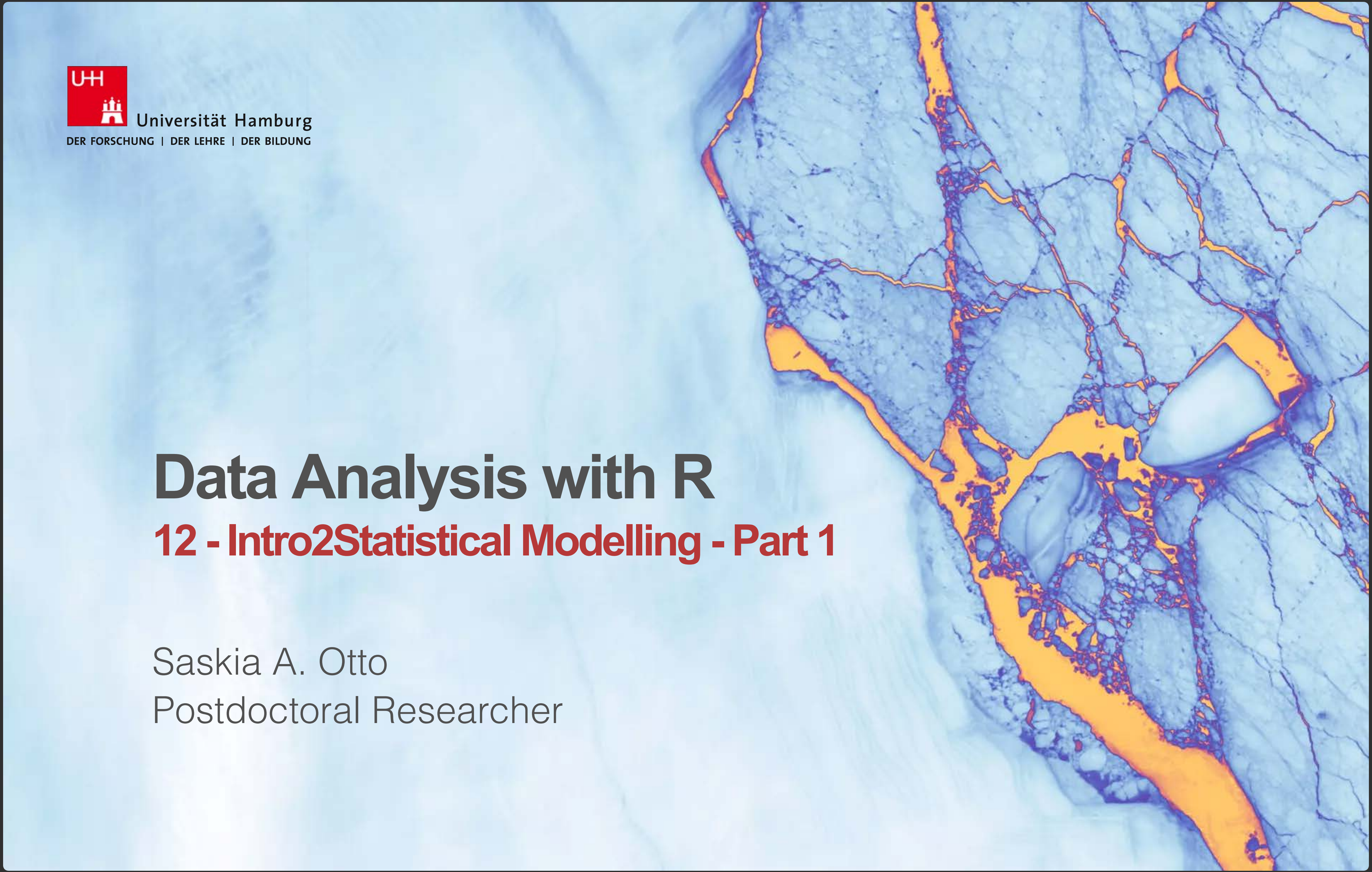
Marine Data Science



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Data Analysis with R

12 - Intro2Statistical Modelling - Part 1

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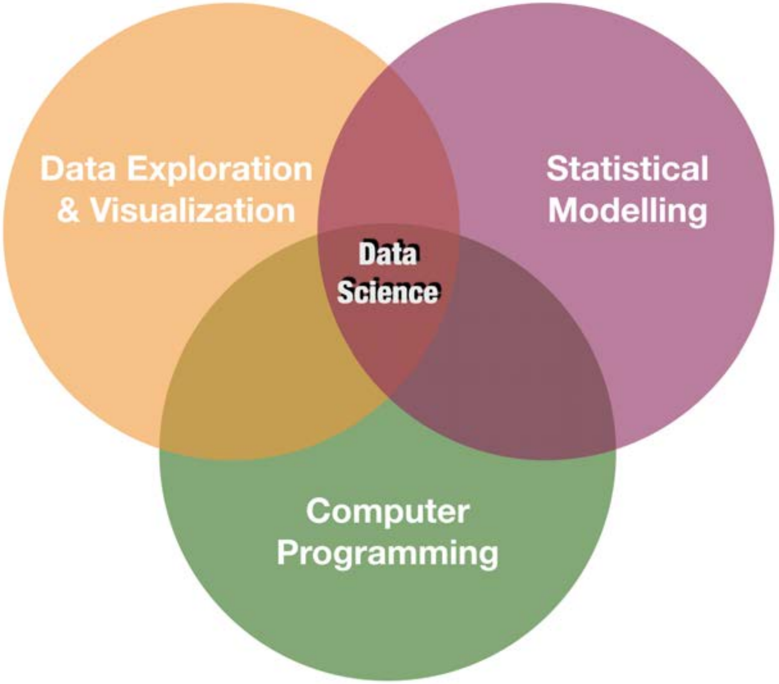
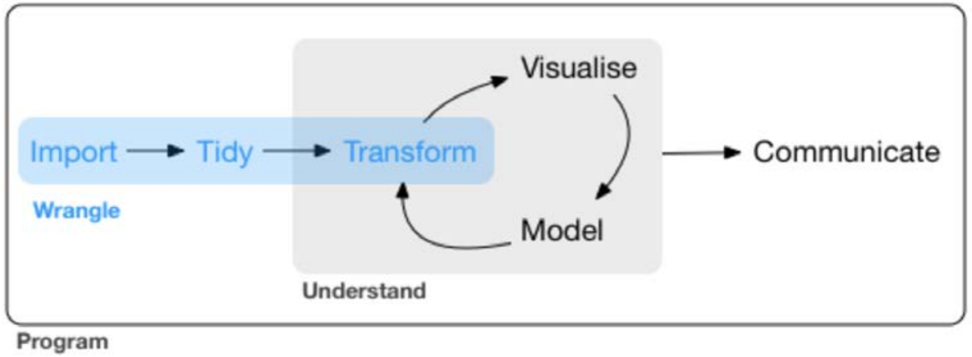
Lets close the loop:





source: R for Data Science by Wickam & Grolemund, 2017 (licensed under CC-BY-NC-ND 3.0 US)

Some basics in descriptive statistics...

Data distribution – the numerical description

PARAMETER vs STATISTIC

Parameter

a measure that describes the entire **population** (e.g. μ, σ^2)

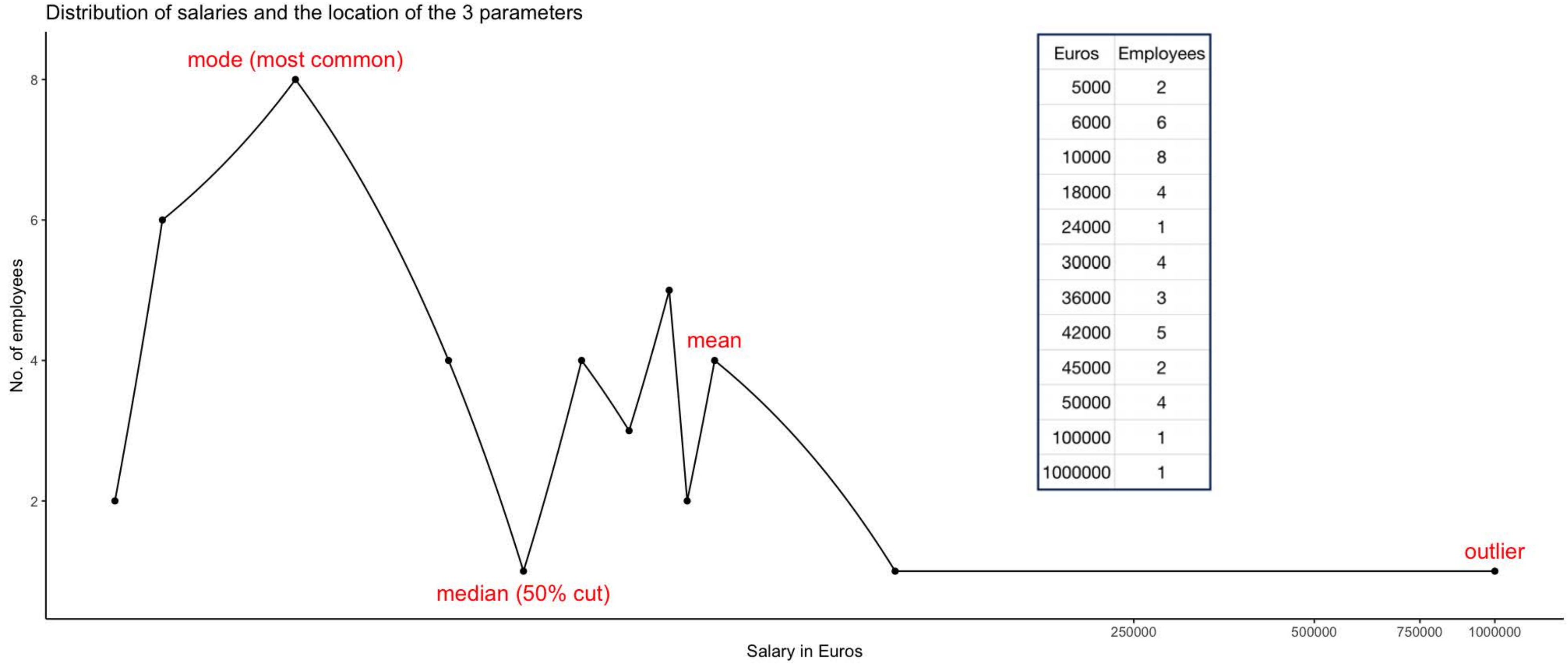
Statistics

Parameters are rarely known and are usually estimated by statistics describing a **sample** (e.g. $\bar{x}, s^2$)

Parameters of location or central tendency

- **Mode**: No built-in function
  - The mode is the most frequently occurring value in a distribution. It is the only measure of central tendency that can be used with nominal data.
- **Median**: `median(x)` or `quantile(x, probs = 0.5)`
  - The middle most value in a data series is called the median: half the scores are above the median and half are below the median. It is also called the "50%-percentile".
- **Arithmetic mean**: `mean()` $\mu; \bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$
  - Often called simply the 'mean' or 'average' but one needs to be specific to differentiate from the other means.
- **Weighted mean**: `weighted.mean(x, w, ...)`
  - Each data value x_i has a weight w_i assigned to it.

Parameters of location or central tendency



Euros	Employees
5000	2
6000	6
10000	8
18000	4
24000	1
30000	4
36000	3
42000	5
45000	2
50000	4
100000	1
1000000	1

The **arithmetic mean** can be a good measure for roughly **symmetric** distributions but can be misleading in skewed distributions since it can be greatly influenced by extreme scores.

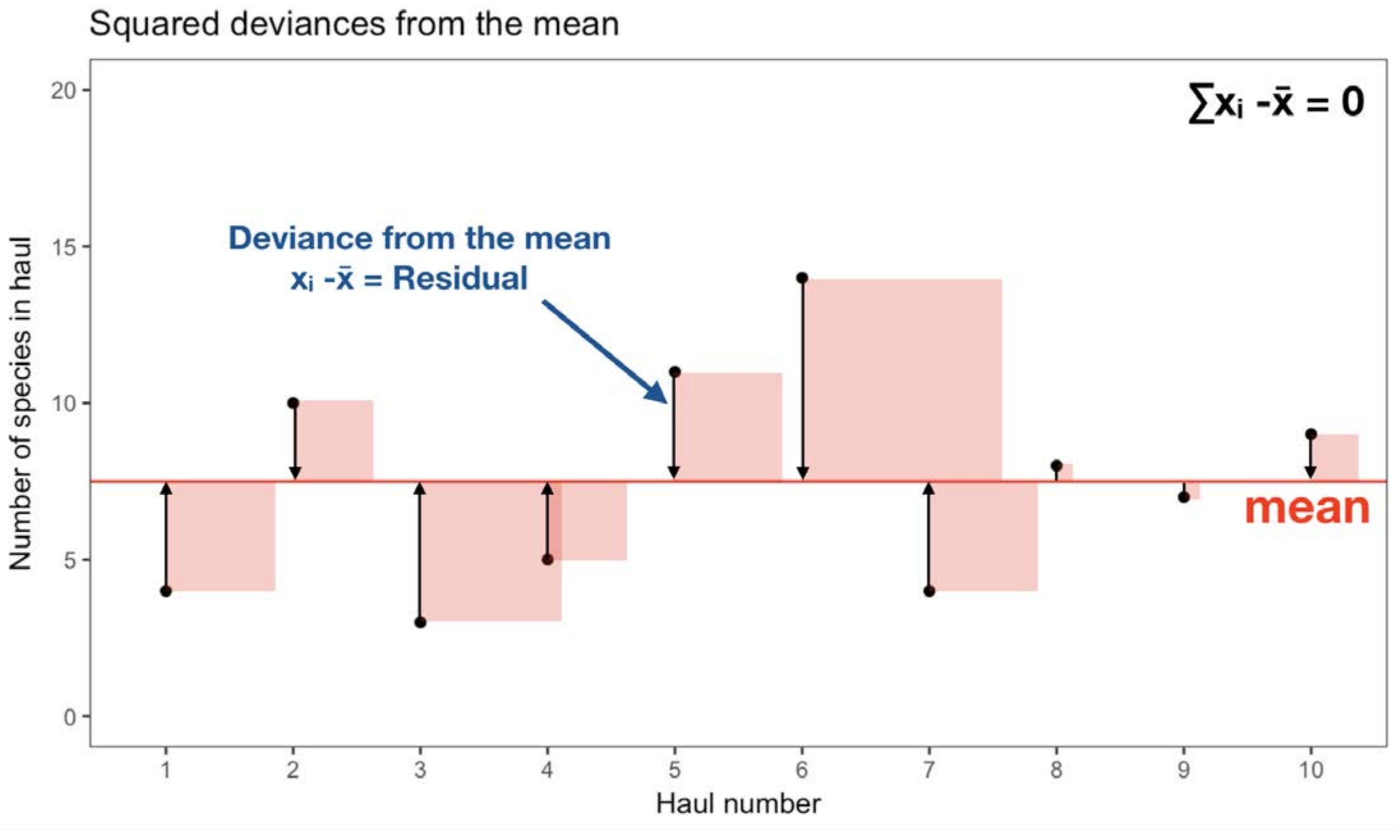
Parameters of spread or variability

- **Range**: `range()`
- **Variance**: `var()`
- **Standard deviation**: `sd()`
- **Standard error**: no built-in function (`sd(x)/sqrt(length(x))`)
- **Coefficient of variation**: no built-in function (`sd(v)/mean(v)`)

Variance σ^2, s^2

The variance is the 'Mean Sum of Squared' **(MSS)** distances of individual measurements from the overall mean:

- Sum of distances from mean equals zero → therefore the square!



