ;
6 If Case 2(c): $X_\Sigma \leftarrow 0$, $\Sigma \leftarrow$ all consumers with types in Σ_2 ;
7 If Case 4:$X_\Sigma \leftarrow \sum_{\vartheta_j \in \Sigma_1 \setminus \{\vartheta_i\}} r_{\vartheta_j}(N-1)E_{\vartheta_j}$, $\Sigma \leftarrow$ all consumer with types in Σ_2 ;
8 $EQP \leftarrow \mathbf{p}$;
9 **for** $iter \leftarrow 1$ **to** N_{iter} **do**
10     $EQT \leftarrow \mathbf{0}_M$;
11     $EQP_{old} \leftarrow EQP$;
12   **for** *each consumer* $i \in \Sigma$ **do**
13   **if** $EQT(\vartheta_i) = 1$ **then**
14   Consumer i retrieves p_{RES,ϑ_i} from $EQP(\vartheta_i)$;
15   **end**
16   **else**
17             $rse^{PA}_{\vartheta_i}(p_{RES,\vartheta_i}) = \frac{E_{\vartheta_i} \cdot \mathcal{ER}}{X_\Sigma + (N-1)r_{\vartheta_i}p_{RES,\vartheta_i}E_{\vartheta_i} + E_{\vartheta_i}}$ $p^*_{RES,\vartheta_i} \leftarrow \arg\min_{p^{PA}_{RES,\vartheta_i}} cost^{PA}(p_{RES,\vartheta_i})$, from Eq. (23);
18             $p^{cap}_{RES,\vartheta_i} \leftarrow f_{cap}(p^*_{RES,\vartheta_i})$, from Eq. (24);
19             $EQT(\vartheta_i) \leftarrow 1$, ;
20             $EQP(\vartheta_i) \leftarrow EQP(\vartheta_i) + p^{cap}_{RES,\vartheta_i}$;
21   Consumer i retrieves p_{RES,ϑ_i} from $EQP(\vartheta_i)$;
22             $X_\Sigma \leftarrow +(N-1)r_{\vartheta_i}p^{cap}_{RES,\vartheta_i}E_{\vartheta_i}$
23   **end**
24   **end**
25   **if** $|EQP - EQP_{old}| \leq tol$ **then**
26   Exit;
27   **end**
28 **end**

Privacy concerns can be handled by encrypting the values of EQT and EQP at each iteration and appropriately authenticating users. Then, if the number of consumers N is much higher than the number of energy demand profiles M, inferring the consumers' energy profiles is difficult. However, if the first and second consumers to play are of the same type, then the second in row consumer may infer the type of the first one and so on. To avoind this we should enforce that the second consumer type to play does not have the same energy profile as the first one, which is a realistic requirement since practically, the number of consumer types will be large. In the special case where $N = M$, communicating the outcomes EQT and EQP is not needed; the computing consumer should only know the current value of X_Σ.

Finally, this algorithm schedules consumers' loads in day and night zones once, in the beginning of a day zone. However, it can be repeated several times in a non-real time scale e.g., in an intra-day time scale (e.g., every 1 h during the day zone) in a Model Predictive Control fashion, where at each repetition (i) the consumers reconsider their energy demand profiles and exclude loads that have been already served, (ii) the forecast of the RES, $\mathcal{ER}$ is updated.

VI. Numerical Evaluations

A. Case Study Setup

We consider a smart grid with $N = 1000$ consumers, divided into 5 distinct consumer types, with a maximum total demand for RESs $D^{Total} = 4250$. Table I summarizes the consumer types parameters. The type distribution and the day-

Type ℓ	0	1	2	3	4
E_ℓ (kWh)	2	3	5	10	15
r_ℓ	0.20	0.40	0.30	0.07	0.03

TABLE I
GAME PARAMETERS FOR RESIDENTIAL SMART-GRID.

time energy demand levels are selected to be consistent with European households [26]. Most households are moderately energy efficient (types 1 and 2), combined with many highly efficient households (type 0) and few inefficient ones (types 3 and 4). Consumers with type 0 are assumed to be risk-seeking ($\epsilon_0 = 1$) and the risk-aversion degrees of all other types are determined by (7), and are close to 1. We set the RES price as $c_{RES} = 1$ €/kWh.

The first numerical evaluation studies the proposed DRP under various values of the price parameters $\beta = \{2, 2.5\}$ and $\gamma = 3$, and varying available RES capacities $\mathcal{ER}$, ranging from 5% to 125% of D^{Total}.

