ADM UE 2.28

$\sqrt{10}$ ist irrational

$a, b \in \mathbb{N}, b \neq 0$ $ggT(a,b)=1$

Ann. 1)

$\sqrt{10} \in \mathbb{Q} \iff \exists a \in \mathbb{Z}, b \in \mathbb{N}, a \wedge b$ sind nicht kürzbar : $\sqrt{10} = \frac{a}{b}$

$\left(\frac{a}{b}\right)^2 = 10$

$n | p^2 \Rightarrow n | p$ $p = p_1^{\alpha_1} \cdot \ldots \cdot p_r^{\alpha_r}$

$\frac{a^2}{b^2} = 10$ funktioniert immer wenn $n \in \mathbb{P}$

$p^2 = p_1^{2\alpha_1} \cdot \ldots \cdot p_r^{2 \cdot \alpha_r}$

$a^2 = b^2 \cdot 10$ $\Rightarrow \frac{a^2}{10} = b^2$ a^2 ist durch 10 teilbar

$n | p^2 \Rightarrow \exists k : p_k = n$

$\Rightarrow n | p$

Ann. $10 | a \Rightarrow \exists c \in \mathbb{Z}: a = 10 \cdot c$

$\underbrace{\frac{a^2}{10}}_{\in \mathbb{Z}} = \frac{(10 \cdot c)^2}{10} = \underbrace{10 \cdot c^2}_{\in \mathbb{Z}} \checkmark$

$10 | p^2 \iff 2 \cdot 5 p^2$

$\iff 2 | p^2 \wedge 5 | p^2$

$\iff 2 | p \wedge 5 | p$

$\Rightarrow 10 | p$

$\Rightarrow 10^2 \cdot c^2 = b^2 \cdot 10$

$10 c^2 = b^2$ $\Rightarrow 10 | b^2$

Ann. $10 | b \Rightarrow \exists d \in \mathbb{Z}: b = 10 \cdot d$

$\underbrace{\frac{b^2}{10}}_{\in \mathbb{Z}} = \frac{(10 \cdot d)^2}{10} = \underbrace{10 \cdot d^2}_{\in \mathbb{Z}} \checkmark$

$\Rightarrow 10 | a \wedge 10 | b$ ↯

Ann. 1) ist falsch
weil $a \wedge b$ nicht teilbar sein
dürfen

ADM VE 2.32

Alle $\sqrt[6]{z}$ in $\mathbb{C}$ für $z = -27 + 0i$ → weil $Re(z) < 0$

$r = |z| = \sqrt{27^2} = 27$ $\varphi = \arctan(0) = 0 + \pi$

$z = 27 \cdot (\cos(\pi) \cdot i\sin(\pi)) = [27, \pi]$

$w^6 = z$

$w_n = \left[\sqrt[n]{|z|}, \frac{\varphi + (n-1)2\pi}{n}\right]$

$w_0 = \left[\sqrt[6]{27}, \frac{\pi}{6}\right]$ $\Rightarrow \frac{\frac{\pi}{6} \cdot 180}{\pi} = \frac{180\pi}{6\pi} = 30° = \varphi$

