1.1 Prove by structural induction on abstract syntax trees that if $\mathcal{X} \subseteq \mathcal{Y}$, then $\mathcal{A}[\mathcal{X}] \subseteq \mathcal{A}[\mathcal{Y}]$

Proof. *Any variables in $\mathcal{A}[\mathcal{X}]$ is also a valid variable in $\mathcal{A}[\mathcal{Y}]$ since $\mathcal{X} \subseteq \mathcal{Y}$. And for the inductive case, Let $o(a_1, \cdots, a_n)$ be a valid AST in $\mathcal{A}[\mathcal{X}]$. If $a_1 \in \mathcal{A}[\mathcal{X}], \cdots a_n \in \mathcal{A}[\mathcal{X}]$, by induction hypothesis $a_1 \in \mathcal{A}[\mathcal{Y}], \cdots a_n \in \mathcal{A}[\mathcal{Y}]$, which means that $o(a_1, \cdots, a_n)$ is also a valid AST in $\mathcal{A}[\mathcal{Y}]$. Thus, by structural induction, if $\mathcal{X} \subseteq \mathcal{Y}$, then $\mathcal{A}[\mathcal{X}] \subseteq \mathcal{A}[\mathcal{Y}]$.* $\square$