The proposed DRP with PA policy is then compared to a DRP with the naive ES policy for reference. Under ES, a so-called *fair share* of RESs capacity is computed as

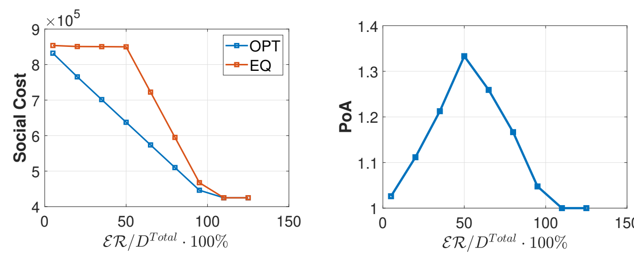
$$sh(\mathbf{p^{NE}}) = \frac{\mathcal{ER}}{N\sum_{\ell=0}^{M-1} r_\ell p^{NE}_{RES,\ell}}. \tag{25}$$

Under ES, consumers of type $\ell \in \Theta$ that play RES and have a day-time demand $E_\ell \leq sh(\mathbf{p^{NE}})$ are allocated their full day-time demand E_ℓ, as well as an extra energy equal to $(sh(\mathbf{p^{NE}}) - E_\ell)$ that will remains unused. On the contrary, the consumers of type $\ell \in \Theta$ that play RES and have a day-time demand E_ℓ larger than the fair share $sh(\mathbf{p^{NE}})$ will be allocated the fair share and their remaining day-time energy demand $E_\ell - sh(\mathbf{p^{NE}})$ will be served by the highly priced peak-load generation. Therefore, the share of RESs received by a consumer i of type $\vartheta_i \in \Theta$ that plays RES is $rse^{ES}_{\vartheta_i}(\mathbf{n_i}) = \min\left(E_{\vartheta_i}, sh(\mathbf{p^{NE}})\right)$. Note that this allocation policy may result in large inefficiencies due to unused RES capacity, even when the total aggregate demand for RESs $D(\mathbf{p^{NE}})$ is higher than $\mathcal{ER}$ and also in the centralized mechanism. Therefore, this allocation policy is solely used as a base-case comparison to the PA allocation considered in this work.

Finally, the convergence properties of the proposed decentralized algorithm under PA are studied for three capping systems, namely, (i) equal cap: cap is constant and equal to 0.1; (ii) random cap: cap is sampled from the uniform distribution $cap \sim U(0,1)$ (evaluated over multiple trials with varying values of cap); and (iii) no cap: equivalent to $cap = 1$.

B. Numerical Results

1) Social Cost and PoA under Varying Parameters: As illustrated in Fig. 1(a), the optimal social cost for the centralized mechanism, which equalize the RES demand and capacity while minimizing the night-time cost, decreases linearly with $\mathcal{ER}$. Indeed, since all risk-aversion degrees are equal

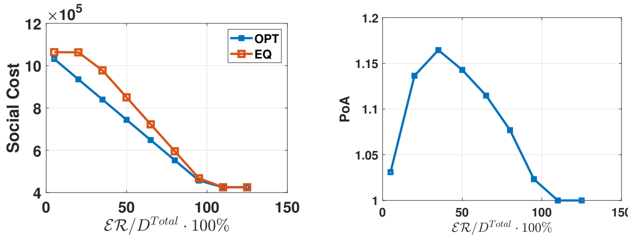


(a) Social Cost (in eurocents). (b) PoA.

Fig. 1. Social cost and PoA under PA rule for residential grid with $\beta = 2$.

or close to 1, the night-time cost can be approximated as $N\sum_{\ell\in\Theta} r_\ell \cdot p_{nonRES,\ell} \cdot \epsilon_\ell \cdot E_\ell \cdot c_{nonRES,n} \approx N\sum_{\ell\in\Theta} r_\ell \cdot p_{nonRES,\ell} \cdot E_\ell \cdot c_{nonRES,n} \approx N\sum_{\ell\in\Theta} r_\ell \cdot E_\ell \cdot c_{nonRES,n} - \mathcal{ER} \cdot c_{nonRES,n}$, which is constant with respect to the competing probabilities and linearly decreasing with $\mathcal{ER}$. Using the second in row approximation, we observe that the night-time cost is mostly affected by the day-time energy demands and its minimization by the centralized mechanism results in "big players" competing for RESs at the expense of smaller ones. It is indeed observed that consumers with lower day-time energy demand compete for RES with non-zero probability only if there is remaining available RES capacity when all consumers with higher day-time energy demand compete for RES with probability 1.