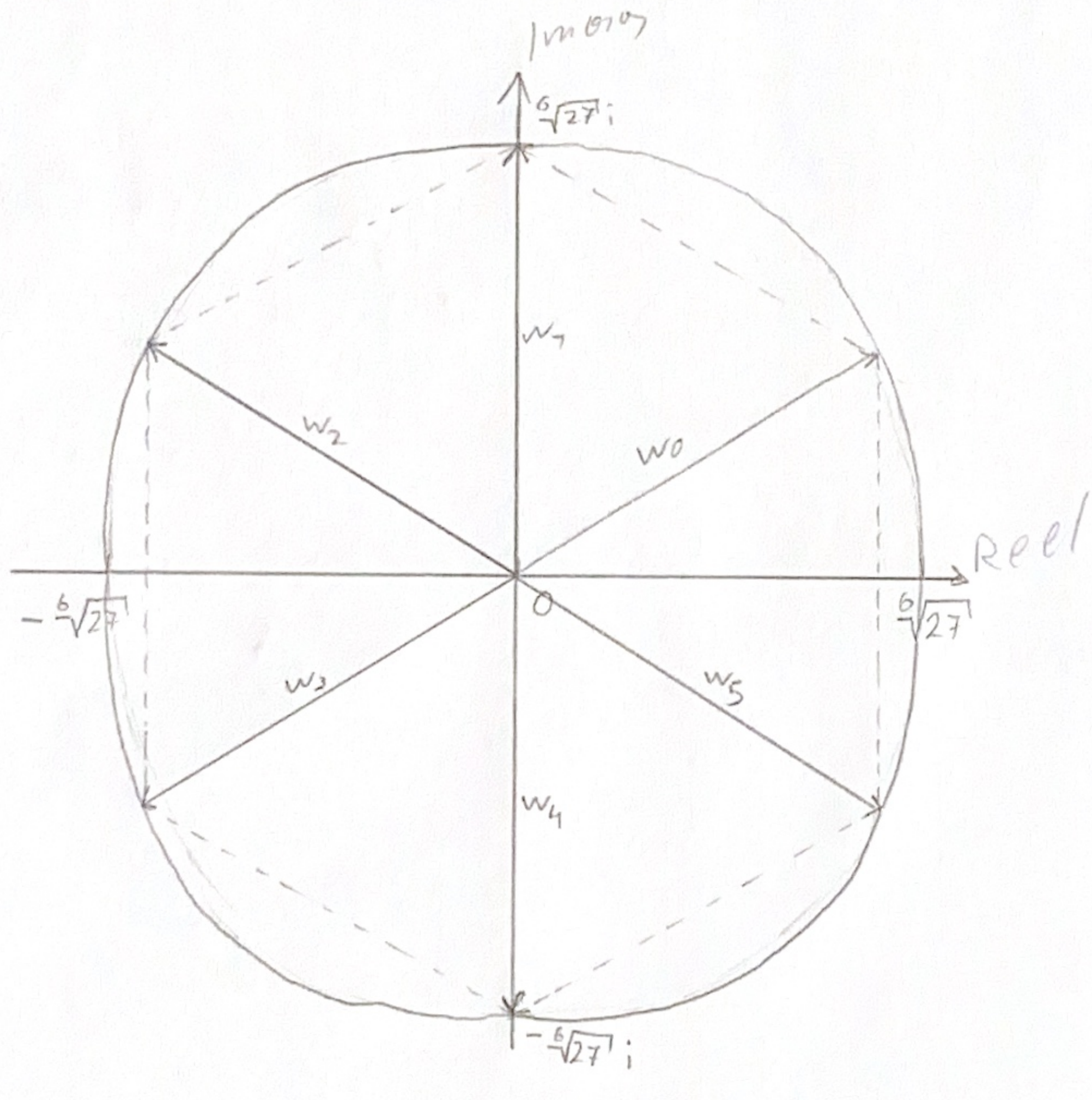
$w_1 = \left[\sqrt[6]{27}, \frac{\pi + 2\pi}{6}\right] = \left[\sqrt[6]{27}, \frac{3\pi}{6}\right]$ $\Rightarrow \frac{3\pi \cdot 180}{6\pi} = 90° = \varphi$

$w_2 = \left[\sqrt[6]{27}, \frac{\pi + 4\pi}{6}\right] = \left[\sqrt[6]{27}, \frac{5\pi}{6}\right]$ $\Rightarrow 150° = \varphi$

