Name: ____________________________

Directions: MAKE SURE TO COPY YOUR ANSWERS TO A SEPARATE SHEET FOR SENDING ME AN ELECTRONIC COPY LATER.

1. (20) Your chemistry professor announces the results of the midterm exam: Mean 18 and standard deviation 5. She says that since the scores were so low, she is going to multiply all scores by 2 and then add 10. What will be the new mean and standard deviation? Answer using R's **c()** notation, e.g. **c(88,-8)** if you think the new mean and standard deviation will be 88 and -8, respectively.

2. (25) Consider the ALOHA example, Sec. 2.5 (and $p = 0.4$, $q = 0.8$). Let O_k denote the number of original messages that are still pending at the end of epoch k, $k = 1, 2, ...$ We are just concerned with $k = 1$. Find $E(O_1)$.

3. (15) Suppose K and L are independent indicator random variables, with event probabilities p and q. Supply the reasons for each step in the following derivation, in which a and b are constants. The reasons should cite equations or properties, maybe algebra, say with Eqns. (2.10)-(2.13) as an example. You will have answers (a), (b) and (c), i.e. 6 lines in your electronic file.

$$
\begin{aligned}
Var(aK + bL) &= Var(aK) + Var(bL) \quad \text{(reason (a))} \\
&= a^2 Var(K) + b^2 Var(L) \quad \text{(reason (b))} \\
&= a^2 p(1-p) + b^2 q(1-q) \quad \text{(reason (c))}
\end{aligned}
$$

4. (20) Say we have a random variable X, of which we simulate many instances, resulting in an R vector **w**. We make the R call

```
mean(w > mean(w))
```

State what quantity this is approximating. Your answer must use math symbols such as E(), P(), Var(), X and punctuation — no code and no English.

5. (20) In the context of p.65, find $Cov(G_1, G_2)$.

Solutions:

1.

```
c(46,10)
```

2.

$$1 \times 2(0.4)(1-0.4) + 2 \times (0.4^2 + (1-0.4)^2) \tag{1}$$

3.a Eqn. (3.75)

3.b Property G

3.c Eqn. (3.79)

4. $P(X > EX)$

5.

$$\begin{aligned} Cov(G_1, G_2) &= E(G_1 G_2) - E(G_1) \cdot E(G_2) \qquad (2) \\ &= r - (\frac{2}{3})^2 \qquad (3) \end{aligned}$$

where

$$r = P(G_1 G_2 = 1) = \frac{2}{3} \cdot \frac{5}{8} \tag{4}$$

Note that $G_1 G_2$ is itself an indicator random variable.

Name: ____________________________

Directions: MAKE SURE TO COPY YOUR ANSWERS TO A SEPARATE SHEET FOR SENDING ME AN ELECTRONIC COPY LATER.

1. (10) Suppose X is the length of a random rod, in inches, and $Var(X) = 2.6$. Let Y denote the length in feet. Find $Var(Y)$.

2. (10) In the board game, Sec. 2.11, suppose we start at square 3 (no bonus, since we *start* there rather than *landing* there). Let X denote the square we land on after one turn. Find EX.

3. This problem concerns the Monty Hall example, pp.40ff.

(a) (15) Give the numbers of the "mailing tubes" in (3.1) and (3.2), respectively. Use a comma and/or spaces to separate the two equation numbers, e.g. "(2.1) (2.3)".

(b) (15) Consider (3.1). Say we change the left-hand side to $P(A = 2 \mid C = 2, H = 1)$. What would be the new numerical value of the numerator on the right-hand side?

4. (20) Look at the simulation code on p.26. Say we wish to find the expected value of S^2, where S is the sum of the **d** dice. Give a line of code, to replace line 11.

5. Consider the Preferential Attachment Graph model, Sec. 2.13.1..

(a) (10) Give the number of the "mailing tube" justifying (2.69).

(b) (10) Find $P(N_3 = 1 \mid N_4 = 1)$.

(c) (10) Find $P(N_4 = 3)$.

Solutions:

1.

$$(\frac{1}{12})^2 \cdot 2.6$$

2.

$$4 \cdot \frac{1}{6} + 5 \cdot \frac{1}{6} + 6 \cdot \frac{1}{6} + 7 \cdot \frac{1}{6} + 0 \cdot \frac{1}{6} + 1 \cdot \frac{1}{6}$$

3.a (2.8), (2.7)

3.b

$$(\frac{1}{3})(\frac{1}{3})(\frac{1}{2})$$

4.

```
mean(sums^2)
```

5.a (2.2)

5.b

$$(1/2)(2/4) \ / \ ((1/2)(2/4) + (1/2)(1/4))$$

5.c

$$(1/2)(1/4) + (1/2)(1/4)$$

Name: ____________________________

Directions: MAKE SURE TO COPY YOUR ANSWERS TO A SEPARATE SHEET FOR SENDING ME AN ELECTRONIC COPY LATER.