On the other hand, as seen in Fig. 1(a), for the decentralized DRP, the social cost decreases linearly with $\mathcal{ER}$, only for $\mathcal{ER} \in [0.5D^{Total}, D^{Total}]$. The social cost is almost constant with the initial increase in RES capacity due to the fact that consumers tend to over-compete for RES, even for lower values of RES capacity, as it is observed in the obtained values of the competing probabilities. However, for $\mathcal{ER} \in [0.5D^{Total}, D^{Total}]$, the social cost starts decreasing when $\mathcal{ER}$ increases because less highly priced day-time non-RES energy is required to cover the excess demand for RES. Note that, the mixed strategy NE solution should satisfy (8), except if this does not give acceptable probability values. Since the maximum value that the right hand side of (8) can take is $A^{Total} = D^{Total} - \sum_{\vartheta_l \in\Theta} r_{\vartheta_l} E_{\vartheta_l}$ (when all



(a) Social Cost (in eurocents). (b) PoA.

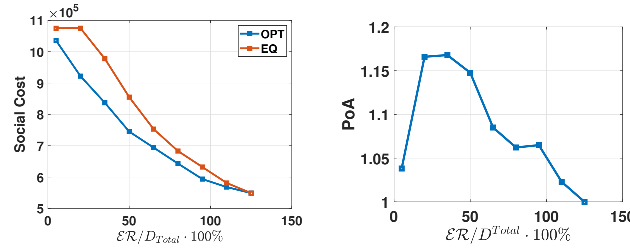
Fig. 2. Social cost and PoA under PA rule for residential grid with $\beta = 2.5$.

competing probablilities are equal to 1), this requirement is only possible if the left-hand side is less or equal than A^{Total}. With the applied parameter values, the left-hand side of (8) is equal to A^{Total} for $\mathcal{ER} \approx 50\% D^{Total}$. Therefore, for $\mathcal{ER} \geq 50\% D^{Total}$ (8) cannot be strictly satisfied and the closest we can get to its satisfaction is by setting the competing probabilities of all consumers equal to 1. Thus, all consumers compete for RES with probability equal to 1 and there is extra demand, which costs the high day-time prices.

As illustrated in Fig. 1(b), the PoA peaks for $\mathcal{ER} \approx 0.5 \cdot D^{Total}$, which is the point that the social cost for the uncoordinated mechanism begins decreasing. This graph can provide valuable insight into how much RES capacity we should install to increase the efficiency of the outcomes of the decentralized DRP. We can identify two zones of high efficiency, namely for low and high RES capacity. In the first zone, this is due to the small gains in cost offered by low RES capacity. In the second, the NE solution has almost converged to the optimal solution and thus social costs are optimal as well.

Finally, the value of $\mathcal{ER}$ at which the PoA reaches its peak (most inefficient outcome) depends on the system model parameters and most importantly on the price parameters β and γ, since the left-hand side of (8) depends on the quantity $\frac{(\gamma-1)}{(\gamma-\epsilon_{\vartheta_i}\beta)}$. In paticular, as illustrated in Fig. 2(a), both centralized and uncoordinated social costs are higher for a value of the parameter $\beta = 2.5$, compared to $\beta = 2$, because it results in higher night-time costs. Moreover, the uncoordinated social cost curve starts decreasing, and the PoA reaches its peak, for lower values of available RES capacity, namely at $\mathcal{ER} = 20\% \cdot D^{Total}$. This is also reflected in the PoA, which peaks at around 1.16, a much lower value than for $\beta = 2$, as seen in Fig. 2(b). Intuitively, this shows that the consumers are less likely to over-compete for RESs when the penalty for non-RES during the day increases.

2) Comparison to ES Policy: As observed in Fig. 3(a), both the centralized mechanism and the uncoordinated DRP achieve higher social costs under the ES than under the PA for all values of RES capacity. This is due to i) the unused RES capacity by consumers' types whose demand is lower than the fair share; and ii) the resulting increased day-time non-RES energy needed to cover the unsatisfied demand of consumers' types whose demand for RES is higher than the fair share. In addition, even for the centralized mechanism the competing probabilities may not be all equal to 1, even for $\mathcal{ER} = 125\% \mathcal{ER}_{max}$. The mechanism may reduce the competing probabilities of smaller players in order to increase



(a) Social Cost (in eurocents). (b) PoA.

