Overview of Direct Sparse Linear Solvers for Power Grid Optimization Problems

Presenter: Leslie Cook

Midwestern State University

ORNL is managed by UT-Battelle LLC for the US Department of Energy

Problem Statement

Solving large sparse linear systems of equations

$$Ax = b$$

- A is a known coefficient matrix with dimension $n \times n$
- b is a known RHS vector with dimension $n \times 1$
- x is an unknown solution vector with dimension $n \times 1$

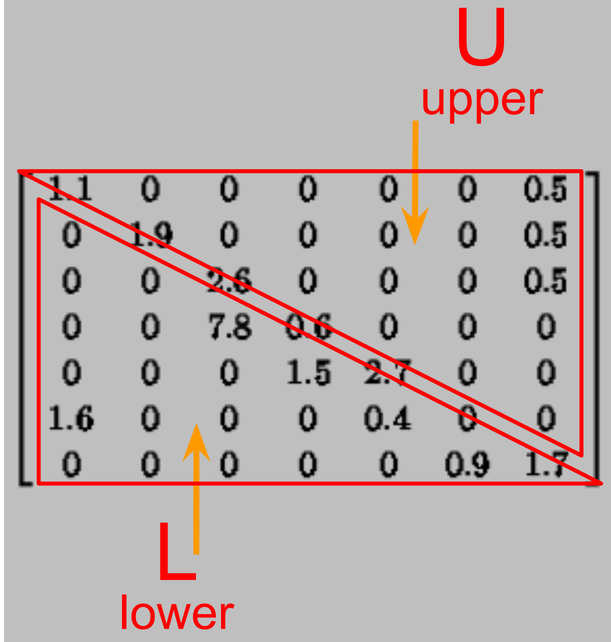
Factorize A into lower L and upper U triangulars, where

$$A = LU$$

Solving $x = A^{-1}b$ directly is too computationally expensive, so we solve by the substitutions,

1. $Ly = b$ solve $y = L^{-1}b$
2. $Ux = y$ solve $\boxed{x = U^{-1}y}$

Sparse Matrix



Problem Statement

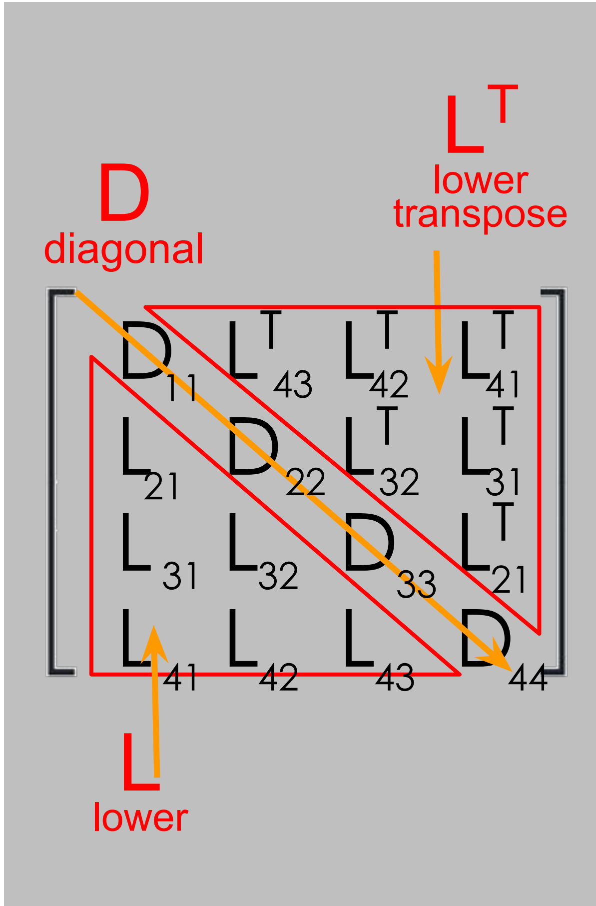
If we know we have a symmetric sparsity pattern, we can exploit matrix symmetry to further reduce computational cost by factoring ***A*** as,

$$\boldsymbol{A = LDL^T}$$

- ***L*** is the lower triangular
- ***D*** is the diagonal block
- $\boldsymbol{L^T}$ is the transpose of the lower triangular

Solve $\boldsymbol{LDL^T x = b}$ with the substitutions,

1. $\boldsymbol{Ly = b}$ solve $\boldsymbol{y = L^{-1}b}$
2. $\boldsymbol{DL^T x = y}$ solve $\boxed{\boldsymbol{x = (DL^T)^{-1} y = D^{-1} L y}}$



Oak Ridge National Laboratory

Problem Statement

The residual vector ***r*** is calculated to determine the accuracy of our solution, the closer to zero the better.