1. Consider Equation (3.64).

(a) (15) List (on one line), the equation number(s) of the mailing tubes used to justify the equality $Var(7 + 2I) = 4Var(I)$.

(b) (15) Give the equation number of the relation that justifies $4Var(I) = 4 \cdot 0.5(1 - 0.5)$.

2. (15) Give the number of the mailing tube that justifies (3.80).

3. Consider the variables G_i, p.56.

(a) (10) Find $P(G_2 = 1 \mid G_1 = 1)$.

(b) (15) Find $P(G_1 = G_2)$.

4. (15) Suppose X and Y are independent random variables, with $EX = 1$, $EY = 2$, $Var(X) = 3$ and $Var(Y) = 4$. Find $Var(XY)$.

5. (15) In a certain game, Person A spins a spinner and wins S dollars, with mean 10 and variance 5. Person B flips a coin. If it comes up heads, Person A must give B whatever A won, but if it comes up tails, B wins nothing. Let T denote the amount B wins. Find $Var(T)$.

Solutions:

1.a (3.47), (3.40)

2. (3.32)

3.a Given the first draw resulted in a man, there will be 5 men and 3 women left, so the probability is 5/8.

3.b The requested probability is that of getting two men or two women, (6/9) (5/8) + (3/9)(2/8).

4. Use the relations $E(UV) = EU \cdot EV$ (for independent U,V) and then use $Var(U) = E(U^2) - (EU)^2$ repeatedly:

$$\begin{aligned} Var(XY) &= E(X^2Y^2) - [E(XY)]^2 && (1) \\ &= E(X^2) \cdot E(Y^2) - (EX \cdot EY)^2 && (2) \\ &= [Var(X) + (EX)^2] \cdot [Var(Y) + (EY)^2] - (EX \cdot EY)^2 && (3) \\ &= (3 + 1^2)(4 + 2^2) - (1 \cdot 2)^2 && (4) \end{aligned}$$

5. Use (**??**), in this case with X = I, where I is an indicator variable for the event that B gets a head, and Y = S. Then T = IS, and I and S are independent, so

$$Var(T) = Var(IS) = [Var(I) + (EI)^2] \cdot [Var(S) + (ES)^2] - (EI \cdot ES)^2 \qquad (5)$$

Then use the facts that I has mean 0.5 and variance 0.5(1-0.5), with S having the mean and variance given in the problem.

Name: ____________________________

Directions: **Work only on this sheet** (on both sides, if needed). MAKE SURE TO COPY YOUR ANSWERS TO A SEPARATE SHEET FOR SENDING ME AN ELECTRONIC COPY LATER.

1. This problem concerns the bus ridership example, which begins in Sec. 2.11 and is analyzed via simulation in Sec. 2.12.4.

(a) (25) Find $E(B_1)$.

(b) (20) Suppose the company charges \$3 for passengers who board at the first stop, but charges \$2 for those who join at the second stop. (The latter passengers get a possibly shorter ride, thus pay less.) So, the total revenue from the first two stops is $T = 3B_1 + 2B_2$. We want to find E(T), and the question is whether we can calculate it by first writing

$$E(T) = 3E(B_1) + 2E(B_2) \tag{1}$$

then using our answer from (a) above, and then reasoning that $E(B_2) = E(B_1)$. Which of the following is correct?

(i) The method proposed above is valid. (If you choose this answer, you must also state the numbers of the relevant "mailing tubes.")

(ii) The above method is invalid, because $E(B_2)$ is not necessarily equal to $E(B_1)$.

(iii) $E(B_2) = E(B_1)$, but the above method is invalid for other reasons.

(c), (d) (20) (Note that the following concerns both part (d) and part (d).) Suppose on p.24 we wish to add code to find $E(L_{10})$, not just $P(L_{10} == 0$ as we are already doing. We'll need to insert two new lines of code for this (not counting another **print()** call after line 17). State what these two lines are, for your answers to (c) and (d). Include a comment, saying *where* the insertions should be made. Example: If the code **x <− y + 3** should go between lines 8 and 9, write

```
x <- y + 3  # insert between lines 8 and 9
```

2. Twenty tickets are sold in a lottery, numbered 1 to 20, inclusive. Five tickets are drawn for prizes.

(a) (25) Find the probability that two of the five winning tickets are even-numbered. (You may call built-in R functions, e.g. **sqrt()** in your answer.)

