Questions

3.2.4 *How many elements does S_3 have?*

$$\begin{aligned}
|S_{i+1}| &= 3 + 3 \times |S_i| + |S_i|^3 \\
|S_0| &= 0 \\
|S_1| &= 3 \\
|S_2| &= 3 + 9 + 27 = 39 \\
|S_3| &= 3 + 39 \times 3 + 39^3 = 59439
\end{aligned}$$

3.5.10 *Rephrase Definition 3.5.9 as a set of inference rules.*

$$\frac{t \longrightarrow t'}{t \longrightarrow^* t'} \qquad \frac{}{t \longrightarrow^* t} \qquad \frac{t \longrightarrow^* t' \quad t' \longrightarrow^* t''}{t \longrightarrow^* t''}$$

3.5.13 1. *Suppose we add a new rule*

$$\text{if true then } t_2 \text{ else } t_3 \longrightarrow t_3 \qquad \text{(E-Funny1)}$$

to the ones in Figure 3-1. Which of the above theorems (3.5.4, 3.5.7, 3.5.8, 3.5.11, and 3.5.12) remain valid?

- 3.5.4 (Determinacy of one-step evaluation) is **invalid** since there are two rules applied to if true then t_2 else t_3
- 3.5.7 (every value is normal form) is still **valid**
- 3.5.8 (If t is in normal form, then t is a value) is still **valid**
- 3.5.11 (Uniqueness of normal forms) is **invalid** because evaluation is no longer deterministic
- 3.5.12 (Termination of Evaluation) is still **valid**

2. *Suppose instead that we add this rule:*

$$\frac{t_2 \longrightarrow t_2'}{\text{if } t_1 \text{ then } t_2 \text{ else } t_3 \longrightarrow \text{if } t_1 \text{ then } t_2' \text{ else } t_3} \quad \text{(E-Funny2)}$$

Now which of the above theorems remain valid? Do any of the proofs need to change?

- 3.5.4 (Determinacy of one-step evaluation) is **invalid**
- 3.5.7 (every value is normal form) is still **valid**
- 3.5.8 (If t is in normal form, then t is a value) is still **valid**
- 3.5.11 (Uniqueness of normal forms) is **valid**. The proof need to change since the single-step evaluation is no longer deterministic.
- 3.5.12 (Termination of Evaluation) is still **valid**

3.5.16 TODO

3.5.17 *Show that the small-step and big-step semantics for this language coincide, i.e. $t \longrightarrow^* v$ iff $t \Downarrow v$*

We need to show both side of an iff relationship.

- Prove $t \longrightarrow^* v$ if $t \Downarrow v$
  TODO
- Prove $t \Downarrow v$ if $t \longrightarrow^* v$
  TODO

3.5.18 *Suppose we want to change the evaluation strategy of our language so that the `then` and `else` branches of an `if` expression are evaluated (in that order) before the guard is evaluated. Show how the evaluation rules need to change to achieve this effect.*

We need to replace all the boolean evaluation rules with the following rules:

$$\frac{t_2 \longrightarrow t_2'}{\text{if } t_1 \text{ then } t_2 \text{ else } t_3 \longrightarrow \text{if } t_1 \text{ then } t_2' \text{ else } t_3} \quad \text{(E-Then)}$$

$$\frac{v_2 \text{ val} \quad t_3 \longrightarrow t_3'}{\text{if } t_1 \text{ then } v_2 \text{ else } t_3 \longrightarrow \text{if } t_1 \text{ then } v_2 \text{ else } t_3'} \quad \text{(E-Else)}$$

$$\frac{v_2 \text{ val} \quad v_3 \text{ val} \quad t_1 \longrightarrow t_1'}{\text{if } t_1 \text{ then } v_2 \text{ else } v_3 \longrightarrow \text{if } t_1' \text{ then } v_2 \text{ else } v_3} \quad \text{(E-If)}$$

$$\frac{v_2 \text{ val} \quad v_3 \text{ val}}{\text{if true then } v_2 \text{ else } v_3 \longrightarrow v_2} \quad \text{(E-IfTrue)}$$

$$\frac{v_2 \text{ val} \quad v_3 \text{ val}}{\text{if false then } v_2 \text{ else } v_3 \longrightarrow v_3} \quad \text{(E-IfFalse)}$$