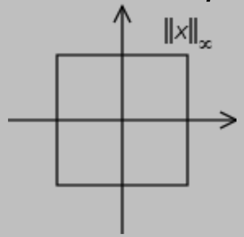
$$\boldsymbol{r = b - Ax}$$

We then compute the norm of the residual vector ***r*** and the norm of the RHS vector ***b*** to obtain the relative residual $\boldsymbol{R_{rel}}$ as a measure of how well our solution converges.

$$\boldsymbol{R_{rel} = ||r|| \;/\; ||b||}$$

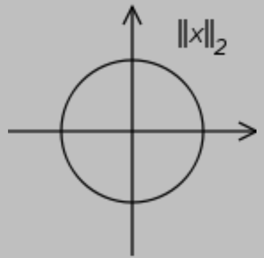
INF-norm

$||x||_\infty = \max_i |x_i|$



2-norm

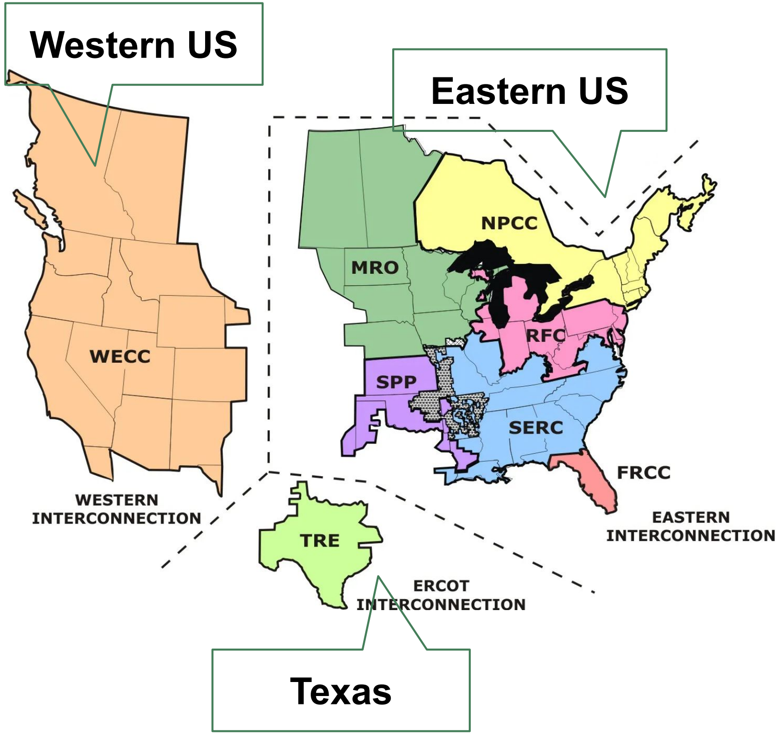
$||x||_2 = \sqrt{\sum |x_i|^2}$



Test Cases

Linear systems extracted from AC power flow analysis for power grids of Texas, Western US, and Eastern US.

Grid Model	rows	columns	NNZ	NNZ un
Texas 2000-bus	55667	55667	173710	291743
Western US 10k-bus	238072	238072	723720	1209368
Eastern US 70k-bus	1640411	1640411	4991820	8343229



Testing Environments
Oak Ridge Leadership Computing Facility

- **Summit**

21 cores and each node consists of 2x IBM POWER9™ CPUs and 6 NVIDIA Volta v100 GPUs

- **Frontier**

64 cores each node consists of 1x HPC and AI Optimized 3rd Gen AMD EPYC CPU and 4 Purpose Built AMD Instinct 250X GPUs