Variance σ^2, s^2

The variance is the 'Mean Sum of Squared' **(MSS)** distances of individual measurements from the overall mean:

- Sum of distances from mean equals zero → therefore the square!
- We don't want our measure of variability to depend on the sample size, so the obvious solution is to **divide by the number of samples** to get the mean squared deviation:

$$\sigma^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}$$

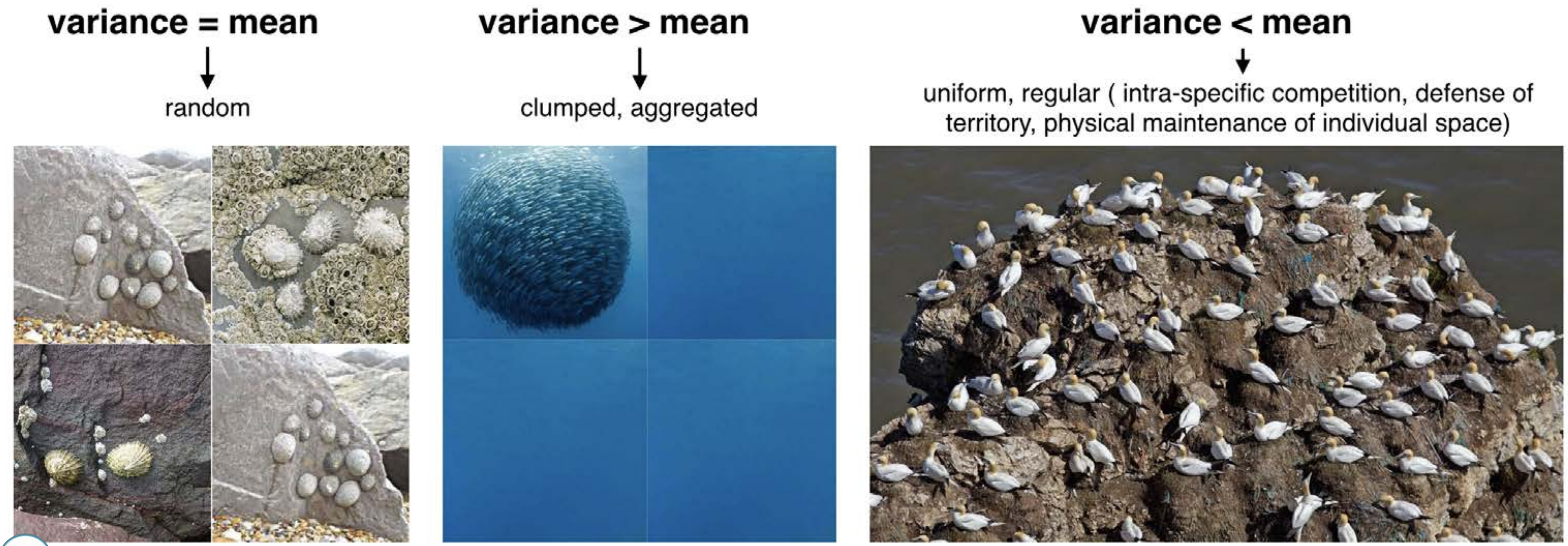
The *n-1* in the equation represents the so-called **degrees of freedom** (df): $df = N - p$ (with p = number of parameter estimated from the data).

Suppose ..

a sample has 5 numbers and a mean of 4 → the sum has to be 20. If the first 4 numbers are 1, 1, 4, and 10, the last number has to be 4: so we have 4 df for 5 numbers.

The variance is

- most commonly used
- measure for the overall spread
- less sensitive to extreme scores
- useful for judging quality of data:



variance = mean

random

variance > mean

clumped, aggregated

variance < mean

uniform, regular (intra-specific competition, defense of territory, physical maintenance of individual space)

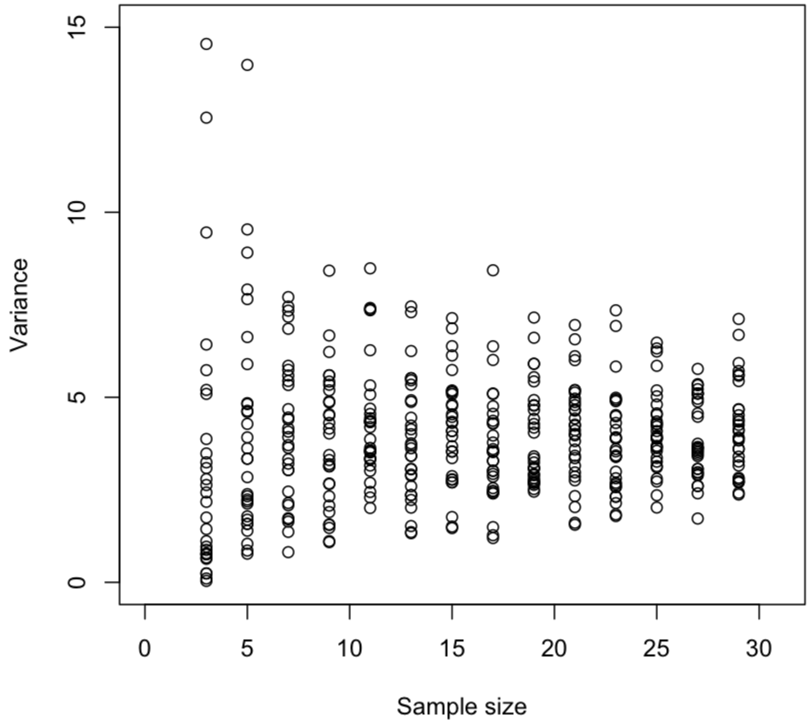
Variance and the role of sample size

Here is a simulation to demonstrate how the variance spread changes with sample size

```r
plot(c(0,32), c(0,15), type = "n",
  xlab = "Sample size",
  ylab = "Variance")

for (df in seq(3,31,2)) {
  for( i in 1:30) {
    x <- rnorm(df, mean = 10, sd = 2)
    points(df, var(x))
  }
}
```

Press 'p' in the html document for more info.



Standard deviation SD (σ, s) and standard error SE

The standard deviation is just the square root of the variance

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n - 1}}$$

The standard error normalizes the SD by the square root of the sample size *n*

$$SD = \frac{\sigma}{\sqrt{n}}$$

Standard deviation **SD** (σ, s) and standard error **SE**

Both parameters are typically used for **error bars** in graphs:

- Standard deviation and error are expressed in the **same units as the data**.
- If data is *normal distributed*: **68%** of all measurements fall within **one standard deviation** of the mean, **95%** of all measurements fall within **two standard deviations** of the mean.
- The greater the sample size *n* the higher s^2 and $s \rightarrow$ *SE* gives an estimate of variability independent of sample size.
- The greater *n*, the smaller *SE*.

Coefficient of variation **CV**

- Allows for comparison of the variation of populations that have significantly different mean values.
- Is defined as the ratio of the standard deviation to the mean: $CV = \frac{s}{\bar{x}}$.
- It is often reported as on a scale of 0 to 100% by multiplying the above calculation by 100.
- Tends to be biased with small sample sizes.

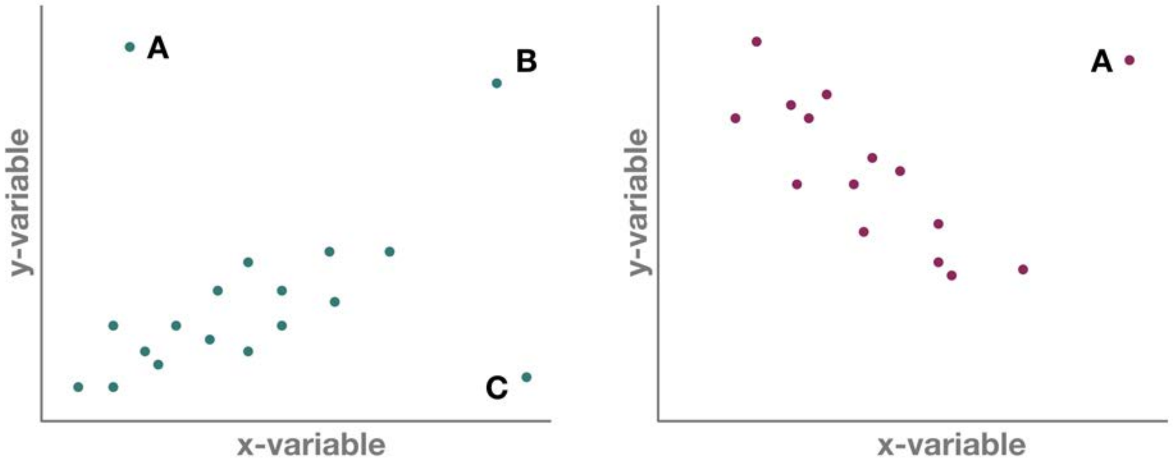
Sizing ocean giants

source: Sizing ocean giants: patterns of intraspecific size variation in marine megafauna by McClain et al., DOI: 10.7717/peerj.715

Something on outliers

- Might incorrectly influence analysis
- 1 outlier only → take it out
- Several tests (e.g. regression) model the center of the data; if you are more interested in the outlier apply outlier-analysis techniques
- Outliers in 1 dimension are not necessarily outliers in 2 or more dimensions

But which one is an outlier? That depends on the dimensional perspective.



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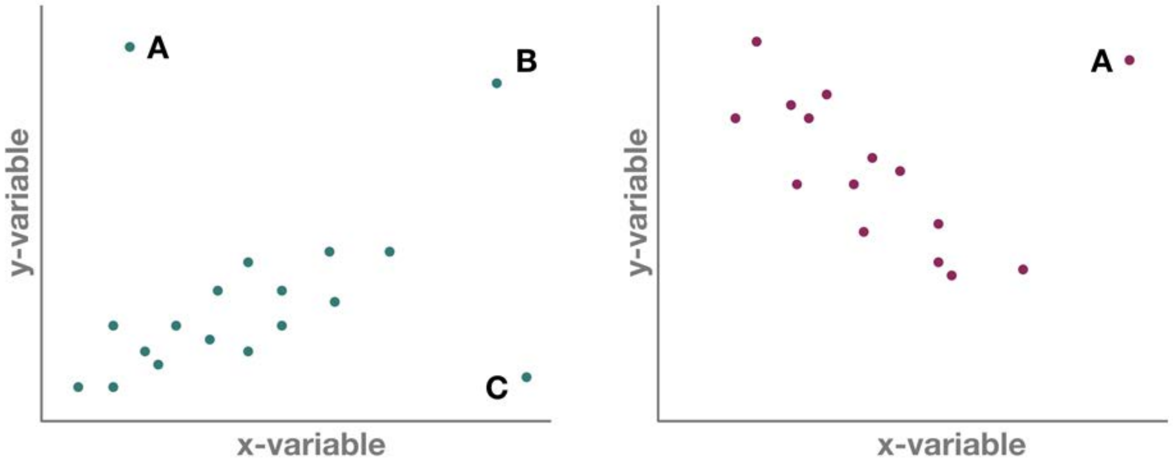


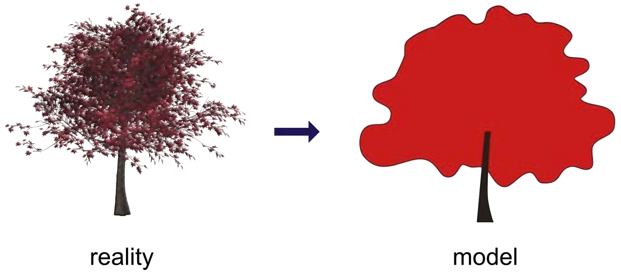
1-dimensional (left panel): Point A might be an outlier of variable y, but not of x; C is an outlier of x but not y, and B is an outlier of both x and y separately

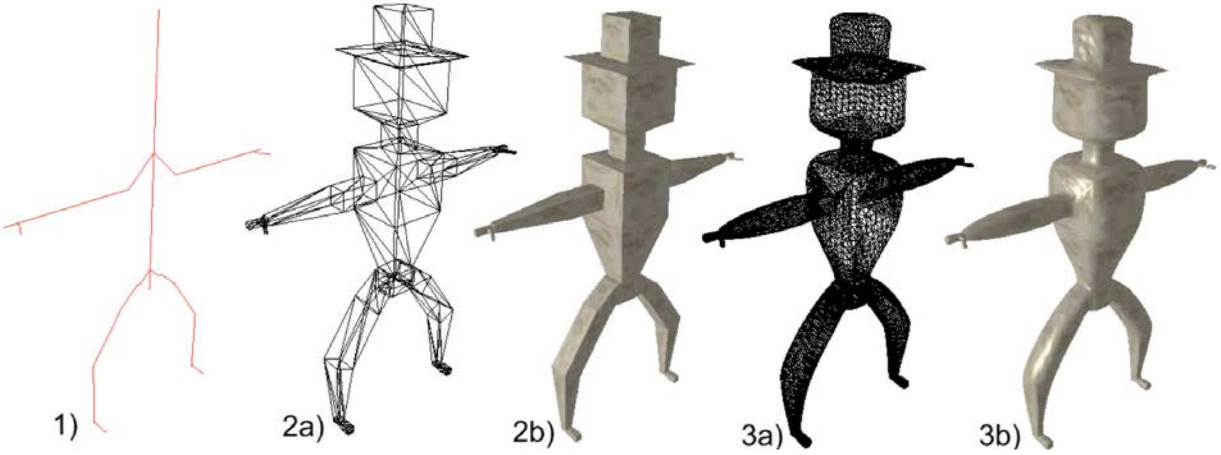
2-dimensional space xy (left panel): A and C could be outliers (both are far away from the regression line) but B is not an outlier anymore, since it lies on the regression line. Right panel: Point A could be an outlier in the x-space but is definitely one in the xy-space

