# MATH363 Class Test 1

## Week 2

> $\hat{\beta} = (X^TX)^{-1}X^Ty$
>
> $E[\hat{\beta}] = \beta$
>
> $Var[\hat{\beta}] = \sigma^2(X^TX)^{-1}$

> $e^Te = Y^TY - Y^TX\hat{\beta}$

> 用MATLAB快速计算:
>
> ```
> % 转置 X.'
> % 矩阵乘法 *
> % 逆矩阵 inv(X)
> ```

## Week 3

> 判断矩阵（design matrix X）是否为full rank:
>
> 1. $det(X) \neq 0$
>
> 行列式如何计算：行列式等于矩阵任意一行或一列的元素与 $A_{ij} = (-1)^{(i+j)}M_{ij}$ 之积.
>
> 2. 看列之间是否存在线性相关

> 判断矩阵（design matrix X）是否为正交orthogonal:
>
> 看 $X^TX$ 是否为对角矩阵

> 多项式回归
>
> 多项式列是观测值 $x$ 的函数，各列之间需要正交：$Col_i^TCol_j = 0$

## Week 4

> Theorem 1
>
> 如果 $X$ full rank，那么 $\hat{\sigma^2} = \frac{1}{n-p}(Y - X\hat{\beta})^T(Y - X\hat{\beta})$ 是 $\sigma^2$ 的无偏估计

> Theorem 2
>
> $Y = X\beta + \varepsilon$ with 独立正态分布的 $\varepsilon \sim N_n(0, \sigma^2 I_n)$ , 如果 $X$ is of rank $p \leq n$, then
>
> 1. $\hat{\beta} \sim N_p(\beta, \sigma^2(X^TX)^{-1})$
> 2. $\hat{\beta}$ & $\hat{\sigma}^2$ 独立
> 3. $\frac{(n-p)}{\sigma^2}\hat{\sigma}^2 \sim \chi^2_{n-p}$

> Test for $\beta_k = b_k$:

$\frac{\hat{\beta}_k-\beta_k}{\sqrt{s_{kk}}\sqrt{\hat{\sigma^2}}} \sim t_{n-p}$

置信区间：$\hat{\beta}_k \pm t_{n-p}(\alpha/2)\sqrt{s_{kk}}\sqrt{\hat{\sigma^2}}$，其中 $s_{kk}$ 是 $(X^TX)^{-1}$ 对角线上的元素

$F-test$ 是否限制条件成立：

$\frac{(SS_{null}-SS_A)/r}{SS_A/(n-p)} \sim F_{r,n-p}$

$SS_A = e^Te = (Y-X\hat{\beta})^T(Y-X\hat{\beta}) = (n-p)\hat{\sigma^2}$

$SS_{null} = (Y-X\hat{\beta}_{null})^T(Y-X\hat{\beta}_{null})$

# Week 5

ANOVA Table

| Source | SS | Df | MS | F |
|---|---|---|---|---|
| Regression | $SS_{Total}-SS_{Resid}$ | $p$ | $(SS_{Total}-SS_{Resid})/p$ | $\frac{(SS_{Total}-SS_A)/p}{SS_A/(n-p-1)}$ |
| Residual | $SS_{Resid}\ (SS_A)$ | $n-p-1$ | $SS_{Resid}/(n-p-1)$ | |
| Total | $SS_{Total}\ (null\ model)$ | $n-1$ | | |

Tips:

$SS_{Total} = Y^TY - n\bar{Y}^2 = \sum_{i=1}^n (y_i-\bar{y})^2$

Coefficient of determination

$R^2 = \frac{SS_{Regre}}{SS_{Tot}}$

Adjusted $R^2$

$R^2_{adj} = 1-(1-R^2)\frac{n-1}{n-p-1}$

The hat matrix

$H = X(X^TX)^{-1}X^T$

Properties:

1. 对称性 $H^T = H$
2. 幂等 $H^2 = H$
3. 迹(对角元素和) $tr(H) = p$

Prediction Intervals

$\hat{Y}^\star \pm t_{n-p}(\phi/2)\sqrt{MS\hat{E}_{pred}}$

$\sqrt{MS\hat{E}_{pred}} = \hat{\sigma}^2(1+x^{\star T}(X^TX)^{-1}x^*)$

Confidence interval for mean response

$\hat{Y}^\star \pm t_{n-p}(\phi/2)\sqrt{MS\hat{E}_{conf}}$

> $$\sqrt{MS\hat{E}_{conf}} = \hat{\sigma}^2(x^{\star T}(X^TX)^{-1}x^*)$$