(b) (10) Find the probability that two of the five winning tickets are in the range 1 to 5, two are in 6 to 10, and one is in 11 to 20. (You may call built-in R functions, e.g. **sqrt()** in your answer.)

Solutions:

1.a

$$E(B_1) = 0 \cdot P(B_1 = 0) + 1 \cdot P(B_1 = 1) + 2 \cdot P(B_1 = 2) = 0.4 + 2 \cdot 0.1 \tag{2}$$

1.b Answer (i) is correct, using (3.13) (taking $U = 3B_1$ and $V = 2B_2$) and then (3.14).

1.c,d

```
totl10 <- 0  # insert between 3 and 4
totl10 <- totl10 + passengers  # insert between 15 and 16
```

2.a

```
choose(10,2) * choose(10,3) / choose(20,5)
```

2.b

```
choose(5,2) * choose(5,2) * choose(10,1) / choose(20,5)
```

Name: ____________________________

Directions: **Work only on this sheet** (on both sides, if needed); do not turn in any supplementary sheets of paper. There is actually plenty of room for your answers, as long as you organize yourself BEFORE starting writing.

Unless otherwise stated, give numerical answers as expressions, e.g. $\frac{2}{3} \times 6 - 1.8$. Do NOT use calculators.

1. (15) Using Equation (1.64), give a numerical expression for $Var(X_1)$.

2. Suppose X and Y are independent random variables with standard deviations 3 and 4, respectively.

(a) (10) Find Var(X+Y).

(b) (10) Find Var(2X+Y).

3. (30) Fill in the blanks in the following simulation, which finds the approximate variance of N, the number of rolls of a die needed to get the face having just one dot.

```
onesixth <- 1/6
sumn <- 0
sumn2 <- 0
for (i in 1:10000) {
   n <- 0
   while(TRUE) {
      ________________________________________
      if (______________________________ < onesixth) break
   }
   sumn <- sumn + n
   sumn2 <- sumn2 + n^2
}
approxvarn <- ____________________________________________
cat("the approx. value of Var(N) is ",approx,"\n")
```

4. (20) Jack and Jill keep rolling a four-sided and three-sided die. The first player to get the face having just one dot wins, except that if they both get a 1, it's a tie, and play continues. Let N denote the number of turns needed. Find $p_N(1)$.

5. (15) Let X be the total number of dots we get if we roll three dice. Find an upper bound for $P(X \geq 15)$, using our course materials.

Solutions:

1. $Var(X_1) = E(X_1)^2 - (EX_1)^2$. The last term is 1.52^2, and the next-to-last is $1^2 \cdot 0.48 + 2^2 \cdot 0.52$.

2. By (1.61), $Var(X+Y) = Var(X) + Var(Y) = 3^2 + 4^2$. By (1.48), $Var(2X) = 2^2 Var(X) = 2^2 \cdot 3^2$, so again by (1.61), $Var(2X+Y) = 2^2 \cdot 3^2 + 4^2$.

3.

```
n <- n + 1
runif(1)
sumn2/10000 - (sumn/10000)^2
```

4.

$$
\begin{aligned}
p_N(1) &= P(N = 1) \\
&= P(\text{Jack gets 1 and Jill doesn't or vice versa}) \\
&= \frac{1}{4} \cdot \frac{2}{3} + \frac{3}{4} \cdot \frac{1}{3}
\end{aligned}
$$

5. Use Markov's Inequality:

$$P(X \geq 15) \leq \frac{EX}{15} = \frac{3(3.5)}{15}$$

(Of course, it's a very poor bound in this case.)

Name: ____________________________

Directions: **Work only on this sheet** (on both sides, if needed). MAKE SURE TO COPY YOUR ANSWERS TO A SEPARATE SHEET FOR SENDING ME AN ELECTRONIC COPY LATER.

Important note: Remember that in problems calling for R code, you are allowed to use any built-in R function, e.g. **choose()**, **sum()**, etc.

1. (15) A class has 68 students, 48 of which are CS majors. The 68 students will be randomly assigned to groups of 4. Find the probability that a random group of 4 has exactly 2 CS majors.

2. This problem concerns the bus ridership example, Sec. 2.11 in our book.

(a) (15) Find $E(L_1)$.