Statistical modelling

Models are

- a simplified description of a complex entity or process.
- derived from an hypothesis → one hypothesis may produce numerous competing models.
- compare models with observed data → assess fit of model(s) to sample data.





sources: reality tree by www.pixabay.com (CCO 1.0); stick man graph on the right J. Nielsen (masterthesis "Conversion of Graphs to Polygonal Meshes", Technical University Copenhagen), www.tchami.com

Aim of statistical modelling

- To **identify** the most **relevant predictor** variables.
- To **determine** the **parameter** values that link predictor and response variable.
- We are looking for a model that
  - "**explains** the **greatest** amount of variation in the data..."
  - "that produces the least amount of non explainable variation"
  - "**fits best** to the data"
- Both Y and X can be continuous and/or categorical data

Linear statistical models

Most known equation:
Y = a + b*X

$$Y = constant + coefficient * X_1 + coefficient2 * X_1 \ldots + error$$

Y: response variable

X_1, X_2: predictor variables

constant: mean or intercept of Y (value when all X‘s equal zero)

coefficient: regression slopes or treatment effects

error: part of Y not explained or predicted by X‘s

Linear statistical models (cont)

- predictor variable and parameters relating predictor to response variable are included in the model as a linear combination (nonlinear terms also possible)

- **term „linear“ refers to the combination of parameters, not the shape of the distribution**: linear combination of series of parameters (regression slope, intercept) where no parameter appears as an exponent or is divided / multiplied by another parameter

All models are linear regression models

$$Y_i = \alpha + \beta_1 * X_i + \beta_2 * X_i^2 + \epsilon_i$$

$$Y_i = \alpha + \beta_1 * (X_i * W_i) + \epsilon_i$$

$$Y_i = \alpha + \beta_1 * log(X_i) + \epsilon_i$$

$$Y_i = \alpha + \beta_1 * exp(X_i) + \epsilon_i$$

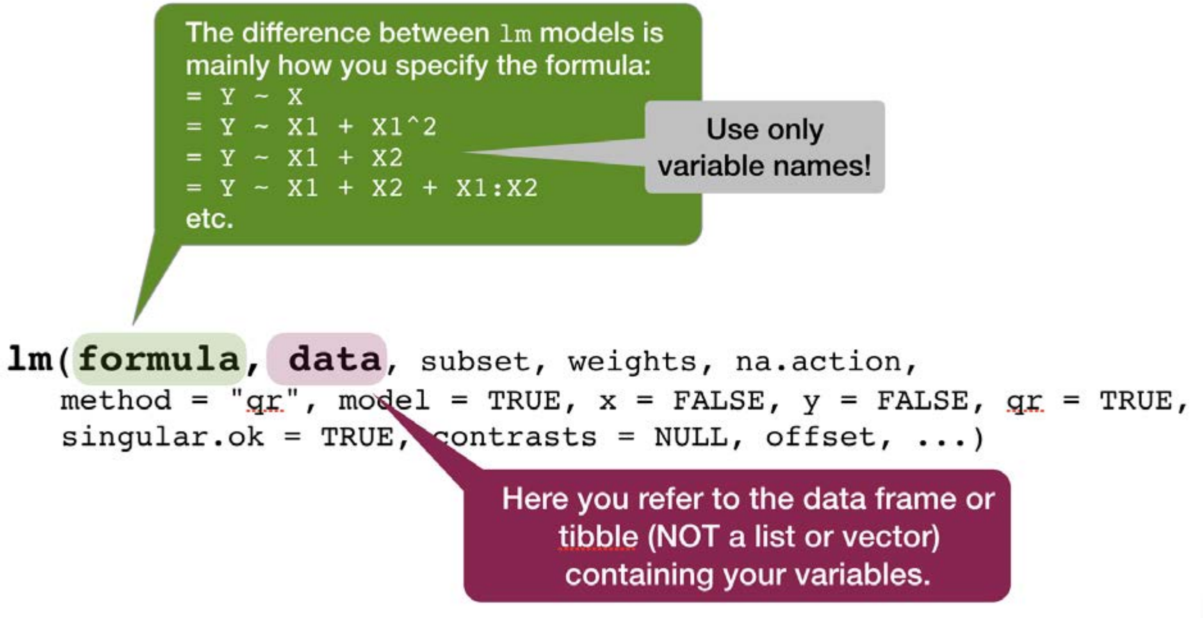
$$Y_i = \alpha + \beta_1 * sin(X_i) + \epsilon_i$$

Why use linear models?

- Linear regression technique **well established** framework
- Advantages:
    - easy to **interpret** (constant and coefficients),
    - easy to **grasp** (also interactions, multiple X)
    - coefficients can be **further used** in e.g. numerical models
    - easy to **extend**: link functions, fixed and random effects, correlation structures,… → GLM, GLMM
- BUT:
    - Dynamics not always linear → transformations do not always help or would lead to information loss

Linear models in R

- use function `lm(formula, data)`
- to look at the estimated coefficients: `coef(model)`
- to look at the complete numerical output: `summary(model)`



The difference between `lm` models is mainly how you specify the formula:

```
= Y ~ X
= Y ~ X1 + X1^2
= Y ~ X1 + X2
= Y ~ X1 + X2 + X1:X2
```

etc.

Use only variable names!

```
lm(formula, data, subset, weights, na.action,
   method = "qr", model = TRUE, x = FALSE, y = FALSE, qr = TRUE,
   singular.ok = TRUE, contrasts = NULL, offset, ...)
```

Here you refer to the data frame or tibble (NOT a list or vector) containing your variables.

Your turn...

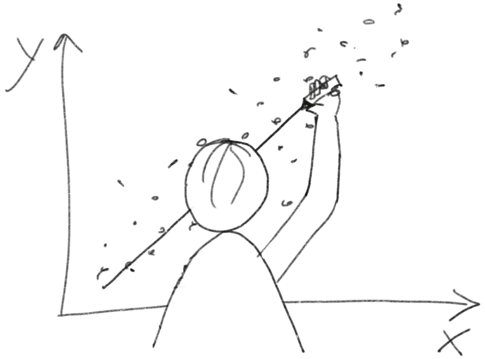
Exercise 1: `Iris` length relationship

Apply a linear regression model to the `iris` dataset and model the petal width as a function of the petal length (`Petal.Width ~ Petal.Length`)

Look at the summary of your model, what would be the corresponding linear regression equation?

Interpreting simple linear regression models

Sample regression equation



Most common way of writing the equation:

$$y = a + b * x$$

- that is the **sample** equation for the **predicted** y values

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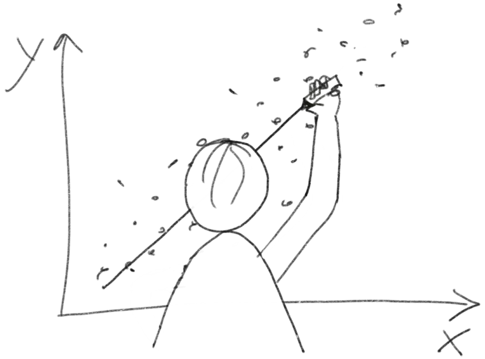


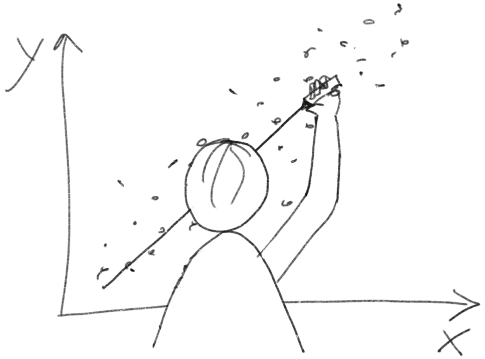
a

sample **intercept** estimates → expected value of y when **x is 0**

b

sample regression **slope** estimates → the expected increase in y associated to a **one unit increase** in x

Sample regression equation



Most common way of writing the equation:

$$y = a + b * x$$

- that is the **sample** equation for the **predicted** y values

Recall:

- **Parameter** = a measure that describes a POPULATION (e.g. μ, σ^2)
- **Statistics** = a measure that describes a SAMPLE (e.g. $\bar{x}, s^2$)

So we need to be specific whether we want to specify the parameters for the population or sample and whether for observed or fitted y values

Population equation:

- for observed Y values: $y_i = \alpha + \beta * x_i + \epsilon_i$
- for predicted Y values: $\mu(y_i) = \alpha + \beta * x_i$

Sample equation:

- for observed Y values: $y_i = a + b * x_i + e_i$
- for predicted Y values: $\hat{y} = a + b * x_i$

coefficents

The population parameters α, β and σ^2 (population variance) are estimated by a, b and s^2 (sample variance).

How are *a* and *b* estimated?

Underlying mathematical tool:

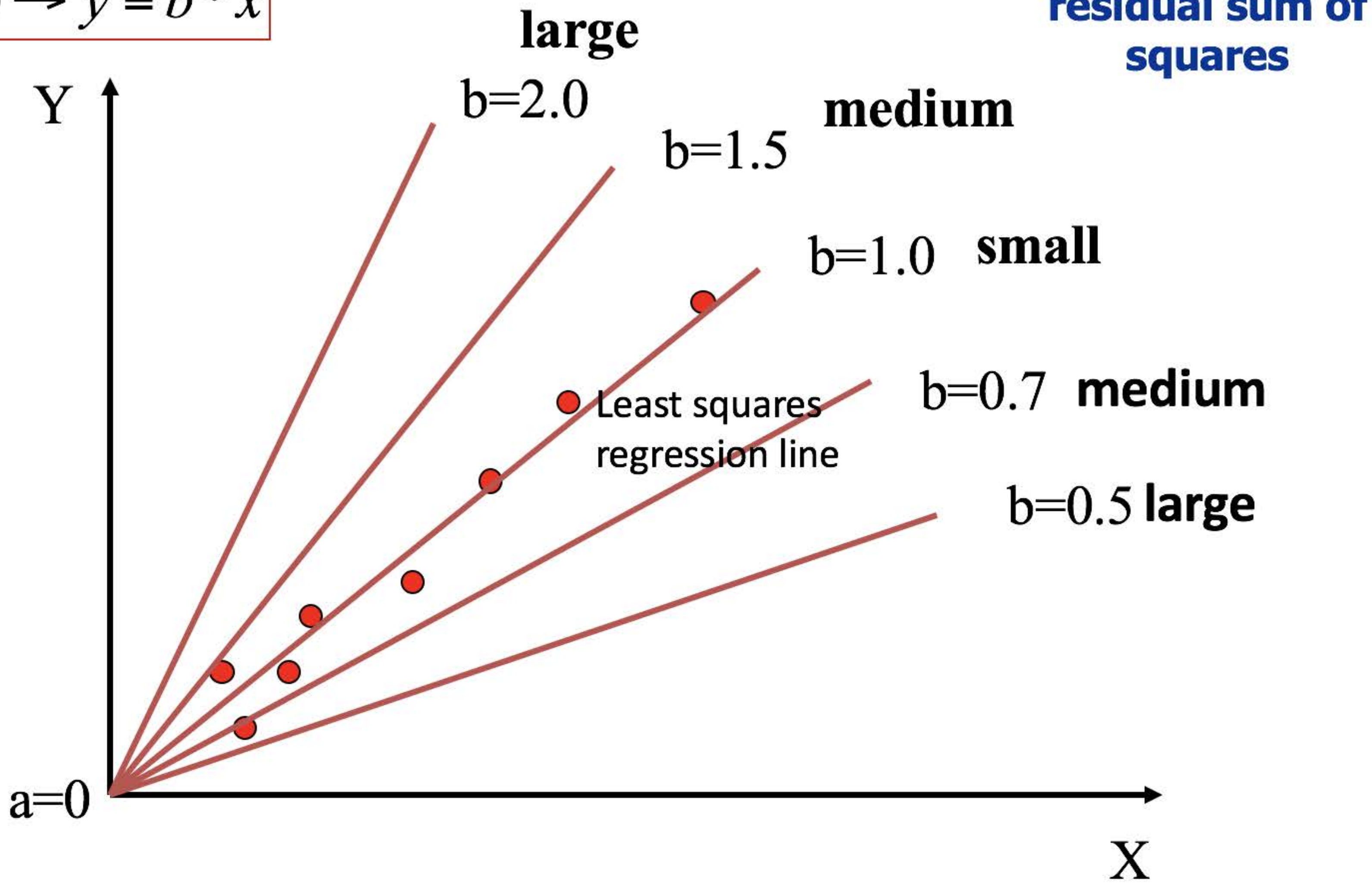
- **Ordinary Least Squares (OLS)**
- OLS finds parameters *a* and *b*, based on minimising the residual sum of squares ($SS_{Residual}$)

$$SS_{\text{Residual}} = \sum e_i^{\,2} = \sum (y_i - (a + b * x_i))^2 \rightarrow MIN$$