Sparse Linear Solver Packages

CPU

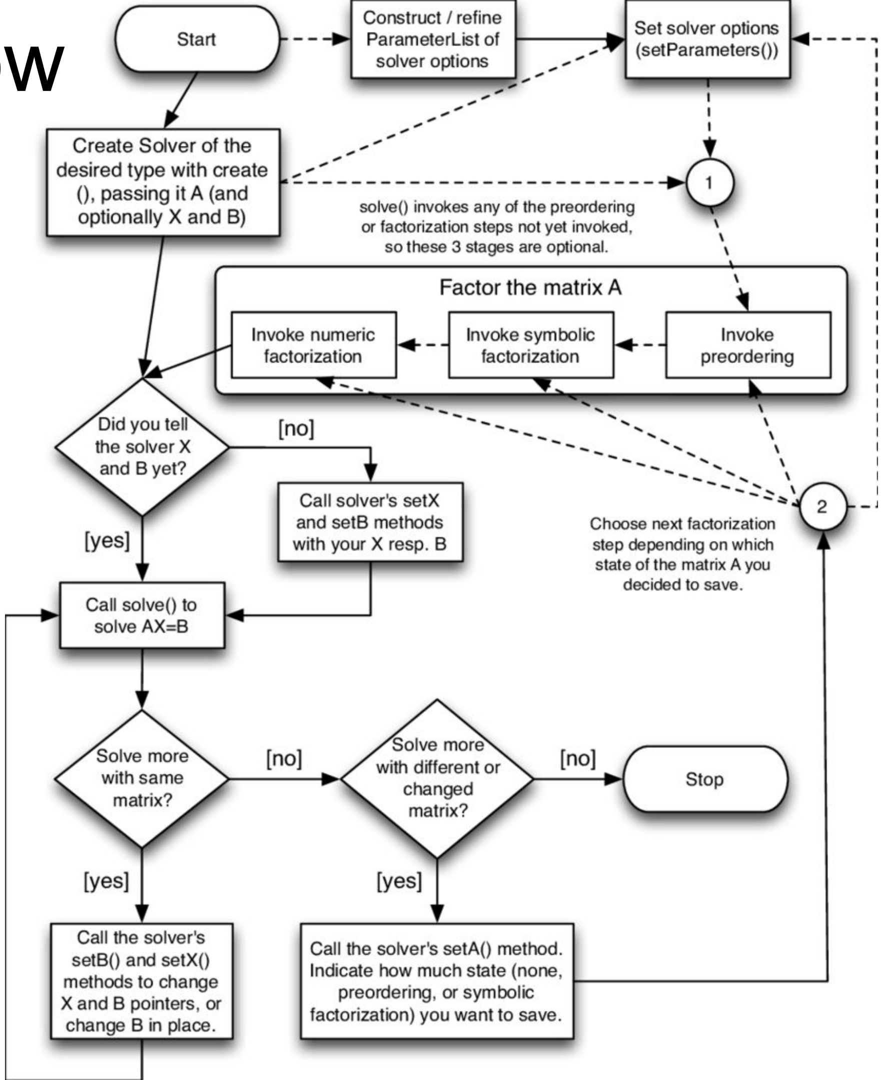
- HSL
  - MA57 (baseline)
- Trilinos Amesos2
  - KLU2
  - ShyLUBasker

GPU

- cuSolver
  - KLU-GLU
  - KLU-RFwFGMRES
- rocSOLVER
  - rocSolver-KLU-RF

Solver	MA 57	KLU2	ShyLU Basker	cuSolver KLU-GLU	cuSolver KLU-RF wGMRES	rocSolver KLU
Exploits symmetry	Yes	No	No	No	No	No
Primary algorithm	LDL^T	LU	(I)LU	LU refact	LU refact	LU refact
Follows with IR	Yes	No	Yes	Yes	Yes	Yes
Norm eq used	INF-norm	2-norm	2-norm	INF-norm	INF-norm	INF-norm
GPU accelerated	No	No	No	Yes	Yes	Yes
compiler	gcc	gcc	gcc	nvcc	nvcc	hipcc
Open source	No	Yes	Yes	No	No	No