(b) (15) Find $Var(L_1)$.

3. This problem again concerns the bus ridership example, but focuses on the simulation, Sec. 2.12.4. Here we are interested in finding $E(L_8)$.

(a) (10) Where should a line

```
tot_l2 <- 0
```

be placed? Answer using a half-line number, e.g. 6.5 if you think this code should be inserted between lines 6 and 7.

(b) (15) What code should be inserted at line 12.5?

(c) (10) Give a print statement to go after line 16, printing the approximate value of $E(L_8)$

4. (10) Say a large research program measures boys' heights at age 10 and age 15. Call the two heights X and Y. So, each boy has an X and a Y. Each boy is a "notebook line", and the notebook has two columns, for X and Y. We are interested in Var(Y-X). Which of the following is true? (Answer with a Roman numeral, e.g. (v).)

(i) $Var(Y - X) = Var(Y) + Var(X)$

(ii) $Var(Y - X) = Var(Y) - Var(X)$

(iii) $Var(Y - X) < Var(Y) + Var(X)$

(iv) $Var(Y - X) < Var(Y) - Var(X)$

(v) $Var(Y - X) > Var(Y) + Var(X)$

(vi) $Var(Y - X) > Var(Y) - Var(X)$

(vii) None of the above.

5. (10) Suppose at some public library, patrons return books exactly 7 days after borrowing them, never early or late. However, they are allowed to return their books to another branch, rather than the branch where they borrowed their books. In that situation, it takes 9 days for a book to return to its proper library, as opposed to the normal 7. Suppose 50% of patrons return their books to a "foreign" library. Find Var(T), where T is the time, either 7 or 9 days, for a book to come back to its proper location. (Hint: Use the concept of indicator random variables.)

Solutions:

1.

$$\frac{\binom{48}{2}\binom{20}{2}}{\binom{68}{4}}$$

2.a

$$EL_1 = EB_1 = 0 \cdot 0.5 + 1 \cdot 0.4 + 2 \cdot 0.1$$

2.b First, note that $Var(L_1) = E(L_1^2) - (EL_1)^2$; then compute $E(L_1^2) = 0^2 \cdot 0.5 + 1^2 \cdot 0.4 + 2^2 \cdot 0.1$.

3.a 3.5 (or earlier)

3.b

```
if(j == 8) tot_l2 <- tot_l2 + passengers
```

3.c

```
print(tot_l2 / nreps)
```

4.

$$Var(Y - X) = Var[Y + (-X)] = Var(Y) + Var(-X) + 2Cov(Y, -X) = Var(Y) + Var(X) - 2Cov(X, Y)$$

Since X and Y are positively correlated, their covariance is positive, so the answer is (iii).

5. $T = 7 + 2I$, where I is an indicator random variable for the event that the book is returned to a "foreign" branch. Then

$$Var(T) = Var(7 + 2I) = 4Var(I) = 4 \cdot 0.5(1 - 0.5)$$

Name: ____________________________

Directions: **Work only on this sheet** (on both sides, if needed). MAKE SURE TO COPY YOUR ANSWERS TO A SEPARATE SHEET FOR SENDING ME AN ELECTRONIC COPY LATER.

Important note: Remember that in problems calling for R code, you are allowed to use any built-in R function, e.g. **choose()**, **sum()**, etc.

1. In the R code at the bottom of p.69, suppose we wish to change it to find $P(Y_6 = X_6)$. Replace each of these lines below. You may remove lines if you wish (do not add any); if so, replace the given line with a comment line,

```
# line removed
```

so that the original line numbers are retained.

(a) (10) Show the new line 2.

(b) (10) Show the new line 3.

(c) (10) Show the new line 4.

2. Coin A has probability 0.6 of heads, Coin B is fair, and Coin C has probability 0.2 of heads. I toss A once, getting X heads, then toss B once, getting Y heads, then toss C once, getting Z heads. Let W = X + Y + Z, i.e. the total number of heads from the three tosses (ranges from 0 to 3).

(a) (10) Find P(W = 1).

(b) (10) Find Var(W).

3. This problem concerns the parking example, pp.59-60.

(a) (15) Find $p_N(3)$.

(b) (10) Find P(D = 1).

(c) (10) Say Joe is the one looking for the parking place. Paul is watching from a side street at the end of the first block (the one before the destination), and Martha is watching from an alley situated right after the sixth parking space in the second block. Martha calls Paul and reports that Joe never went past the alley, and Paul replies that he did see Joe go past the first block. They are interested in the probability that Joe parked in the second space in the second block. Fill in the blank, using only math and probability symbols, N and D—no English except for *and*, *or* and *not*: The probability they wish to find is P(__________________________).