Minimisation: obtain partial derivatives (of *a* and *b*) and set equal to zero

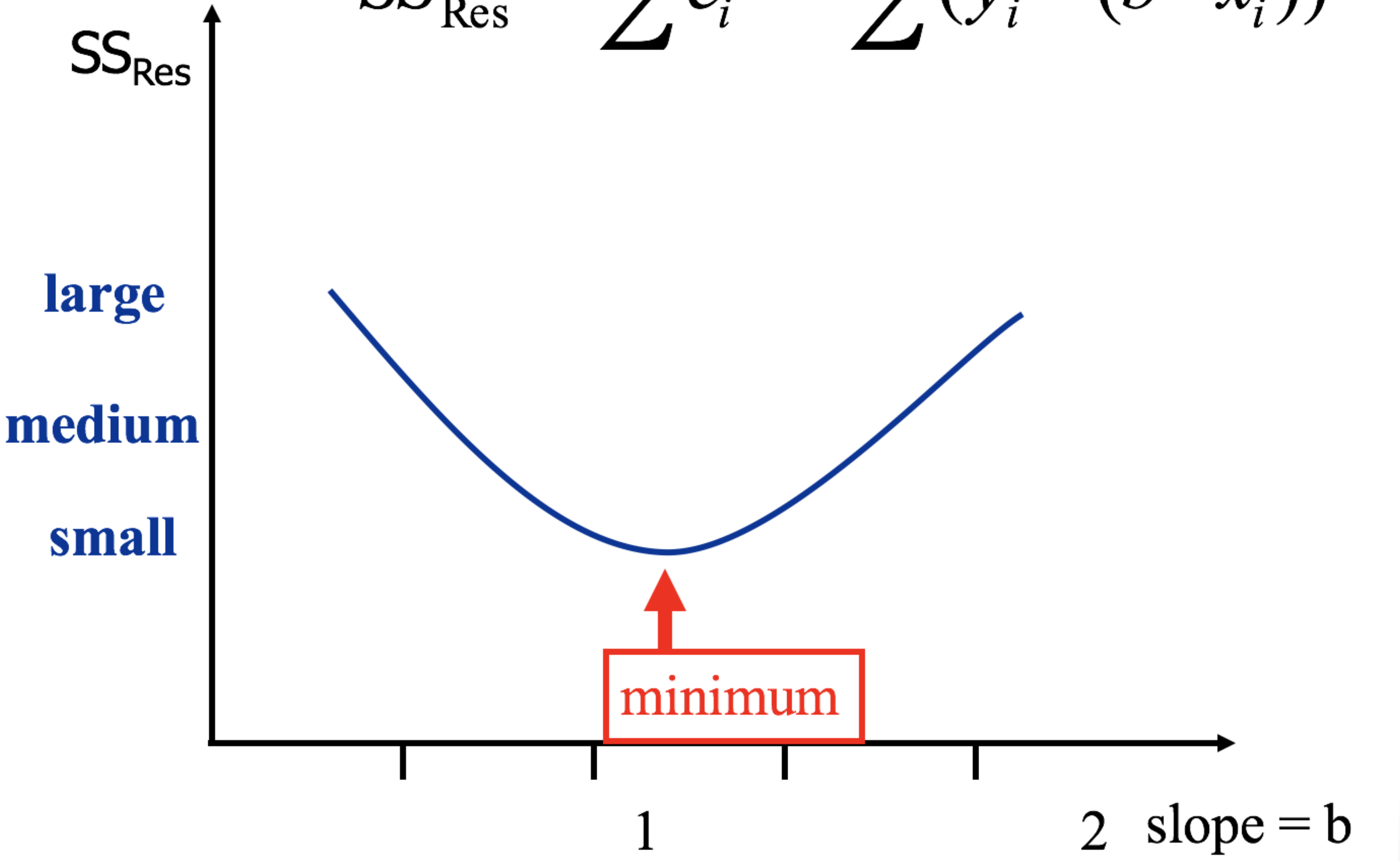
Regression through origin

$$\boxed{a = 0 \rightarrow y = b * x}$$

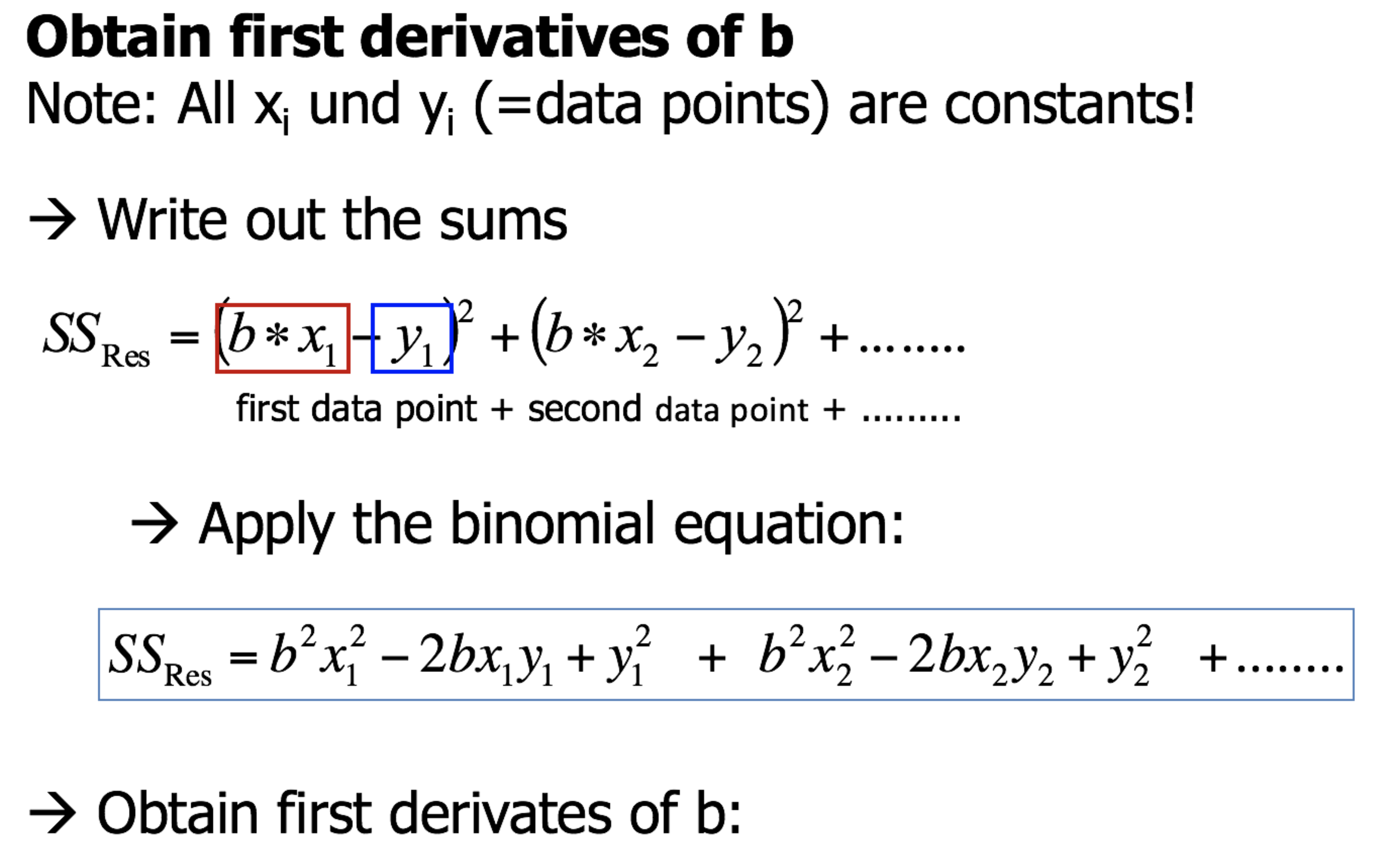


Residual sum of squares (with a = 0)

$$SS_{\mathrm{Res}} = \sum e_i^{\,2} = \sum (y_i - (b * x_i))^2$$



Minimising $SS_{Residual}$ numerically



Obtain first derivatives of b

Note: All x_i und y_i (=data points) are constants!

→ Write out the sums

$$SS_{\text{Res}} = \left(\boxed{b * x_1} - \boxed{y_1}\right)^2 + \left(b * x_2 - y_2\right)^2 + \ldots\ldots\ldots$$

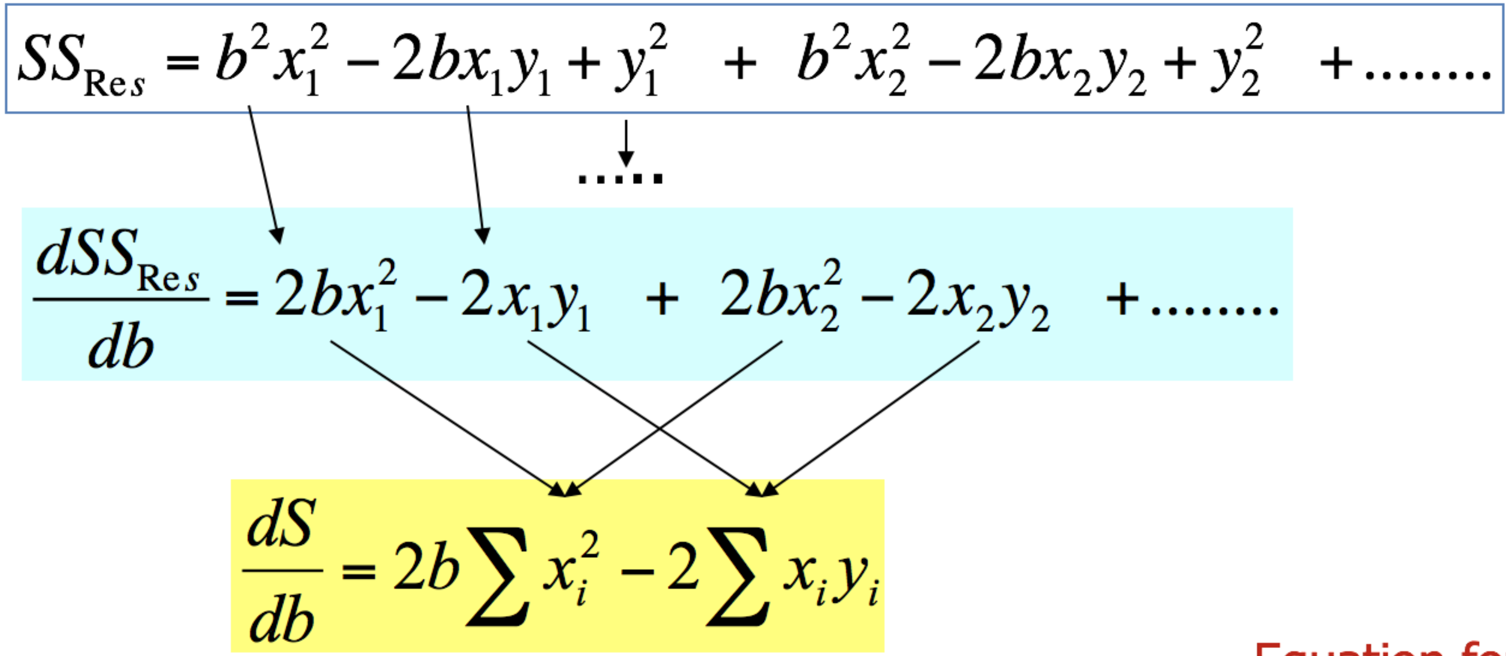
first data point + second data point + ...

→ Apply the binomial equation:

$$\boxed{SS_{\text{Res}} = b^2x_1^2 - 2bx_1y_1 + y_1^2 \quad + \quad b^2x_2^2 - 2bx_2y_2 + y_2^2 \quad + \ldots\ldots\ldots}$$

→ Obtain first derivates of b:

Minimising $SS_{Residual}$ numerically



$$SS_{\text{Res}} = b^2x_1^2 - 2bx_1y_1 + y_1^2 \quad + \quad b^2x_2^2 - 2bx_2y_2 + y_2^2 \quad + ...$$

.....

$$\frac{dSS_{\text{Res}}}{db} = 2bx_1^2 - 2x_1y_1 \quad + \quad 2bx_2^2 - 2x_2y_2 \quad + ...$$

$$\frac{dS}{db} = 2b\sum x_i^2 - 2\sum x_iy_i$$

→ Set equal to zero and solve for b

$$2\sum x_iy_i = 2b\sum x_i^2 \quad \Longrightarrow \quad \frac{\sum x_iy_i}{\sum x_i^2} = b$$

Equation for linear regression where a=0 (regression through origin) → SS_{Res} is minimal!

Coefficient estimations

Analogous are the following equations derived:

Estimation of the slope *b*:

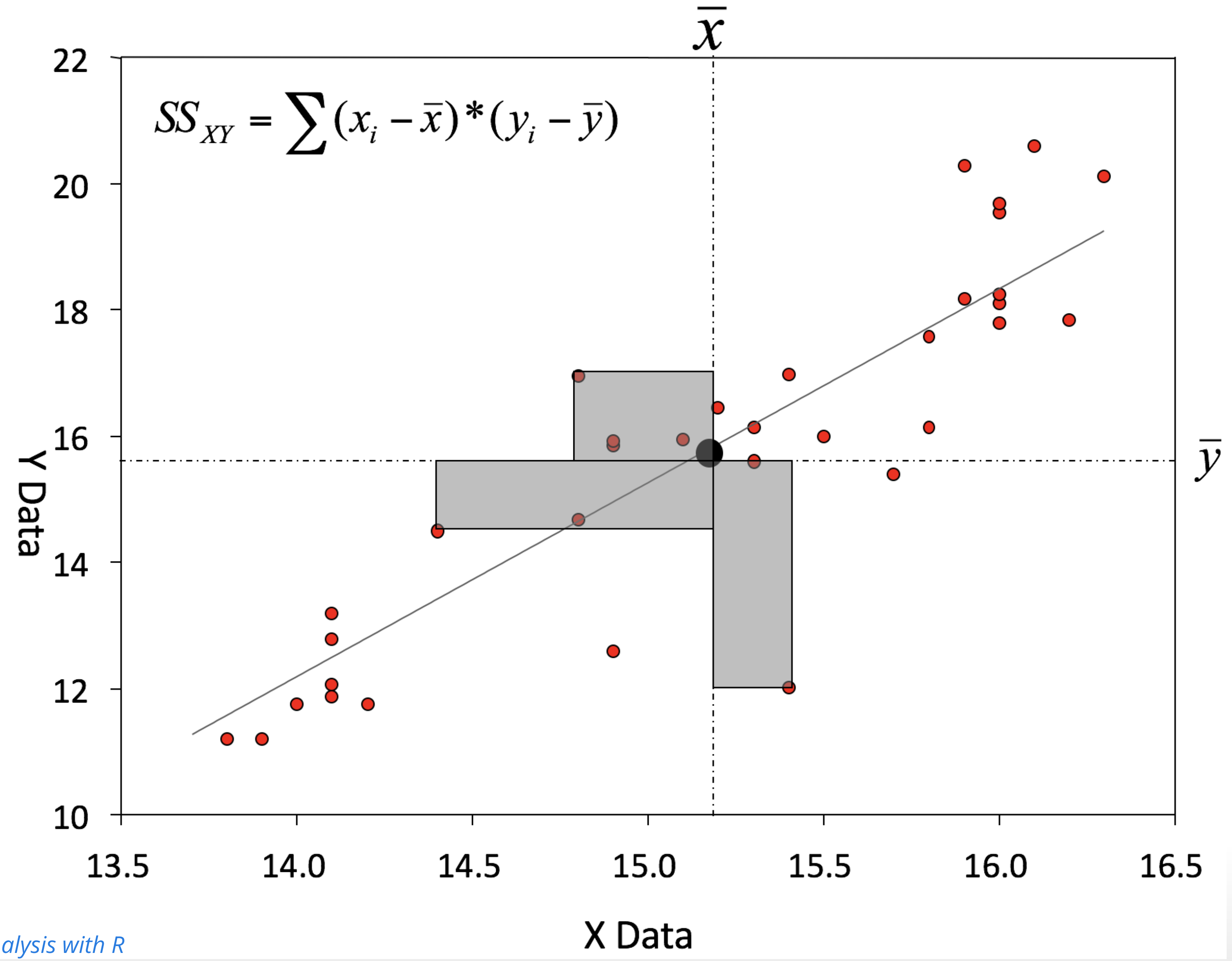
$$b = \frac{\sum\left[\left(x_i - \bar{x}\right) * \left(y_i - \bar{y}\right)\right]}{\sum\left(x_i - \bar{x}\right)^2} = \frac{SS_{XY}}{SS_X}$$