Fig. 3. Social cost and PoA under ES for residential smart grid with $\beta = 2$.

the RES utilization. Both centralized and uncoordinated curves decrease with increasing $\mathcal{ER}$, but not in a linear way contrary to the PA policy, due to the non-linearity of the cost functions with respect to $\mathcal{ER}$ under the ES policy. Moreover, we observe that the uncoordinated social cost under ES follows a similar trend as under PA, namely, it is constant for small values of $\mathcal{ER}$ and then starts to decrease. This sows that, similarly to the PA rule, consumers tend to over-compete for RES under the ES policy.

Additionally, as seen in Fig. 3(b), the ES policy achieves lower PoA that the PA policy for most values of the RES capacity. However, the decentralized DRP under ES achieves 100% efficiency only when the RES capacity reaches $\mathcal{ER} = 125\% D^{Total}$, whereas, for the PA policy, the PoA is equal to 1 for lower values of RES capacity $\mathcal{ER} \geq 110\% D^{Total}$. Hence, using the ES policy over the PA policy may be more expensive as more RES capacity should be installed for ensuring efficiency of the decentralized DRP. Furthermore, due to the non-linearity of the social cost function with $\mathcal{ER}$, the PoA curve does not decrease monotonously after the initial peak.

3) Evaluation of Distributed Algorithm: Here, we evaluate the performance and convergence of the distributed algorithm 1. For better visualization purposes, we have implemented the algorithm in a smart grid with $N = 500$ consumers divided into two consumer types, and using the following parameter values: $E_0 = 100$ kWh, $E_1 = 200$ kWh, $D^{Total} = 65000$ kWh, $r_0 = 0.7$, $r_1 = 0.3$, $\epsilon_0 = 1$, $\epsilon_1 = 1.004$, $c_{RES} = 100$ €/kWh, $\beta = 2$, $\gamma = 4$, and $\mathcal{ER} = 25\% \cdot D^{Total} = 16250$ kWh.

Tables II summarizes the evaluation results on the social cost, the demand for RESs and the PoA for the optimal centralized solution as well as for the solution of the distributed algorithm for the three capping systems.

	Social Cost (10^6)	**Demand**	**PoA**	**Iterations**
Centralized	11.37	16250	1	-
Equal cap	13.00	24321	1.14	18
Random cap	12.99	24286	1.14	17-27
No cap	13.01	24324	1.14	14

TABLE II
SOCIAL COST, DEMAND, PoA, AND NUMBER OF ITERATIONS UNTIL CONVERGENCE UNDER THE PA RULE FOR CENTRALIZED AND DISTRIBUTED ALGORITHMIC SOLUTIONS FOR DIFFERENT CAPPING SYSTEMS.

All three capping methods lead to similar social cost and PoA values. Thus, the choice of capping method can only

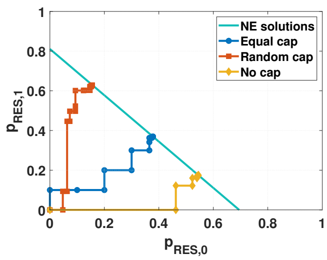


Fig. 4. Decentralized algorithmic solutions for different capping systems.

influence the competing probabilities to introduce a sense of fairness for sharing the RES capacity among the consumer types, without affecting the social cost. Furthermore, Table II highlights that if we do not apply a capping scheme the algorithm has the fastest convergence[3] at the expense of fairness to the allocation of RES. This happens, because we do not restrict the rate at which the solution reaches the NE line. Introducing a constant capping system renders convergence slightly worse, but within the same order of magnitude. Lastly, the random capping system provides no control over the convergence speed, and we observe a large variance in the required number of iterations to convergence. Note that lower cap values increase the number of iterations until convergence. Most importantly, for all three capping systems, we observe that the number of iterations until convergence is much lower than the number of players ($N = 500$), which showcases the efficiency of the algorithm.

Fig 4 shows the solution paths given by the distributed algorithm for all three capping systems. It can be observed that all solution paths converge to a theoretically proven NE, represented by the blue line. For the constant cap ($cap = 0.1$), the solution path oscillates around the 45° line. Therefore, the achieved NE solution consists of similar competing probability values for both consumer types. Lower constant cap values increase fairness among consumer types, and greatly dampen any bias towards any type. If the random cap method is implemented, the solution path is naturally random. Lastly, with the no cap system, the consumer type that plays first gains a considerable advantage.

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[3]The tolerance is set to $tol = 10^{-4}$.