(d) (15) Add to the simulation code on p.60, so that it finds and prints (the latter via **print()**) the approximate value of P(we park in the first block). You must use only one R statement, though it will probably consist of nested function calls. Hint: See p. 21, bottom.

4. (15) March, April, May and June[1] each roll a die until an event occurs: For March, the event is to roll a 3; for April, a 4; for May, a 5; and for June, a 6. Let T denote the total number of rolls they make. Find P(T = 28).

Solutions:

1.

```
for (i in 0:4)
   # line removed
   prob <- prob + dbinom(i,4,0.5) * dbinom(i,6,0.5)
```

2a.

$$
\begin{aligned}
P(W=1) &= P(X=1 \text{ and } Y=0 \text{ and } Z=0 \text{ or } ...) \\
&= 0.6 \cdot 0.5 \cdot 0.8 + 0.4 \cdot 0.5 \cdot 0.8 + 0.4 \cdot 0.5 \cdot 0.2
\end{aligned}
$$

2b. Var(W) = Var(X) + Var(Y) + Var(Z), by independence. Since X is an indicator random variable, $Var(X) = 0.6 \cdot 0.4$, etc. The answer is

$$0.6 \cdot 0.4 + 0.5 \cdot 0.5 + 0.2 \cdot 0.8$$

3a.

$$p_N(3) = P(N=3) = (1-0.15)^{3-1} 0.15$$

3b.

$$
\begin{aligned}
P(D=1) &= P(N=10 \text{ or } N=12) \\
&= (1-0.15)^{10-1} 0.15 + (1-0.15)^{12-1} 0.15
\end{aligned}
$$

3c. $P(N = 12 \mid N > 10 \text{ and } N < 16)$

3d.

```
print(mean(nvalues <= 10))
```

4. Actually, it doesn't matter what the different women's numerical goals are; the probability would be the same even if each woman was rolling until she got, say, a 5. The random variable T is then a sum of 4 independent geometrically-distributed random variables, each having the parameter p = 1/6. As noted in the material surrounding (3.109), such a sum has a negative binomial distribution. Thus P(T = 28) is computed as

```
choose(28-1,4-1) * (1-1/6)^(28-4) (1/6)^4
```

[1] Each of these is a common woman's name, by the way.

Name: ____________________________

Directions: **Work only on this sheet** (on both sides, if needed); do not turn in any supplementary sheets of paper. There is actually plenty of room for your answers, as long as you organize yourself BEFORE starting writing.

1. (25) Consider the simple board game, pp.15ff, starting at 0. Change the game so that it has 16 squares, numbered 0 to 15, but is otherwise identical to the original one. Let X denote the square the player lands on after the first turn. Find E(X), expressing your answer as a sum of fractions, e.g. 3/2 + 1/5 (-3/8).

2. (25) Supppose we have a random variable X, and define a new random variable Y, which is equal to X if X > 8 and equal to 0 otherwise. Assume X takes on only a finite number of values (just a mathematical nicety, not really an issue). Which one of the following is true:

(i) $EY \leq EX$.

(ii) $EY \geq EX$.

(iii) Either of EY and EX could be larger than the other, depending on the situation.

(iv) EY is undefined.

3. This problem concerns the binary tree model in our homework.

(a) (25) Find the probability that the root has exactly 1 grandchild, expressing your answer in terms of p, algebraically simplified.

(b) (25) Fill in the blanks in the following code simulating the function r(k,p):

```
sim1tree <- function(k,p) {
   if (k == 0) return(              )                    # blank
   prevlevelbranches <- 1
   for (m in 1:       ) {                                # blank
      newbranches <- 0
      for (i in 1:                    ) {                # blank
         for (j in 1:2) {
            if (sample(0:1,1,prob=c(1-p,p)) == 1) {
               newbranches <- newbranches + 1
            }
         }
      }
      if (newbranches == 0) return(0)
                                                         # blank
   }
   return(1)
}

treesim <- function(p,k,nreps) {
   count <- 0
   for (i in 1:nreps) {
      treetok <- sim1tree(k,p)
      count <- count + treetok
   }
   return(            )                                  # blank
}
```

Solutions:

1. X can take on the values 1 through 9. Then

$$EX = 1 \cdot P(X = 1) + 2 \cdot P(X = 2) + ... + 9 \cdot P(X = 9) \tag{1}$$

$P(X = 1) = \frac{1}{6}$, $P(X = 4) = \frac{1}{6} + \frac{1}{36}$, etc.

