Homework #4

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1 PROBLEM A

Consider the family of densities $\frac{3}{2*(c^{1.5}-1)}t^{0.5}$, for 1 < t < c, 0 elsewhere. This is a one-parameter family, indexed by c > 1. Let's call this the ucd family. Develop functions with the following call forms:

- ducd(x,c) density at the values in x
  Since the ducd function returns the density of the value at x, we simply need to check the input parameters to make sure that 1 < x < c. If it is true, then we plug x, c into the family density function $\frac{3}{2*(c^{1.5}-1)}x^{0.5}$ and return the calculated value. If the parameters don't meet the requirements, then the ducd function returns 0.

- pucd(q,c) cdf at the values in q
  The pucd function returns the cdf at the values in q. To find the cdf we can use mailing tube (7.14)
  $$f_W(t) = \frac{d}{dt}F_W(t), -\infty < t < \infty$$
  So we see that the cdf function is found by taking the integral of the density function for the restraints of t. Therefore we can find the cdf equals:
  $$F_W(t) = \frac{t^{3/2}}{2*(c^{1.5}-1)} - \frac{1}{c^{1.5}-1}$$

From here we can simply plug in the user's input substituting x for t.

- qucd(q,c) quantiles at the values of q

  To find the quantiles at values of q, we need to look at our cdf function and substitute q for $F_W(t)$ since a quantile function is defined by (7.64)

  $$Q_X(s) = F_X^{-1}(s)$$

  So we think of the quantile function as the question: at what value of X do we have cumulative probability of q? This leads us to the equation:

  $$\begin{aligned} Q_X(q) &= F_X^{-1}(q) \\ &= (q + \frac{1}{c^{1.5} - 1}) * (c^{3/2} - 1)^{2/3} \end{aligned}$$

  Then similarly to what we do above, we simply plug in the values q,c into this equation to calculate qucd and return.

- rucd(n,c) generate n random variates

  We use runif() function to generate random probabilities from 0 to 1.

  Then, we pass these random probabilities to qucd() function because qucd() function will take probability and output value at which the random variable's probability is less than or equal to the given probability. P(X <= c) = q , c is outputted.

  In this way, we can generate n values of the random variable.

2 Problem B

We first used rpois(nreps, lambda) to generate random values of N with Poisson distribution and stored as a vector called 'boxes'. Using rbeta(nreps, alpha, beta), we generate random values of defectives probability with beta distribution and stored as a vector called 'p'. Then, we multiply the results together to obtain the number of defective boxes and stored as a vector called 'd'.

Calling the mean() function, we obtain the expected value of D and store it in a variable called 'ed'. And then, we solved for variance with the mailing tube: $Var(D) = E(D^2) - (ED)^2$.

To obtain $E(D^2)$, we call mean(d^2), and to obtain $(ED)^2$, we just square ed. Subtracting both values will obtain the variance of D.