$w_3 = \left[\sqrt[6]{27}, \frac{7\pi}{6}\right]$ $\Rightarrow 210° = \varphi$

$w_4 = \left[\sqrt[6]{27}, \frac{9\pi}{6}\right]$ $\Rightarrow 270° = \varphi$

$w_5 = \left[\sqrt[6]{27}, \frac{11\pi}{6}\right]$ $\Rightarrow 330° = \varphi$



ADM UE 2.37

$\sqrt[5]{18-6\sqrt{3}i} \Rightarrow w^5 = z \qquad z = 18-6\cdot\sqrt{3}i$

$r' = \sqrt{18^2+(6\cdot\sqrt{3})^2} = \sqrt{432} = 12\sqrt{3}$

$\tan(\varphi) = -\frac{6}{18}\cdot\sqrt{3} = -\frac{1}{3}\cdot\sqrt{3} = \arctan(-\frac{\sqrt{3}}{3}) = -30^\circ = \varphi$

$\varphi = \frac{-30}{180}\cdot\pi = \frac{-\pi}{6}$ bleibt so weil z im 4. Quadr. ist

$z = [12\sqrt{3}; -\frac{\pi}{6}]$

$w_k = [\sqrt[n]{12\cdot\sqrt{3}}, \frac{-\frac{\pi}{6}+k\cdot 2\pi}{5}]$ für $k \in \mathbb{N}: 0 \le k \le n-1$

$= [\sqrt[2n]{432}, \frac{\pi\cdot(12k-1)}{30}]$

$w_0 = [\sqrt[10]{432}, \frac{-\pi}{30}]$

$w_1 = [\sqrt[10]{432}, \frac{11\pi}{30}]$

$w_2 = [\sqrt[10]{432}, \frac{23\pi}{30}]$

$w_3 = [\sqrt[10]{432}, \frac{35\pi}{30}] = [\sqrt[10]{432}, \frac{7\pi}{6}]$

$w_4 = [\sqrt[10]{432}, \frac{47\pi}{30}]$

ADM UE 2.43

Lösungen für $z^2 + 4z + 8 = 0$

$$z_{1,2} = -\frac{4}{2} \pm \sqrt{\frac{4^2}{4} - 8}$$

$$z_{1,2} = -2 \pm \sqrt{4-8} = -2 \pm \sqrt{-4} \qquad \sqrt{-4} = \sqrt{-1} \cdot \sqrt{4} = i \cdot \sqrt{4} = 2i$$

$$z_{1,2} = -2 \pm 2i$$

$$|z_{1,2}| = \sqrt{2^2+2^2} = \sqrt{8} = \sqrt{2} \cdot 2 = r_{1,2}$$

$$\arg(z_{1,2}) = \arctan\left(\pm \frac{2}{2}\right) = \arctan(\pm 1) \Rightarrow \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

