5. Optical Resonator

The radiation emitted by most lasers contains several discrete optical frequencies, separated from each other by frequency differences which can be associated with different modes of the optical resonator. Each mode is defined by the variation of the electromagnetic field perpendicular and along the axis of the resonator.

It is common practice to distinguish two types of resonator modes: "Longitudinal" modes differ from one another only in their oscillation frequency; "transverse" modes differ from one another not only in their oscillation frequency, but also in their field distribution in a plane perpendicular to the direction of propagation. Corresponding to a given transverse mode are a number of longitudinal modes which have the same field distribution as the given transverse mode but which differ in frequency.

The symbols TEM_{mnq} and TEM_{plq} are used to describe the transverse and longitudinal mode structure of a wave inside the resonator for rectangular and cylindrical coordinates, respectively. The capital letters stand for "transverse electromagnetic waves" and the first two indices identify a particular transverse mode, whereas q describes a longitudinal mode. Because resonators that are used for typical lasers are long compared to the laser wavelength, they will, in general, have a large number of longitudinal modes. Therefore, the index q which specifies the number of modes along the axis of the cavity will be very high. The indices for the transverse modes, which specify the field variations in the plane normal to the axis, are very much lower and typically may be only the first few integers.

Although a resonator mode consists of a transverse and axial field distribution, it is useful to consider these two components separately because they are responsible for different aspects of laser performance. The spectral characteristics of a laser, such as line width and coherence length, are primarily determined by the longitudinal modes, whereas beam divergence, beam diameter, and energy distribution are governed by the transverse modes. In general, lasers are multimode oscillators unless specific efforts are made to limit the number of oscillating modes. The reason for this lies in the fact that a very large number of longitudinal resonator modes fall within the bandwidth exhibited by the laser transition, and a large number of transverse resonator modes can occupy the cross section of the active material.

5.1 Transverse Modes

It was the work of Fox and Li [5.1] that had the greatest impact on the development of the resonator theory. They utilized an integral equation technique to calculate the field distributions in a resonator that are reproduced over many successive round trips. Their homogeneous Fredholm integral equation has eigen functions that describe the field dependence in the transverse direction, and the associated eigenvalues determine the diffraction loss and the phase shift of the field distribution during each transit.

A laser resonator will oscillate in what is called an eigen mode. This is the transverse intensity distribution that occurs when a round trip through the cavity ends with the same distribution (mode) with which it started, except for amplitude losses due to diffraction.

The theory of modes in optical resonators has been treated in [5.1–3]; comprehensive reviews of the subject can also be found in [5.4, 5].

5.1.1 Intensity Distribution

In an optical resonator, electromagnetic fields can exist whose distribution of amplitudes and phases reproduce themselves upon repeated reflections between the mirrors. These particular field configurations comprise the transverse electromagnetic modes of a passive resonator.

Transverse modes are defined by the designation TEM_{mn} for Cartesian coordinates. The integers m and n represent the number of nodes of zeros of intensity transverse to the beam axis in the vertical and horizontal directions. In cylindrical coordinates, the modes are labeled TEM_{pl} and are characterized by the number of radial nodes p and angular nodes l. The higher the values of m, n, p, and l, the higher the mode order. The lowest order mode is the TEM_{00} mode, which has a Gaussian intensity profile with its maximum on the beam axis. For modes with subscripts of 1 or more, intensity maxima occur that are off-axis in a symmetrical pattern. To determine the location and amplitudes of the peaks and nodes of the oscillation modes, it is necessary to employ higher order equations which involve either Hermite (H) or Laguerre (L) polynomials. The Hermite polynomials are used when working with rectangular coordinates, while Laguerre polynomials are more convenient when working with cylindrical coordinates.

In cylindrical coordinates, the radial intensity distribution of allowable circularly symmetric TEM_{pl} modes is given by the expression

$$I_{pl}(r,\phi,z) = I_0 \varrho^l [\mathrm{L}_p^l(\varrho)]^2 (\cos^2 l\phi) \exp(-\varrho) \tag{5.1}$$

with $\varrho = 2r^2(z)/w^2(z)$, where z is the propagation direction of the beam and r, ϕ are the polar coordinates in a plane transverse to the beam direction. The radial intensity distributions are normalized to the spot size of a Gaussian profile; that is, $w(z)$ is the spot size of the Gaussian beam, defined as the radius at which the intensity of the TEM_{00} mode is $1/e^2$ of its peak value on the axis. L_p is the generalized Laguerre polynomial of order p and index l.

The intensity distribution given in (5.1) is the product of a radial part and an angular part. For modes with $l = 0$ (i.e., TEM_{p0}), the angular dependence drops out and the mode pattern contains p dark concentric rings, each ring corresponding to a zero of $\mathrm{L}_p^0(\varrho)$. The radial intensity distribution decays because of the factor $\exp(-\varrho)$.