Estimation of the intercept *a*:

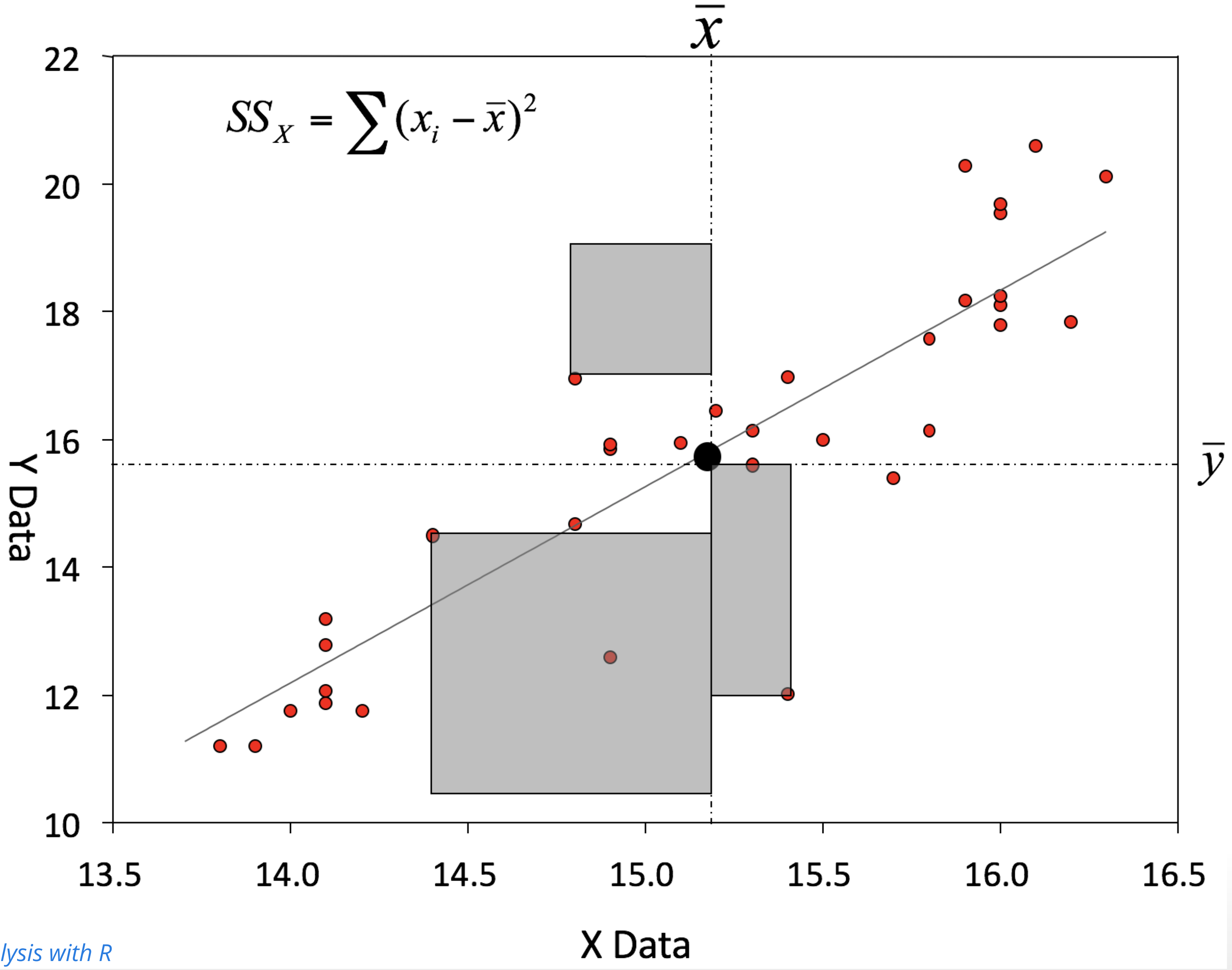
$$a = \bar{y} - b\bar{x}$$

→ Since regression lines go through the "center" of the data cloud, i.e. through the mean of x and y, you just need to transform the equation to get *a*:

$$\bar{y} = a - b * \bar{x}$$



$$SS_{XY} = \sum (x_i - \bar{x}) * (y_i - \bar{y})$$



$\bar{x}$
$SS_X = \sum (x_i - \bar{x})^2$
$\bar{y}$
22
20
18
16
14
12
10
Y Data
13.5
14.0
14.5
15.0
15.5
16.0
16.5
X Data

An example: Salinity as a function of depth

(taken from the ICES hydro dataset, station 0076, 2015-02-01)

```r
mod <- lm(sal ~ pres, data = sal_profile)
coef(mod) # same as: coefficients(mod)
```

```
## (Intercept)        pres
##  5.57662340  0.01282616
```

How to interpret these coefficients?

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- Salinity at the surface (when **depth is 0m**) is around 5.58

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```

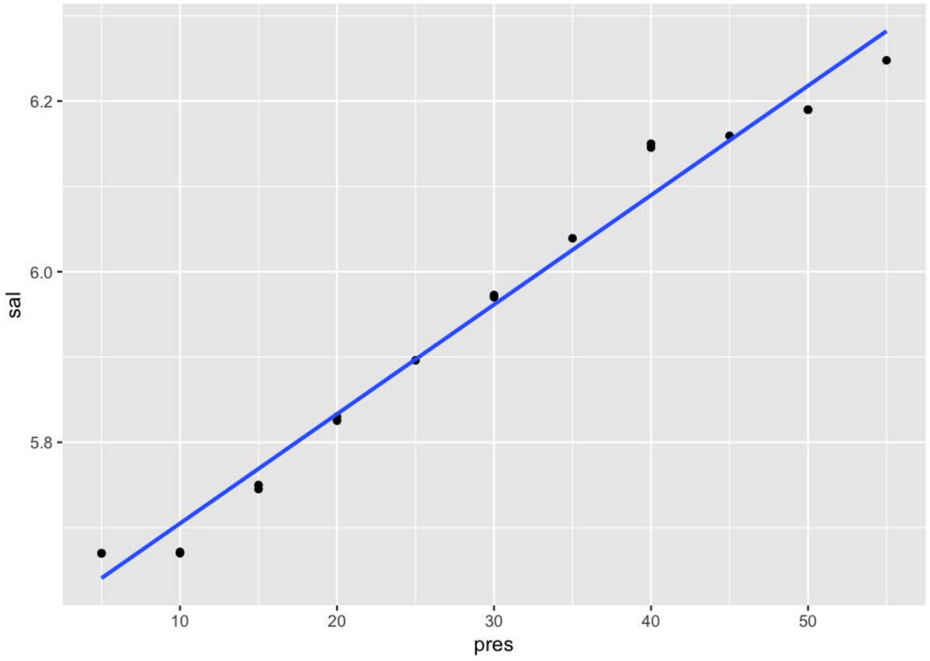
How to interpret these coefficients?

- Salinity at the surface (when **depth is 0m**) is around 5.58
- Each **1m** increase in depth is associated with an INCREASE in salinity by **0.013 units**.

Visualize your linear regression

Use `geom_smooth` and set the method argument to the name of the modelling function (in this case 'lm'). If you don't want to show the standard error additionally, set `se = FALSE`:

```r
p <- ggplot(sal_profile,
  aes(x = pres, y = sal)) +
  geom_point() +
  geom_smooth(method = "lm",
    se = FALSE)
p
```

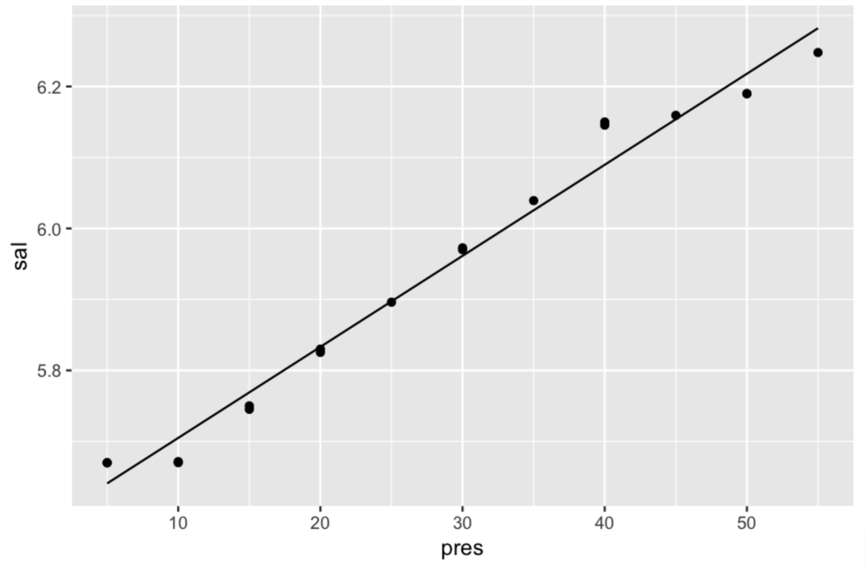


Visualize your linear regression manually

You can also **compute the predicted values** for your model using

- `predict(model)` from the *stats* package → simply returns a vector, which you can add to your data using the `dplyr::mutate()` function
- `add_predictions(data, model, var = "pred")` from the *modelr* package (in tidyverse) → adds the variable 'pred' containing the predicted values to your data frame; useful when piping operations!

```
modelr::add_predictions(
  sal_profile, mod) %>%
ggplot(aes(x = pres)) +
  geom_point(aes(y = sal)) +
  geom_line(aes(y = pred))
```



Predictions are best visualised from an **evenly spaced grid** of *X* values that covers the region where your data lies

- Add to the argument `newdata` in a data frame that includes an even X sequence:
    - `predict(model, newdata = data.frame(x = seq(min(x), max(x), 0.1)))`
- **Easier**: Use the function `modelr::data_grid(data, X1, X2)` to generate this grid. If you have more than one X, it finds the unique variables and then generates all combinations.
    - Use this grid then for the `add_predictions(data = grid, model)` function

```
library(modelr)
grid <- sal_profile %>%
  data_grid(pres = seq_range(pres, 20)) # 20 evenly spaced values between min and max
add_predictions(grid, mod) %>%
  ggplot(aes(x = pres)) + geom_line(aes(y = pred))
```

Visualize your regression including some statistics

- You can extract statistics from the model object directly or from functions applied on the model object such as `coef()` or `summary()` (as a help: look at the **names** or structure to see what you could extract).

- Lets extract **R-sq** and the **coefficients** to show the equation in the plot

```r
a <- round(coef(mod)[1], 3)
b <- round(coef(mod)[2], 3)
mod_summ <- summary(mod)
names(mod_summ)
```

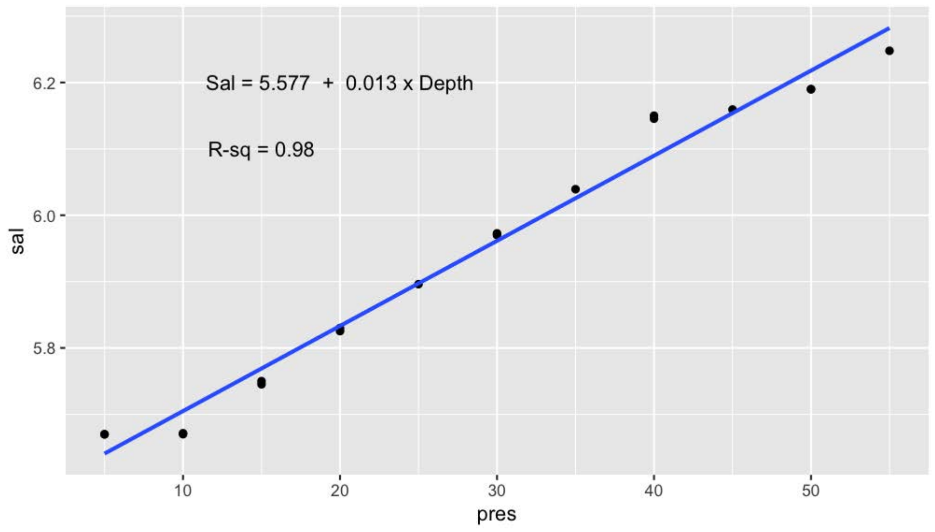
```
##  [1] "call"          "terms"         "residuals"     "coefficients"
##  [5] "aliased"       "sigma"         "df"            "r.squared"
##  [9] "adj.r.squared" "fstatistic"    "cov.unscaled"
```

```r
r_sq <- round(mod_summ$r.squared, 3)
```

Visualize your regression including some statistics

- Add model statistics as text in your ggplot using the `annotate("text", x, y, label)` function.
- You can use the `paste()` function to concatenate text and values stored in objects:

```r
p +
  annotate("text", x = 20, y = 6.2,
    label = paste("Sal =",a ,
      " + ",b , "x Depth")) +
    annotate("text", x = 15, y = 6.1,
    label = paste("R-sq =", r_sq))
```



Your turn...