2. Answer (iii) is correct. If we'd had the additional condition $X \geq 0$, then (i) would have been right. But without that condition, then for instance suppose X were always negative; then Y would always be larger, etc.

3.a The root will have exactly one grandchild iff it has two children, and one of them has one child and the other has none. Thus the queried probability is

$$p^2 \cdot [2p(1-p)] = 2p^3(1-p) \tag{2}$$

3.b

```
sim1tree <- function(p,k) {
   if (k == 0) return(1)
   prevlevelbranches <- 1
   for (m in 1:k) {  # levels
      newbranches <- 0
      for (i in 1:prevlevelbranches) {
         for (j in 1:2) {  # account for left, right outlinks
            if (sample(0:1,1,prob=c(1-p,p)) == 1) {
               newbranches <- newbranches + 1
            }
         }
      }
      if (newbranches == 0) return(0)
      prevlevelbranches <- newbranches
   }
   return(1)
}

treesim <- function(p,k,nreps) {
   count <- 0
   for (i in 1:nreps) {
      treetok <- sim1tree(p,k)
      count <- count + treetok
   }
   return(count/nreps)
}
```

Name: ____________________________

Directions: **Work only on this sheet** (on both sides, if needed); do not turn in any supplementary sheets of paper. There is actually plenty of room for your answers, as long as you organize yourself BEFORE starting writing.

1. X be the number of dots we get in rolling a three-sided die once. (It's cylindrical in shape.) The die is weighted so that the probabilities of one, two and three dots are 1/2, 1/3 and 1/6, respectively. Note: Express all answers in this problem as common fractions, reduced to lowest terms, such as 2/3 and 9/7.

(a) (10) State the value of $p_X(2)$.

(b) (10) Find EX and Var(X).

(c) (15) Suppose you win \$2 for each dot. Find EW, where W is the amount you win.

2. This problem concerns the **REVISED** version of the committee/gender example.

(a) (10) Find $E(D^2)$. Express your answer as an *unsimpilfied* expression involving combinatorial quantities such as $\binom{168}{28}$.

(b) (15) Find $P(G_1 = G_2 = 1)$. Express your answer as a common fraction.

3. (15) State the (approximate) return value for the function below, in terms of w. **You must cite an equation number in the book to get full credit.**

```r
xsim <- function(nreps,w) {
   sumn <- 0
   for (i in 1:nreps) {
      n <- 0
      while (TRUE) {
         n <- n + 1
         u <- runif(1)
         if (u < w) break
      }
      sumn <- sumn + n
   }
   return(sumn/nreps)
}
```

4. (15) Consider the parking space example on p.48. (NOT the variant in the homework.) Let N denote the number of empty spaces in the first block. State the value of Var(N), expressed as a common fraction.

5. (10) Suppose X and Y are independent, with variances 1 and 2, respectively. Find the value of c that minimizes Var[cX + (1-c)Y].

Solutions:

1.a 1/3

1.b

$$EX = 1 \cdot (1/2) + 2 \cdot (1/3) + 3 \cdot (1/6) = 5/3$$

$$\begin{aligned} Var(X) &= E(X^2) - (EX)^2 \\ &= 1^2 \cdot (1/2) + 2^2 \cdot (1/3) + 3^2 \cdot (1/6) - 25/9 \\ &= 5/9 \end{aligned}$$

1.c EW = E(2X) = 2 EX = 10/3

2.a

$$E(D^2) = (-2)^2 \frac{\binom{6}{1}\binom{3}{3}}{\binom{9}{4}} + ...$$

2.b

$$\begin{aligned} P(G_1 = G_2 = 1) &= P(G_1 = 1)P(G_2 = 1|G_1 = 1) \\ &= \frac{6}{9} \cdot \frac{5}{8} \\ &= \frac{5}{12} \end{aligned}$$

3. 1/w, by (3.74)

4. 10(0.2)(1-0.2) = 8/5, by (3.82)

5.

$$\begin{aligned} 0 &= \frac{d}{dc} Var[cX + (1-c)Y] \\ &= \frac{d}{dc}\left[c^2 Var(X) + (1-c)^2 Var(Y)\right] \\ &= \frac{d}{dc}[c^2 + 2(1-c)^2] \\ &= 2c - 4(1-c) \end{aligned}$$

So, the best c is 2/3.