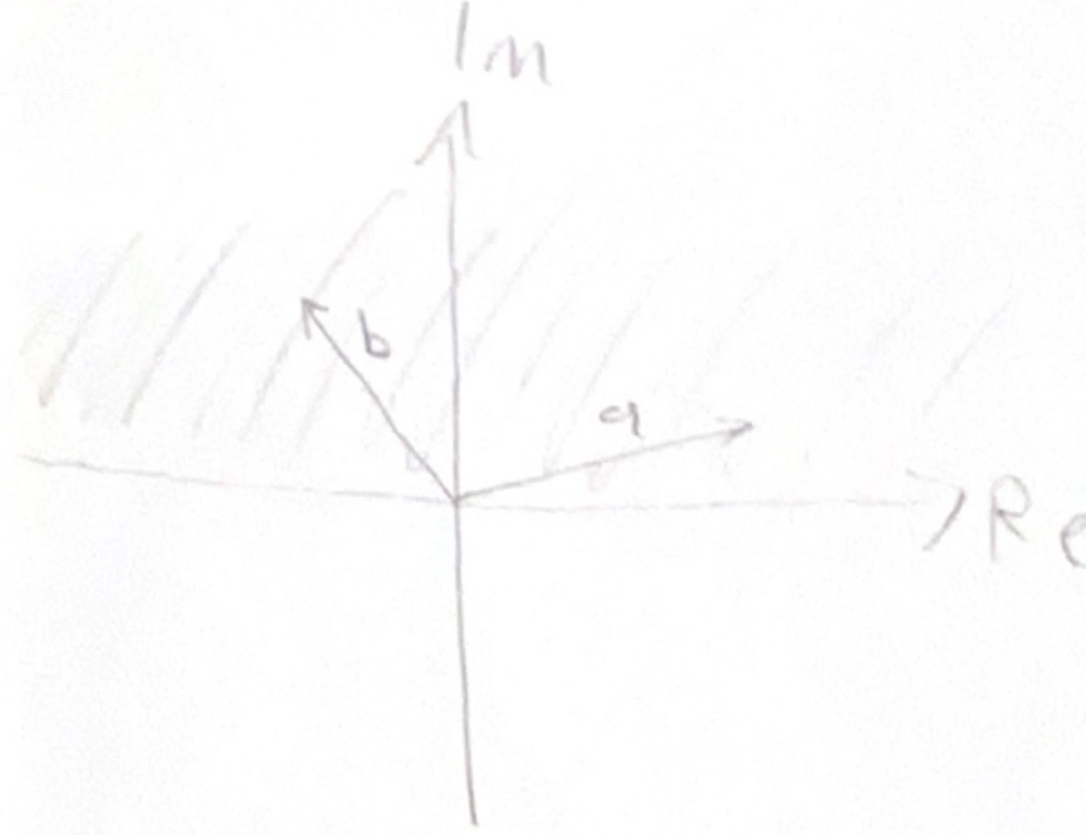
(i) weil $Re(z_{1,2}) < 0$ ist muss $\frac{\pi}{2} \leq \varphi \leq \frac{3\pi}{2}$ sein