Exercise 2: `Iris` length relationship

Visualize your `Petal.Width ~ Petal.Length` model using

- `geom_smooth()`
- `predict()` or `modelr::add_predictions()` to add a line manually.
- Try also to include the regression equation somewhere in the plot.

Aside: intercept terms

R includes an intercept term in each model by **default**

$$y = a + bx$$

```
y ~ x
```

Aside: intercept terms (cont)

You can **include** an intercept explicitly by **adding a 1** to the formula term. If you want to **exclude** the intercept, add a 0 or **substract a 1**:

```r
# includes intercept
mod1 <- lm(sal ~ pres, data = sal_profile)
mod1 <- lm(sal ~ 1 + pres, data = sal_profile)
```

```r
# excludes intercept
mod2 <- lm(sal ~ 0 + pres, data = sal_profile)
mod2 <- lm(sal ~ pres - 1, data = sal_profile)
```

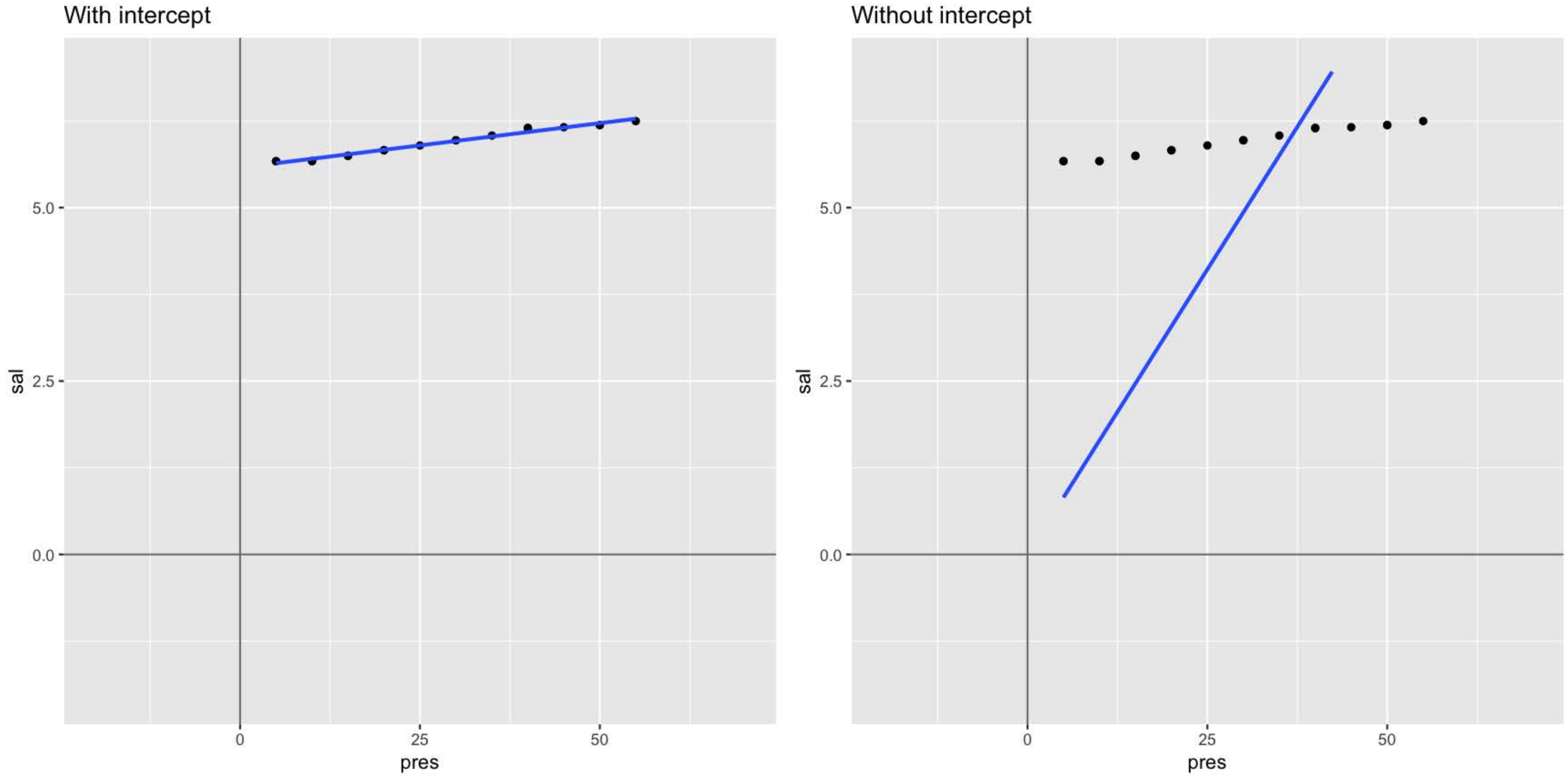
mod1

```
##
## Call:
## lm(formula = sal ~ 1 + pres, data = sal_profile)
##
## Coefficients:
## (Intercept)         pres
##     5.57662      0.01283
```

mod2

```
##
## Call:
## lm(formula = sal ~ pres - 1, data = sal_profile)
##
## Coefficients:
##   pres
## 0.1644
```

Not including **a** forces the intercept through the origin of coordinates



Your turn...

Exercise 3: `Iris` length relationship

Compare your `Petal.Width ~ Petal.Length` model including an intercept with the same without an intercept

- how do the slope coefficients differ?
- which regression line seems to fit better (in the plot)

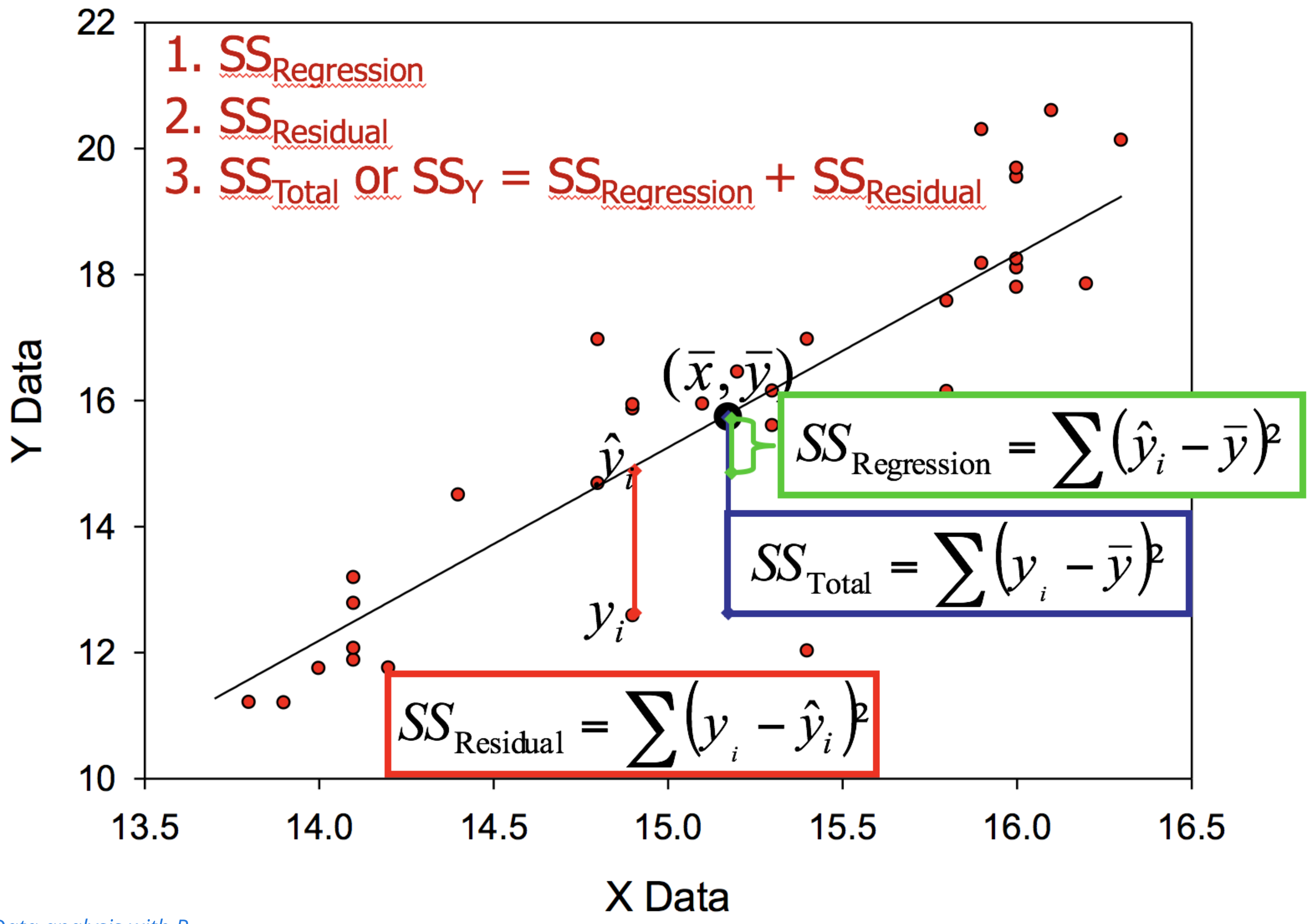
Simply add both regression lines to the same plot. Or if you want to show both plots next to each other, save each plot under, e.g., p1 and p2 and use the following line of code

```
gridExtra::grid.arrange(p1,p2, nrow = 1)
```

Partitioning variation in linear models

Total variation in response variable (SS_Y or SS_{Total}) can be partitioned into two components

- Variation explained by predictor: $SS_{Regression}$
  - Effect of continous variable (Regression)
  - Difference between categorical groups (ANOVA)
- Variation not explained (residual or error variation): $SS_{Residual}$



1. $SS_{\text{Regression}}$
2. SS_{Residual}
3. SS_{Total} or $SS_Y = SS_{\text{Regression}} + SS_{\text{Residual}}$

$$SS_{\text{Regression}} = \sum(\hat{y}_i - \bar{y})^2$$

$$SS_{\text{Total}} = \sum(y_i - \bar{y})^2$$

$$SS_{\text{Residual}} = \sum(y_i - \hat{y}_i)^2$$

Coefficient of determination - R^2

- is a **statistical measure** of how close the data are to the fitted regression line: it represents the **proportion of variation in Y explained by linear relationship**
- R^2 tells you how good the model is:
    - **0%** indicates that the model **explains none** of the variability of the response data around its mean.
    - In **ecology, 50%** explained variance is already considered as **very good**.

$$R^2 = 1 - \frac{SS_{\text{Residual}}}{SS_{\text{Total}}} = 1 - \frac{\sum (y_i - \hat{y}_i)^2}{\sum (y_i - \bar{y})^2} \rightarrow \infty \quad R^2 \rightarrow 1$$

The total variance in the denominator increases with an increasing number of covariates, even if they do not contribute to the explained variance.

Coefficient of determination - R^2

When the model has more than one X variable: **adjusted** R^2

= corrected coefficient of determination

$$R_{adj}{}^{2} = 1 - (1 - R^2)\left(\frac{(N-1)}{(N-p-1)}\right)$$

degrees of freedom also considered (p = number of independent variables

Your turn...

Exercise 4: `Iris` length relationship

How much variation in `Sepal.Length` is explained by `Petal.Length` in your model? Do you consider it a good model?

Mean Squares (MS) = Variances

$$\sigma^2 = \frac{\sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2}{n-1}$$

- SS depends on the sample size
- MS accounts for the sample size by dividing SS by the df → it represents the **sample variance**
- Degrees of freedom: $\text{df}_{\text{Total}} = \text{df}_{\text{Regression}} + \text{df}_{\text{Residual}}$

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- MS accounts for the sample size by dividing SS by the df → it represents the **sample variance**
- Degrees of freedom: $\mathrm{df}_{\mathrm{Total}} = \mathrm{df}_{\mathrm{Regression}} + \mathrm{df}_{\mathrm{Residual}}$

$$MS_{Regression} = \frac{SS_{Regression}}{df_{Regression}} = \frac{\sum(\hat{y} - \bar{y})^2}{n-1}$$

$$MS_{Residual} = \frac{SS_{Residual}}{df_{Residual}} = \frac{\sum(y_i - \hat{y})^2}{n-2}$$

$$MS_{Total} = \frac{SS_{Total}}{df_{Total}} = \frac{\sum(y_i - \bar{y})^2}{n-1}$$

Reliability of estimated parameters: SE_a and SE_b

The uncertainty of the estimated **slope and intercept**

- increases with increasing *variance* and declines with increasing *sample size*
- is also greater when the *range of X values* is small (as measured by SS_X)

The uncertainty of the **intercept** estimate additionally increases

- with the *square of the distance* between the origin and the mean value of X (as measured by $\sum(x_i^2)$.

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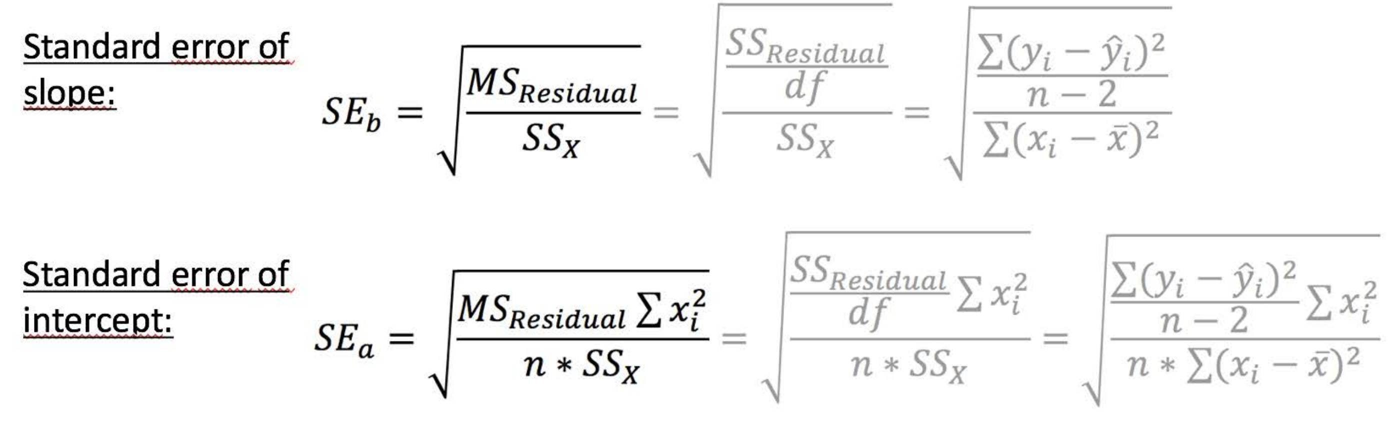


Standard error of slope:

$$SE_b = \sqrt{\frac{MS_{Residual}}{SS_X}} = \sqrt{\frac{\frac{SS_{Residual}}{df}}{SS_X}} = \sqrt{\frac{\frac{\sum(y_i-\hat{y}_i)^2}{n-2}}{\sum(x_i-\bar{x})^2}}$$

Standard error of intercept:

$$SE_a = \sqrt{\frac{MS_{Residual}\sum x_i^2}{n * SS_X}} = \sqrt{\frac{\frac{SS_{Residual}}{df}\sum x_i^2}{n * SS_X}} = \sqrt{\frac{\frac{\sum(y_i-\hat{y}_i)^2}{n-2}\sum x_i^2}{n * \sum(x_i-\bar{x})^2}}$$

Lets look at the standard errors in the salinity-depth model...

```
summary(mod)
```

```
##
## Call:
## lm(formula = sal ~ pres, data = sal_profile)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.034885 -0.026902 -0.002162  0.013069  0.060330
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 5.5766234  0.0145514  383.24  < 2e-16 ***
## pres        0.0128262  0.0004551   28.18 4.58e-15 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.03057 on 16 degrees of freedom
## Multiple R-squared:  0.9803, Adjusted R-squared:  0.979
## F-statistic: 794.2 on 1 and 16 DF,  p-value: 4.583e-15
```

Your turn...

