



Meetup



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CATEGORY THEORY  
FOR PROGRAMMERS



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Category  
Theory  
for  
Programmers  
Chapter 4:  
Kleisli Categories

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```
string logger;
```

```
bool negate(bool b) {  
    logger += "Not so! ";  
    return !b;  
}
```



```
pair<bool, string> negate(bool b, string logger) {  
    return make_pair(!b, logger + "Not so! ");  
}
```



```
pair<bool, string> negate(bool b) {  
    return make_pair(!b, "Not so! ");  
}
```



```
Writer<vector<string>> process(string s) {  
    auto p1 = toUpper(s);  
    auto p2 = toWords(p1.first);  
    return make_pair(p2.first, p1.second + p2.second);  
}
```





```
type Writer a = (a, String )
```

```
( >=> ) :: (a -> Writer b) -> (b -> Writer c) -> (a -> Writer c)
```

```
m1 >=> m2 = \ x ->
```

```
    let (y, s1) = m1 x
```

```
        (z, s2) = m2 y
```

```
    in (z, s1 ++ s2)
```

```
return :: a -> Writer a
```

```
return x = (x, "")
```

```
upCase :: String -> Writer String
```

```
upCase s = (map toUpper s, "upCase ")
```

```
toWords :: String -> Writer [ String ]
```

```
toWords s = (words s, "toWords ")
```

```
process :: String -> Writer [ String ]
```

```
process = upCase >=> toWords
```



A function that is not defined for all possible values of its argument is called a partial function. It's not really a function in the mathematical sense, so it doesn't fit the standard categorical mold. It can, however, be represented by a function that returns an embellished type optional:



```
template<class A> class optional {  
    bool _isValid;  
    A _value;  
public:  
    optional()    : _isValid(false) {}  
    optional(A v) : _isValid(true), _value(v) {}  
    bool isValid() const { return _isValid; }  
    A value() const { return _value; }  
};
```

For example, here's the implementation of the embellished function `safe_root`:

```
optional<double> safe_root(double x) {  
    if (x >= 0) return optional<double>{sqrt(x)};  
    else return optional<double>{};  
}
```



```
template<class A> class optional {  
    bool isValid;
```

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History

C++Utilities librarystd::optional

## std::optional

Defined in header `<optional>`

```
template< class T >           (since C++17)  
class optional;
```

The class template `std::optional` manages an *optional* contained value, i.e. a value that may or may not be present. A common use case for `optional` is the return value of a function that may fail. As opposed to other approaches, such as `std::pair<T, bool>`, `optional` handles expensive-to-construct objects well and is more readable, as the intent is expressed explicitly.

Any instance of `optional<T>` at any given point in time either *contains a value* or *does not contain a value*.

If an `optional<T>` *contains a value*, the value is guaranteed to be allocated as part of the `optional` object footprint, i.e. no dynamic memory allocation ever takes place. Thus, an `optional` object models an object, not a pointer, even though `operator*()` and `operator->()` are defined.

When an object of type `optional<T>` is *contextually converted to bool*, the conversion returns `true` if the object *contains a value* and `false` if it *does not contain a value*.

```
}
```



```
template<class A> class optional {  
    bool _isValid;  
    A _value;  
public:  
    optional() : _isValid(false) {}  
};
```

<b>has_value</b>	(public member function)
<b>value</b>	returns the contained value (public member function)
<b>value_or</b>	returns the contained value if available, another value otherwise (public member function)

```
optional<double> safe_root(double x) {  
    if (x >= 0) return optional<double>{sqrt(x)};  
    else return optional<double>{};  
}
```



Here's the challenge:

1. Construct the Kleisli category for partial functions (define composition and identity).
2. Implement the embellished function `safe_reciprocal` that returns a valid reciprocal of its argument, if it's different from zero.
3. Compose the functions `safe_root` and `safe_reciprocal` to implement `safe_root_reciprocal` that calculates  $\sqrt{1/x}$  whenever possible.



```
// Question 1
```

```
auto compose(auto f, auto g) {  
    return [f, g] (auto x) {  
        auto const res = f(x);  
        return res.has_value() ? g(res.value()) : std::nullopt;  
    };  
}
```



```
// From Book (modified)
```

```
auto safe_root(double n) -> std::optional<double> {  
    return n >= 0 ? std::optional{sqrt(n)} : std::nullopt;  
}
```

```
// Question 2
```

```
auto safe_reciprocal(int n) -> std::optional<double> {  
    return n != 0 ? std::optional{1.0 / n} : std::nullopt;  
}
```





// Question 3

```
auto safe_root_reciprocal(int n) -> std::optional<double> {  
    auto const r = safe_reciprocal(n);  
    return r.has_value() ? safe_root(r.value()) : std::nullopt;  
}
```

```
auto safe_root_reciprocal2(int n) -> std::optional<double> {  
    return compose(  
        [] (auto x) { return safe_reciprocal(x); },  
        [] (auto x) { return safe_root(x); }  
    ) (n);  
}
```



```
// Question 3
```

```
auto safe_root_reciprocal(int n) -> std::optional<double> {  
    auto const r = safe_reciprocal(n);  
    return r.has_value() ? safe_root(r.value()) : std::nullopt;  
}
```

```
auto safe_root_reciprocal2(int n) -> std::optional<double> {  
    return compose(safe_reciprocal, safe_root)(n);  
}
```



nullopt	←	0	$\theta$
has_value	←	$\supset$	
value	←	$\supset \phi$	
make_optional	←	1,	$\vdash$



```
nullopt      ← 0  θ  
has_value    ← ⊃  
value        ← ⊃φ  
make_optional ← 1, ⊢
```

```
safe_root      ← { ω ≥ 0 : make_optional ω*.5 ♦ nullopt }  
safe_reciprocal ← { ω ≠ 0 : make_optional ÷ω   ♦ nullopt }
```



```
nullopt      ← 0  θ
has_value    ← ⊃
value        ← ⊃φ
make_optional ← 1, ⊢
```

```
safe_root      ← { ω ≥ 0 : make_optional ω*.5 ♦ nullopt }
safe_reciprocal ← { ω ≠ 0 : make_optional ÷ω    ♦ nullopt }
```

```
safe_root_reciprocal ← {
  r ← safe_reciprocal ω
  has_value r : safe_root value r ♦ nullopt
}
```



Bartosz Milewski

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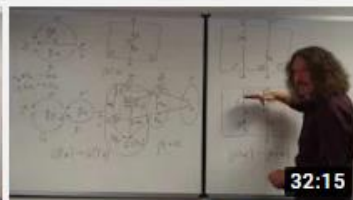
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string log = "";

bool negate (bool x) {

log += "not!";

return !x;

}

function < c(a) >

compose (function < b(a) > f, function < c(b) > g) {

return [f, g](a x) {

f(x)

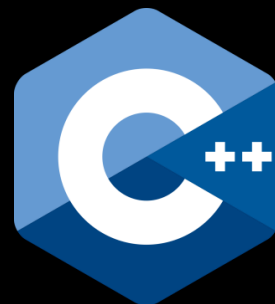
pair < bool, string >

negate (bool x) {

return make\_pair (!x,  
"Non!");

}

set



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