$$\varphi_1 = \frac{3\pi}{4} \qquad \varphi_2 = \frac{5\pi}{4}$$

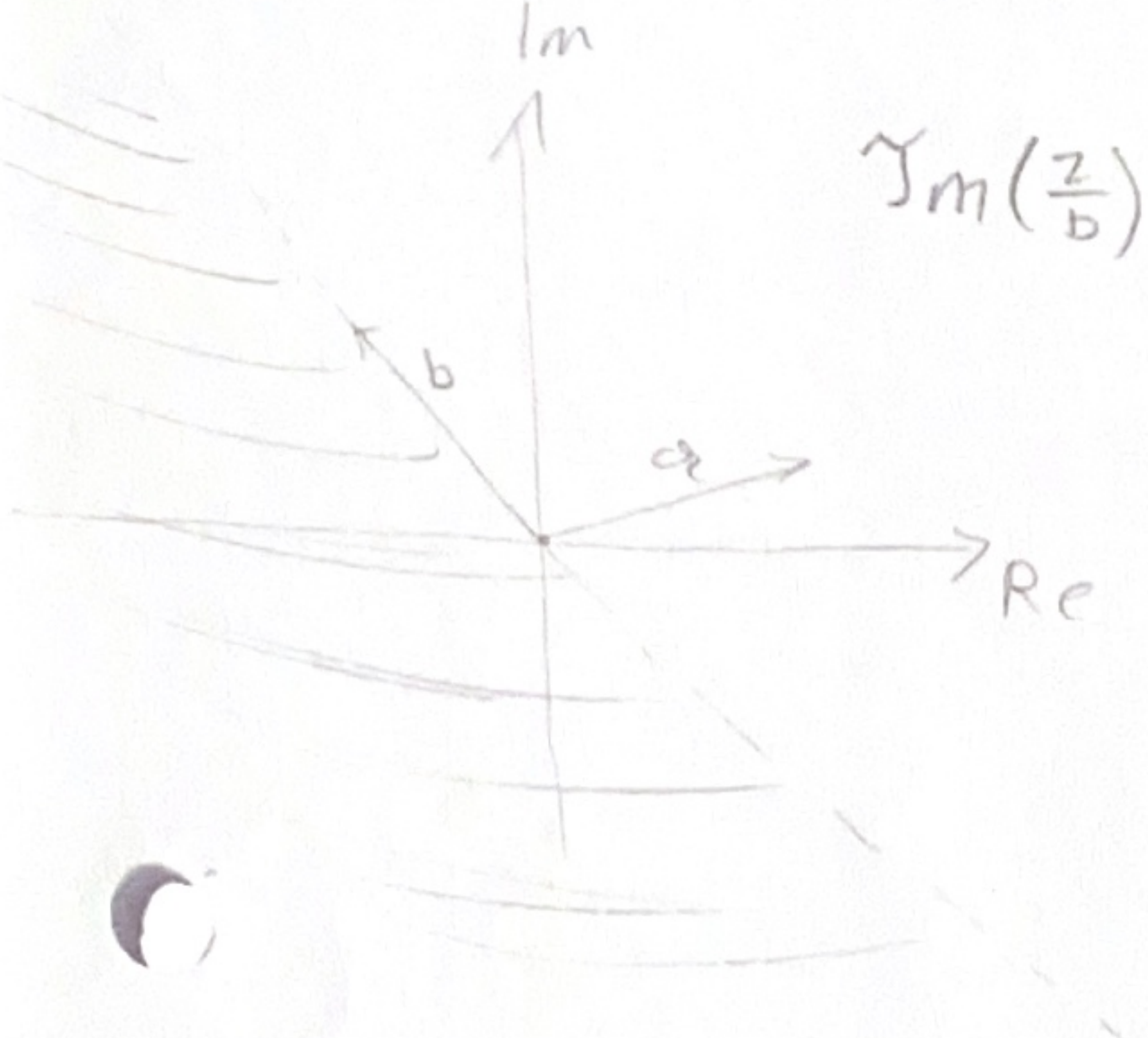
$$z_1 = -2 + 2i = \left[2\sqrt{2}, \frac{3\pi}{4}\right]$$