Exercise 5: `Iris` length relationship

Look at the standard errors of your model.

- Would you consider these values close to the estimated coefficients?
- Do the standard errors indicate that the estimated slope of another sample would be similar to your iris sample or could they be totally different?

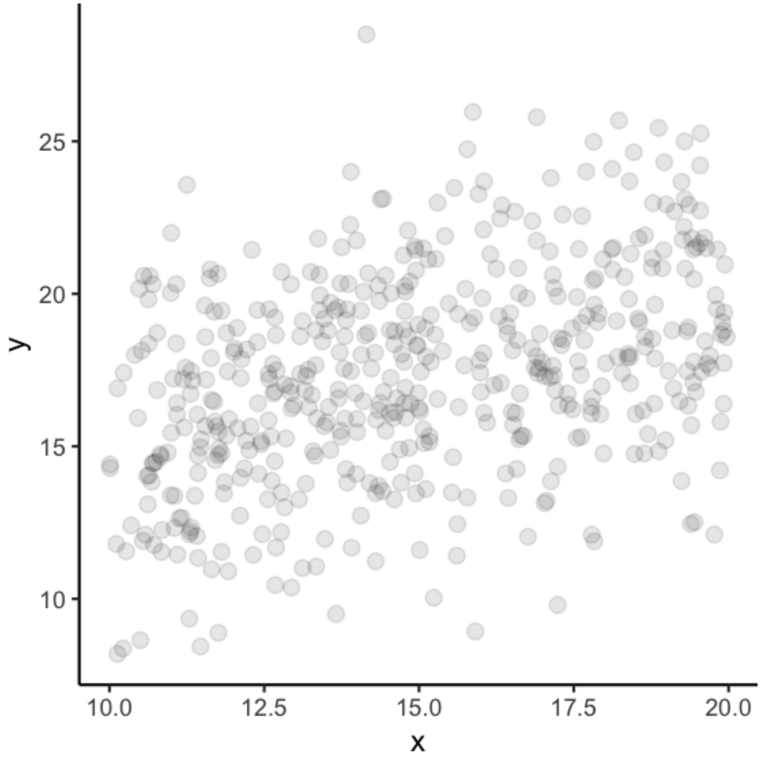
Visualize the standard error: simply set the `se`argument in `geom_smooth()` to `TRUE`.

- What do you think now about the reliability of your estimated coefficient?

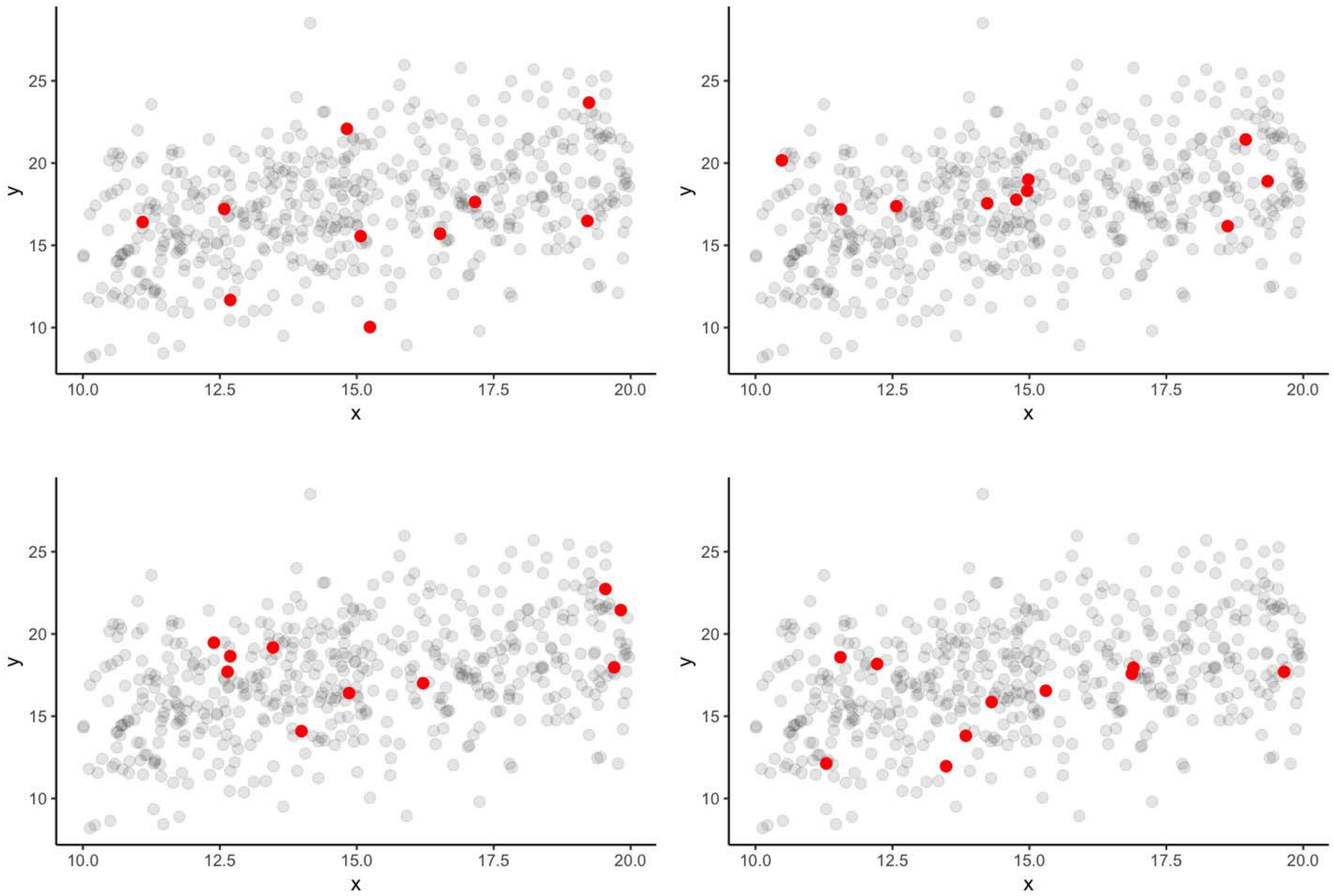
Outlook into model inference

Reproducability of estimated coefficients

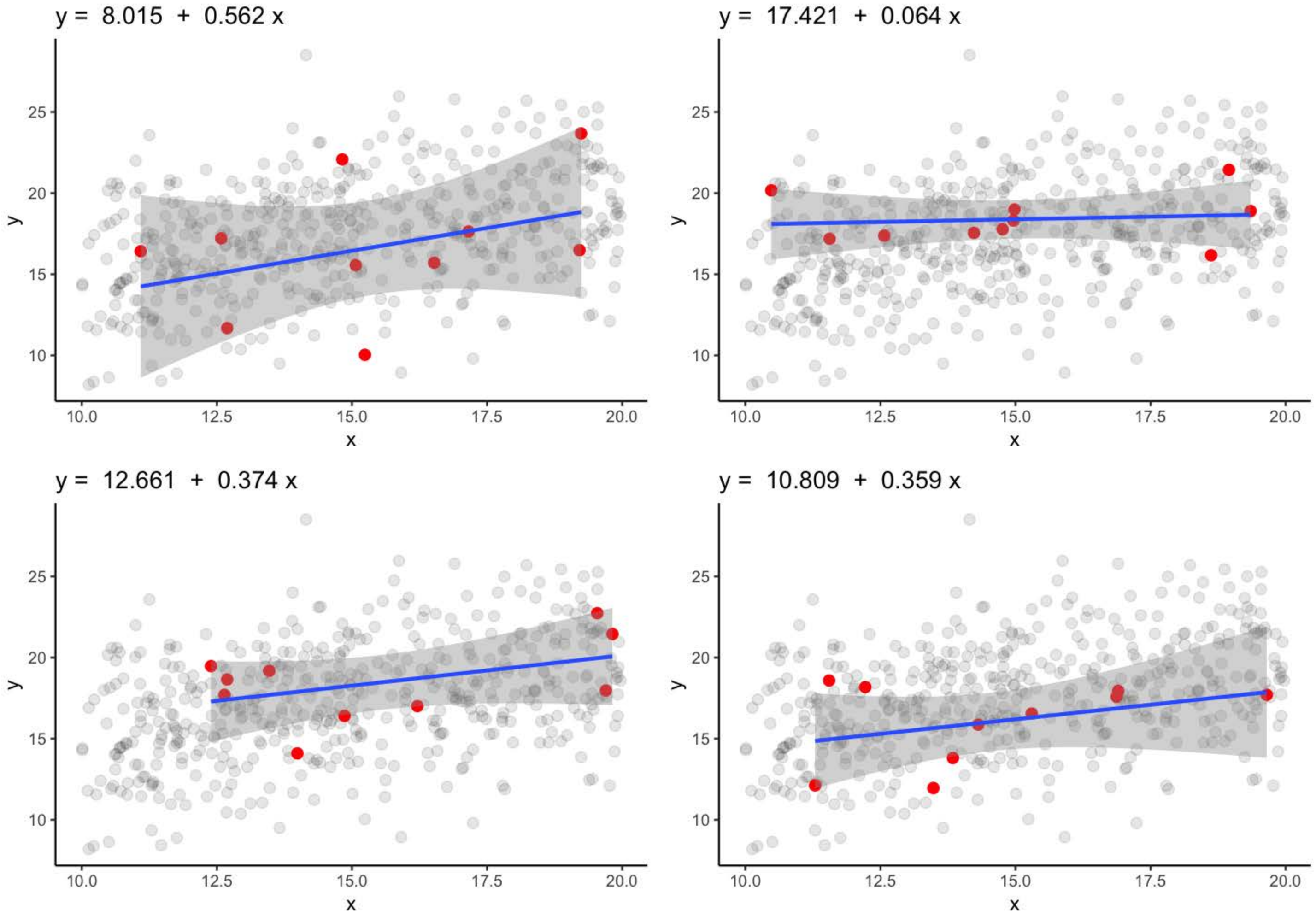
Imagine, each data point represents an individual fish and all data points together represent the entire population.



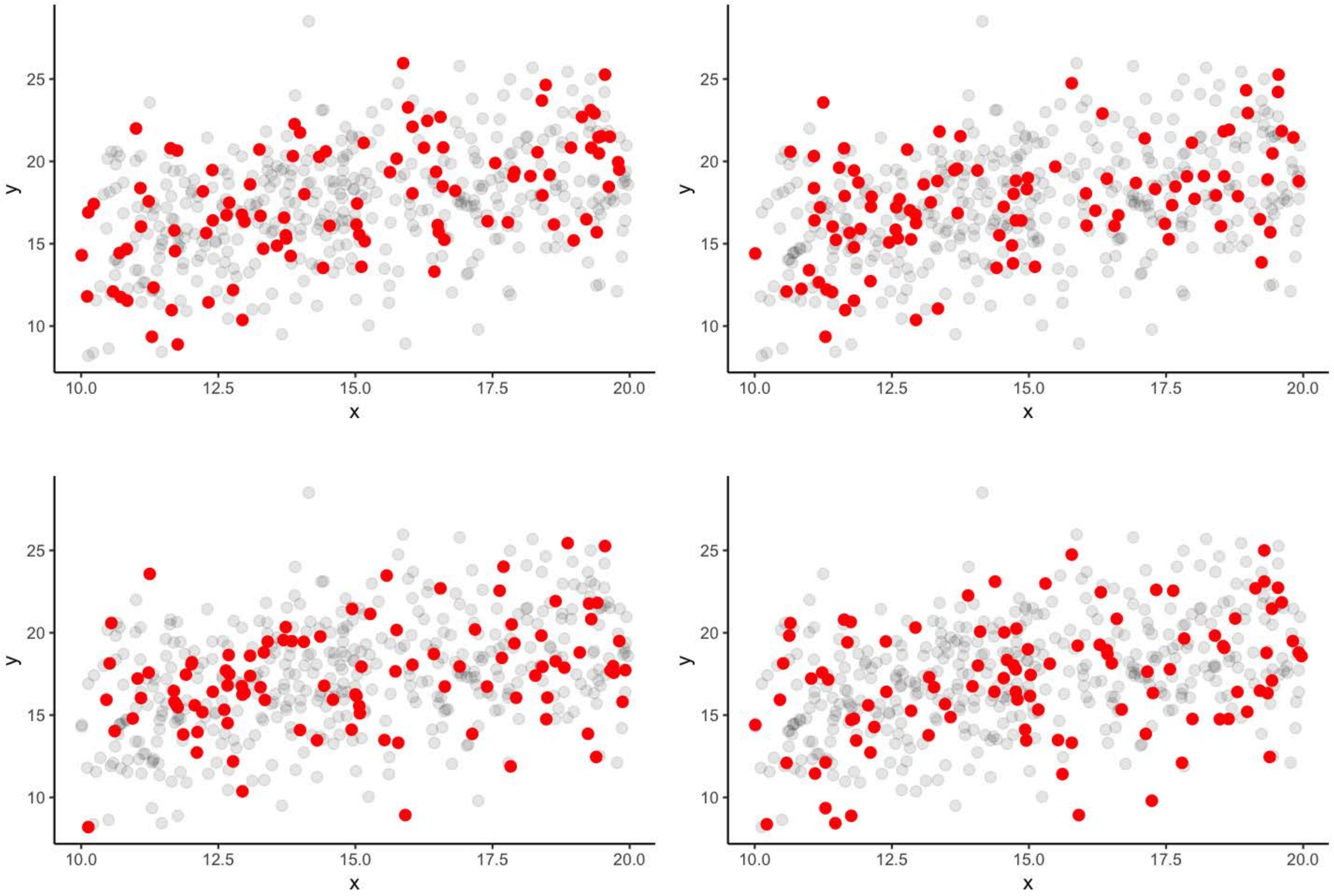
If you take a **random sample of 10 fish**, how similar would the estimated coefficients be?