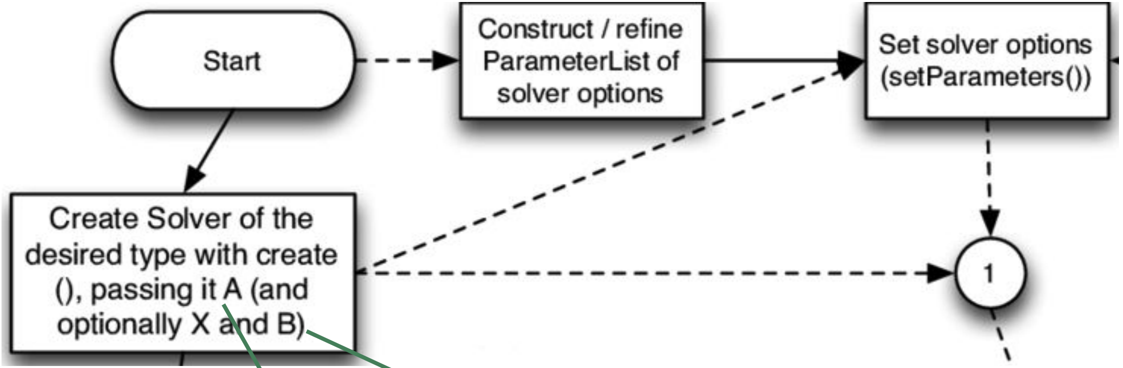
Workflow



AMESOS2

- Tasked with building a testbench for Amesos2 solvers in the ORNL Linear-Solver-Testing GitLab repository

Workflow

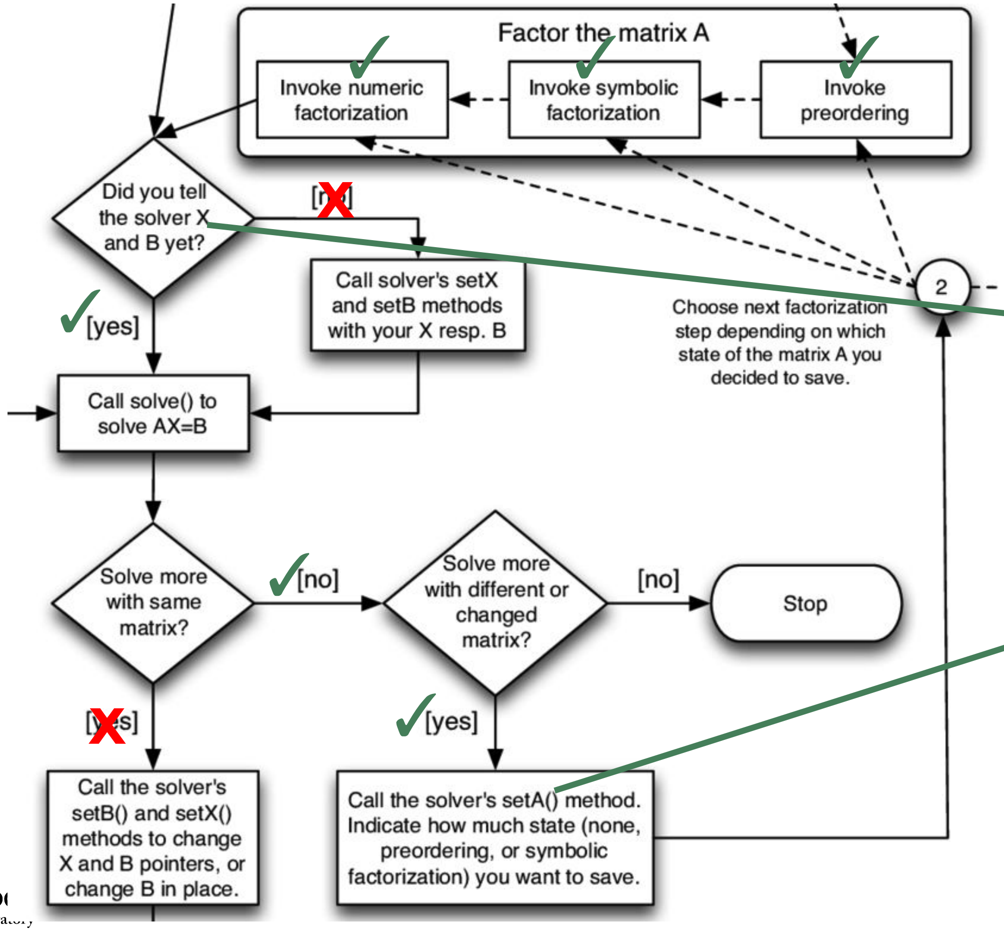


matrix A	vector B
reads in a series of given matrix market files	reads in a series of matching RHS matrix market files

Parameters

KLU2	**ShyLUBasker**
Equilibrate before solve	Set num threads (64)
Is contiguous	Use pivot
	Replace tiny pivot
	Use metis