$$z_2 = -2 - 2i = \left[2\sqrt{2}, \frac{5\pi}{4}\right]$$

ADM UE 2.47

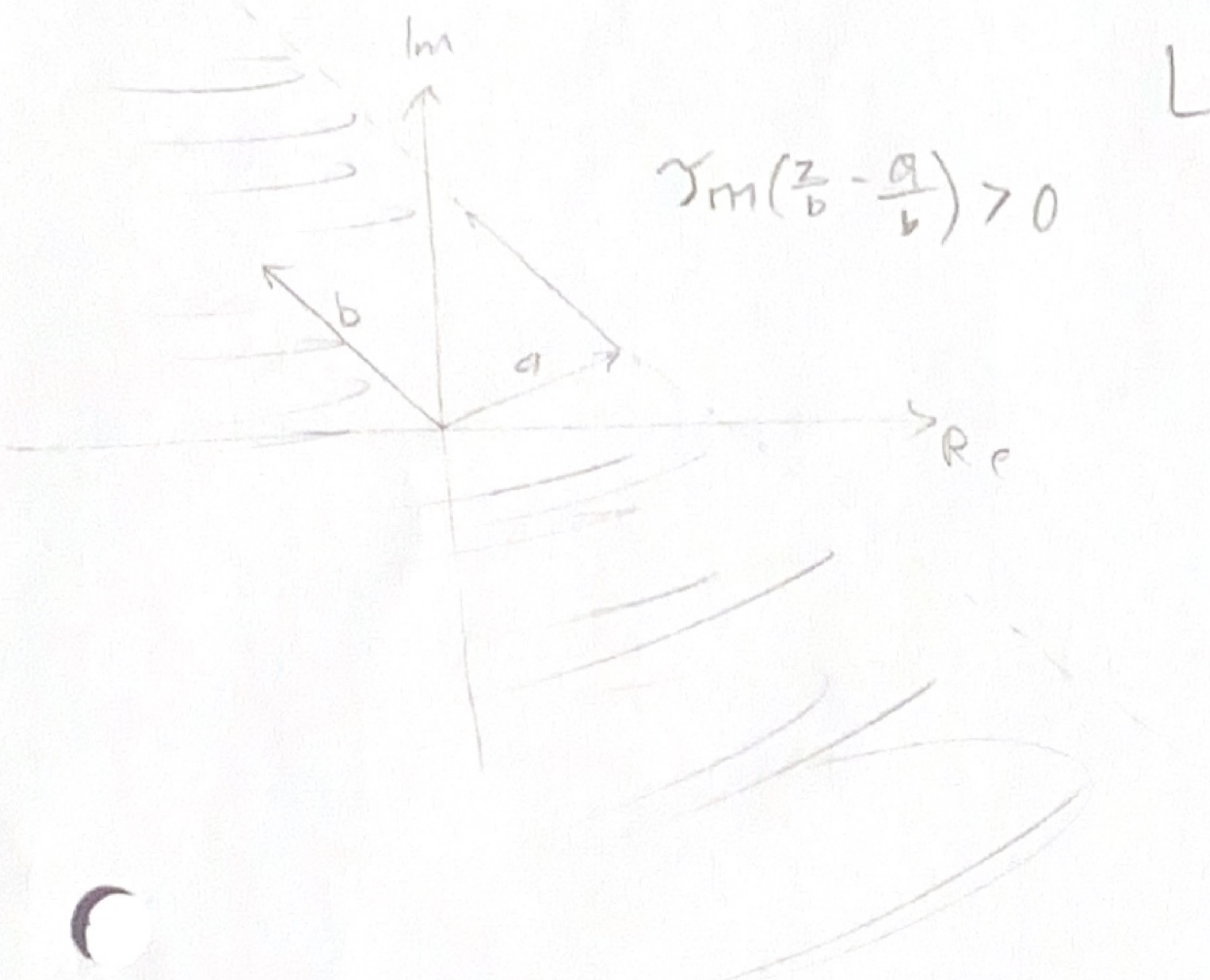


$\Im(z) > 0$



$\Im\left(\frac{z}{b}\right) > 0$

↳ Halbebene um $+\varphi_b$ drehen
weil $\frac{z}{b}$ subtraktion der Winkel bewirkt



$\Im\left(\frac{z}{b} - \frac{a}{b}\right) > 0$

↳ $\frac{a}{b}$ addieren weil anschließend
a subtrahiert wird.
Die Halbebene in der $\Im(a) > 0$ ist
muss auch um φ_b gedreht werden

ADM UE 2.58

man finde zwei Zahlen $x, y \in \mathbb{Z}$: $451x + 176y = 11$

(i) Lemma von Bèzout

$d = ggT(a,b)$, $a,b \in \mathbb{Z}\setminus\{0\} \Rightarrow \exists e,f \in \mathbb{Z}: e \cdot a + f \cdot b = d$

$\Rightarrow ggT(451, 176) = 11$

$11 = 451 \cdot x + 176 \cdot y$

$451 = 176 \cdot 2 + 99$

$176 = 99 \cdot 1 + 77$

$99 = 77 \cdot 1 + 22$

$77 = 22 \cdot 3 + \boxed{11} \rightarrow ggT$

$11 = 77 - 3 \cdot 22$

$= 77 - 3 \cdot (99 - 1 \cdot 77) = 4 \cdot 77 - 3 \cdot 99$

$= 4 \cdot (176 - 1 \cdot 99) - 3 \cdot 99 = 4 \cdot 176 - 7 \cdot 99$

$= 4 \cdot 176 - 7 \cdot (451 - 2 \cdot 176) = -7 \cdot 451 + 18 \cdot 176$

$x = -7$

$y = 18$

$-7 \cdot 451 + 18 \cdot 176 = 11$ ✓