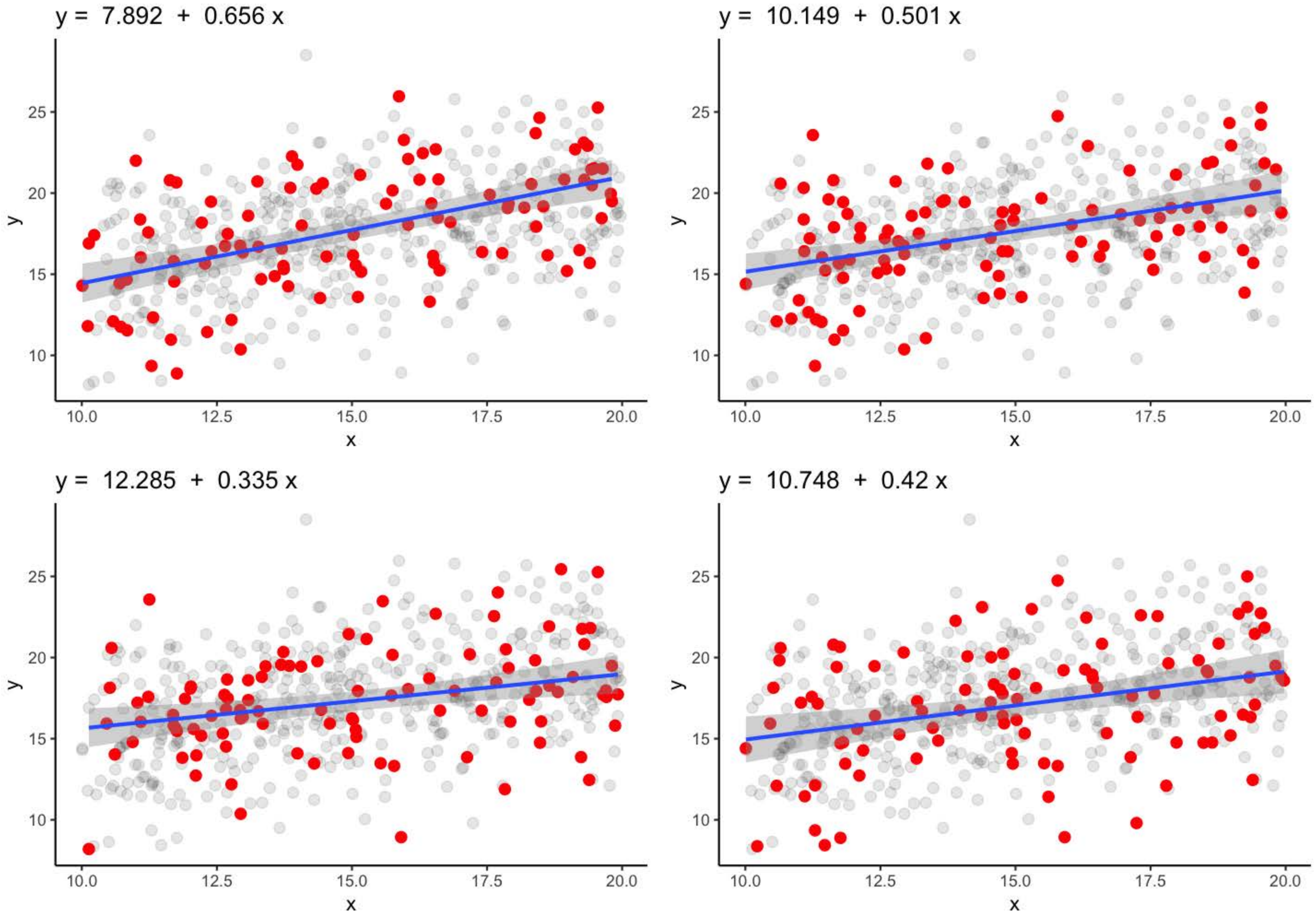
Guess: Which ones are more similar and which ones might be closer to the true population parameters?



True parameters: α = 10.269 and β = 0.475



Lets increase the sample size to 100 fish!



True parameters: α = 10.269 and β = 0.475

Inference

2 approaches for reasoning about uncertain model parameters:

- Nonparametric statistics (bootstrapping)
- Parametric statistics
    - If a model meets a set of assumptions, it is easy to calculate the probability of observing a given b when the true β is 0

Probabilities

If a p-value is very low (< 0.05), it suggests that either

1. you have an unusual sample

2. the true β does not equal 0

3. your model assumptions are wrong

Confidence intervals

Knowing probabilities also lets us calculate confidence intervals for β

```r
# Models with n = 10
confint(mod_list[[1]], level = 0.95)
```

```
##                  2.5 %    97.5 %
## (Intercept) -9.5084037 25.537982
## x           -0.5626634  1.686557
```

```r
confint(mod_list[[2]], level = 0.95)
```

```
##                  2.5 %     97.5 %
## (Intercept) 11.2339388 23.6086437
## x           -0.3394683  0.4677254
```

```r
# Models with n = 100
confint(mod_list[[5]], level = 0.95)
```

```
##                 2.5 %     97.5 %
## (Intercept) 4.7328994 11.0508714
## x           0.4490422  0.8620645
```

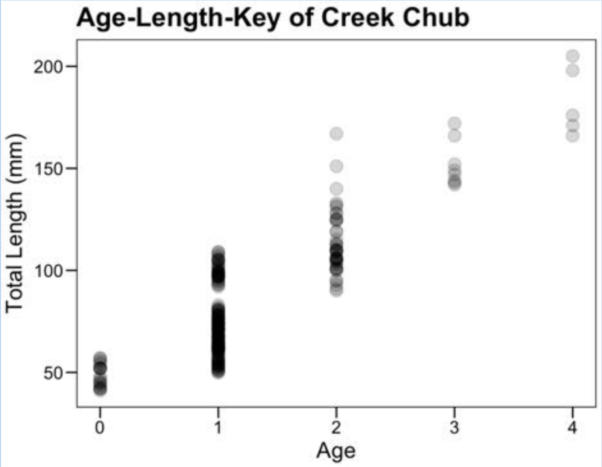
```r
confint(mod_list[[7]], level = 0.95)
```

```
##                 2.5 %     97.5 %
## (Intercept) 9.0867047 15.4839002
## x           0.1259973  0.5433739
```

Your turn...

Exercise 6: Interpret this summary output

```
##
## Call:
## lm(formula = len ~ age, data = cc_fn1)
##
## Residuals:
##     Min       1Q   Median       3Q      Max
## -26.452 -12.315  -2.452  13.003  55.094
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   40.997      2.058   19.92   <2e-16 ***
## age           35.454      1.408   25.18   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 15.84 on 216 degrees of freedom
## Multiple R-squared:  0.746,  Adjusted R-squared:  0.7448
## F-statistic: 634.3 on 1 and 216 DF,  p-value: < 2.2e-16
```



Age-Length-Key of Creek Chub

exercise is based on IFAR by D. Ogle, 2016 descriptive stats: `median()`, `quantile()`, `mean()`, `weighted.mean()`, `range()`, `var()`, `sd()`

linear regression model: `lm()`, `coef()`, `summary()`, `confint()` `geom_smooth(method = "lm")`, `predict()`, `modelr::add_predictions()`

Overview of functions you learned today

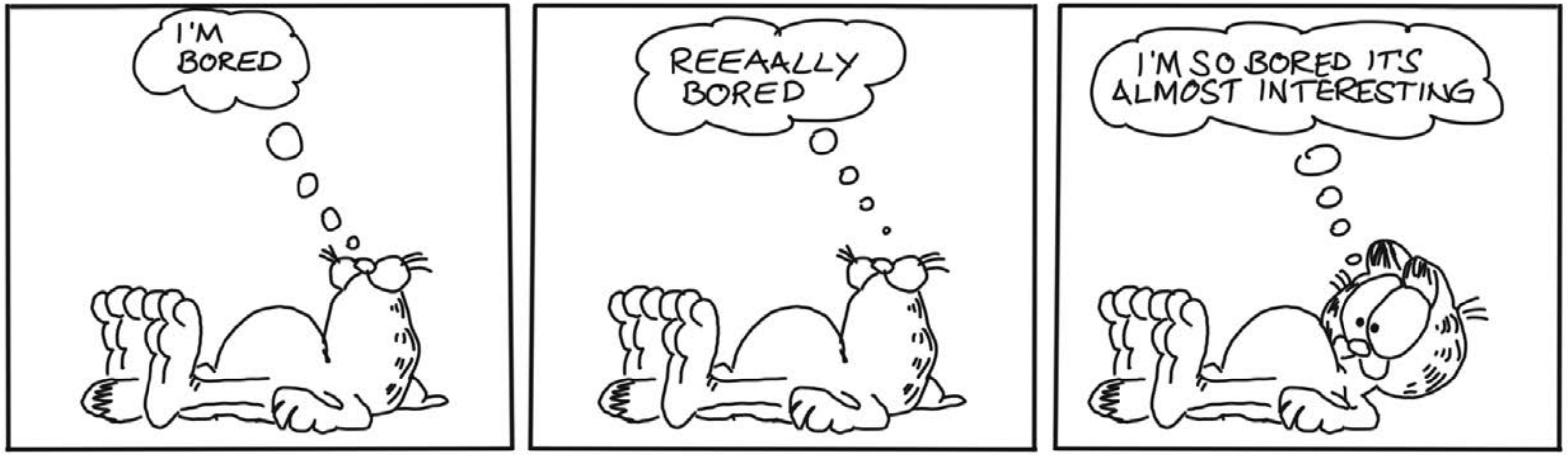
How do you feel now.....?

Totally confused?

Try out the exercises and read up on linear regressions in

- chapter 23 on model basics in 'R for Data Science'
- chapter 10 (linear regressions) in "The R book" (Crawley, 2013, 2nd edition) (an online pdf version is freely available here)
- or any other textbook on linear regressions

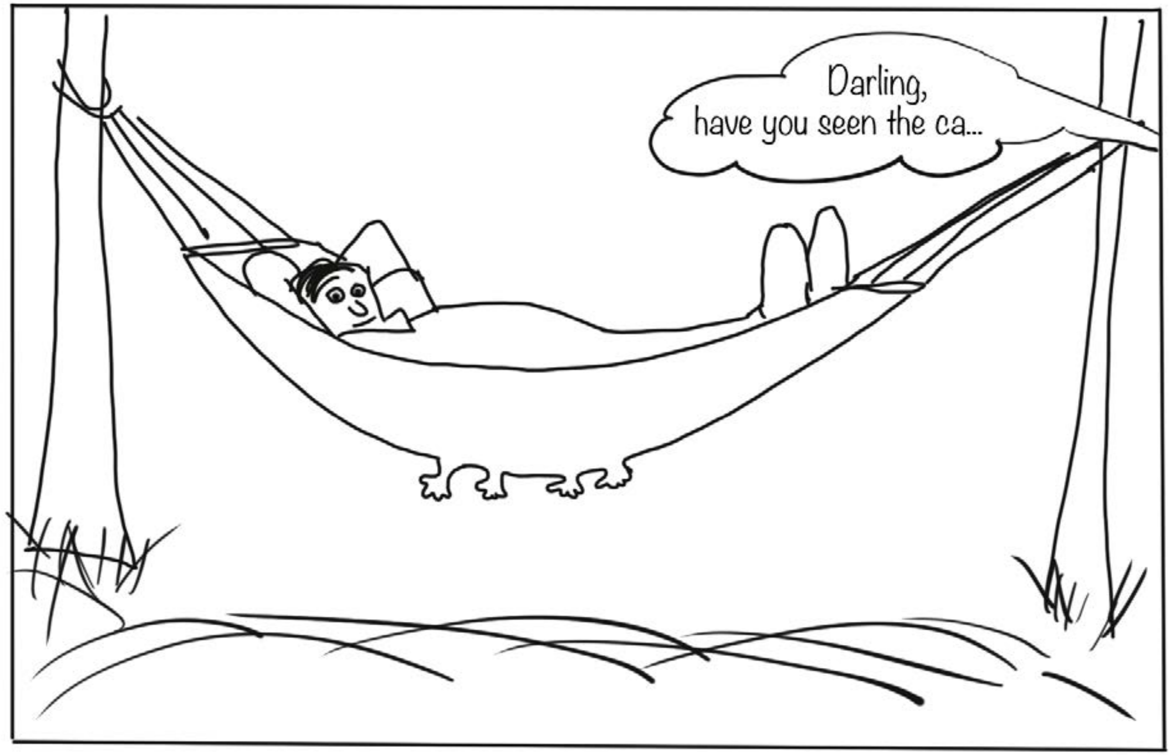
Totally bored?



Apply a model for the weight variables in the `iris` dataset. Load the hydro data and model the temperature or oxygen as a function of depth. Choose a single stations or model across all stations.

Totally content?

Then go grab a coffee, lean back and enjoy the rest of the day...!



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DER FORSCHUNG | DER LEHRE | DER BILDUNG

Thank You

For more information contact me: saskia.otto@uni-hamburg.de

http://www.researchgate.net/profile/Saskia_Otto
http://www.github.com/saskiaotto

Image on title and end slide: Section of an infrared satallite image showing the Larsen C ice shelf on the Antarctic Peninsula - USGS/NASA Landsat: A Crack of Light in the Polar Dark, Landsat 8 - TIRS, June 17, 2017
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