Workflow



X is set to randomized values

setA(A, Amesos2::SYMBFACT)
reuse symbolic factorization

RESULTS

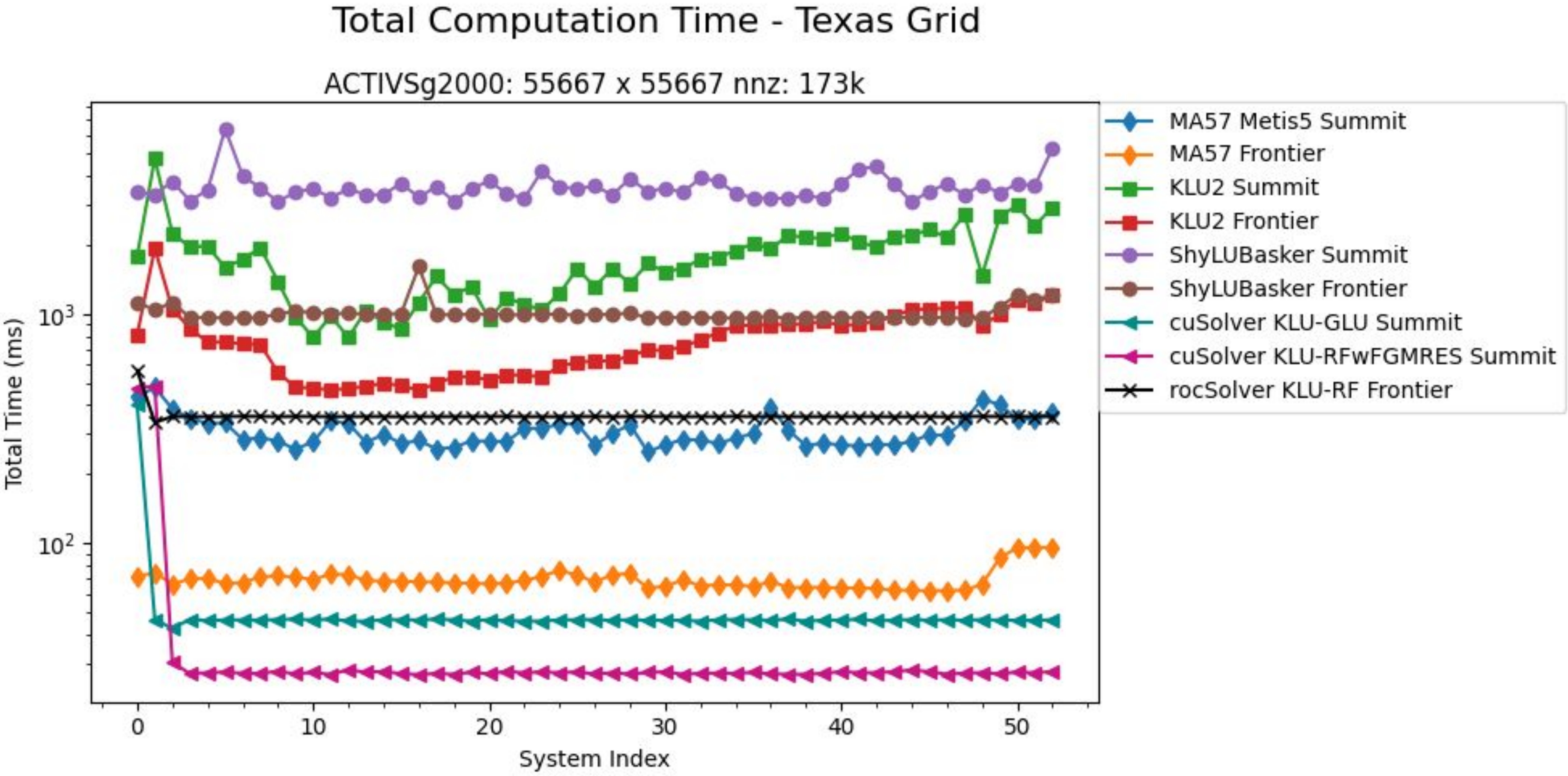


Figure 1. Total runtimes for the Texas (2000-bus) test case. ShyLUBasker performed significantly worse on Summit compared to Frontier and KLU2 performs better than ShyLUBasker. MA57 is much better on Frontier than Summit for the Texas grid test case. CUDA cuSolvers perform the best overall.

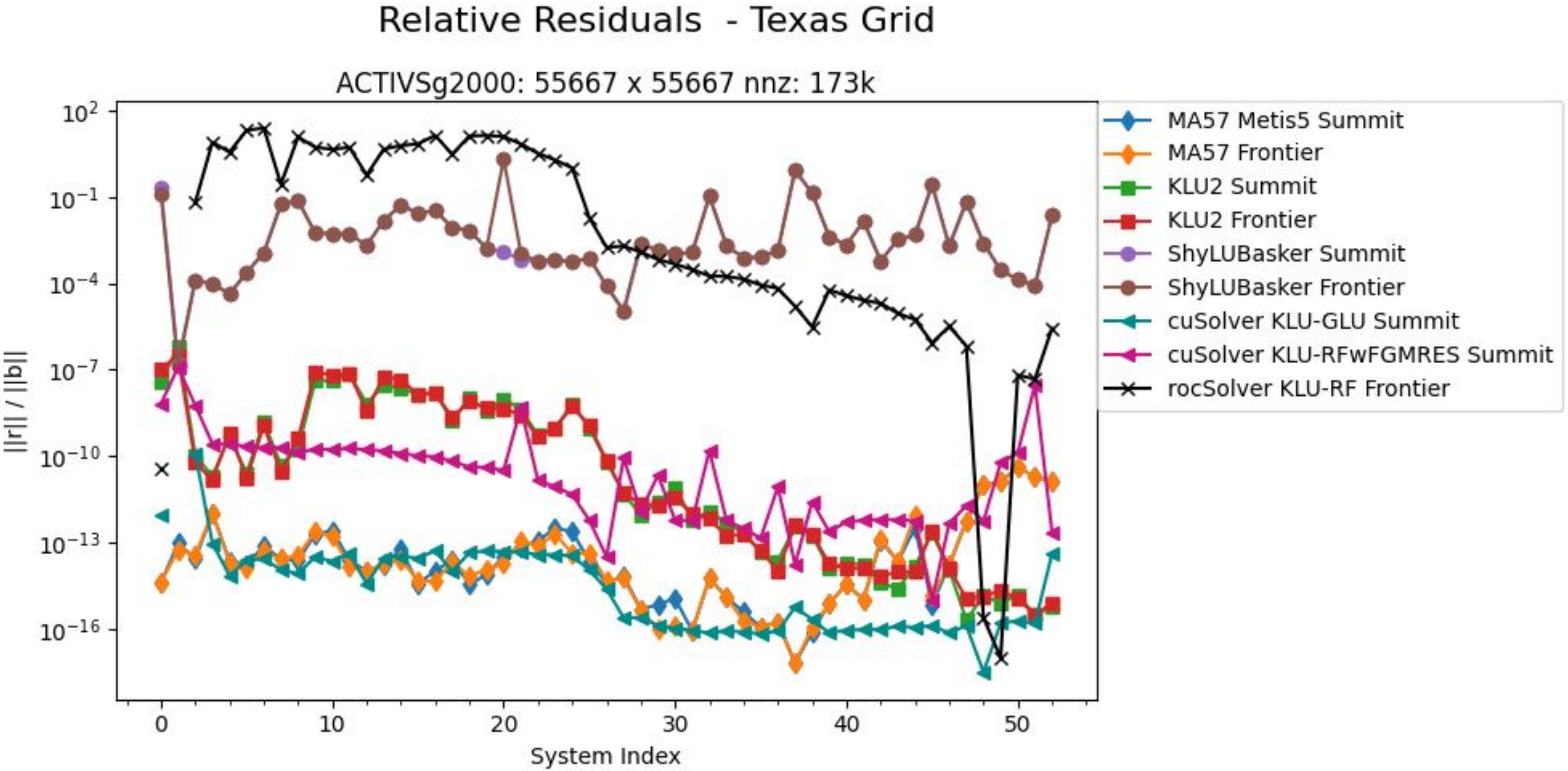


Figure 2. Relative residuals for the Texas (2000-bus) matrix market files. Data points for rocSolver and cuSolver KLU-GLU at index 2 were omitted where the solution failed to converge. ShyLUBasker and rocSolver have very poor accuracy, while MA57 and cuSolver KLU-GLU are the most stable for the Texas grid test case.

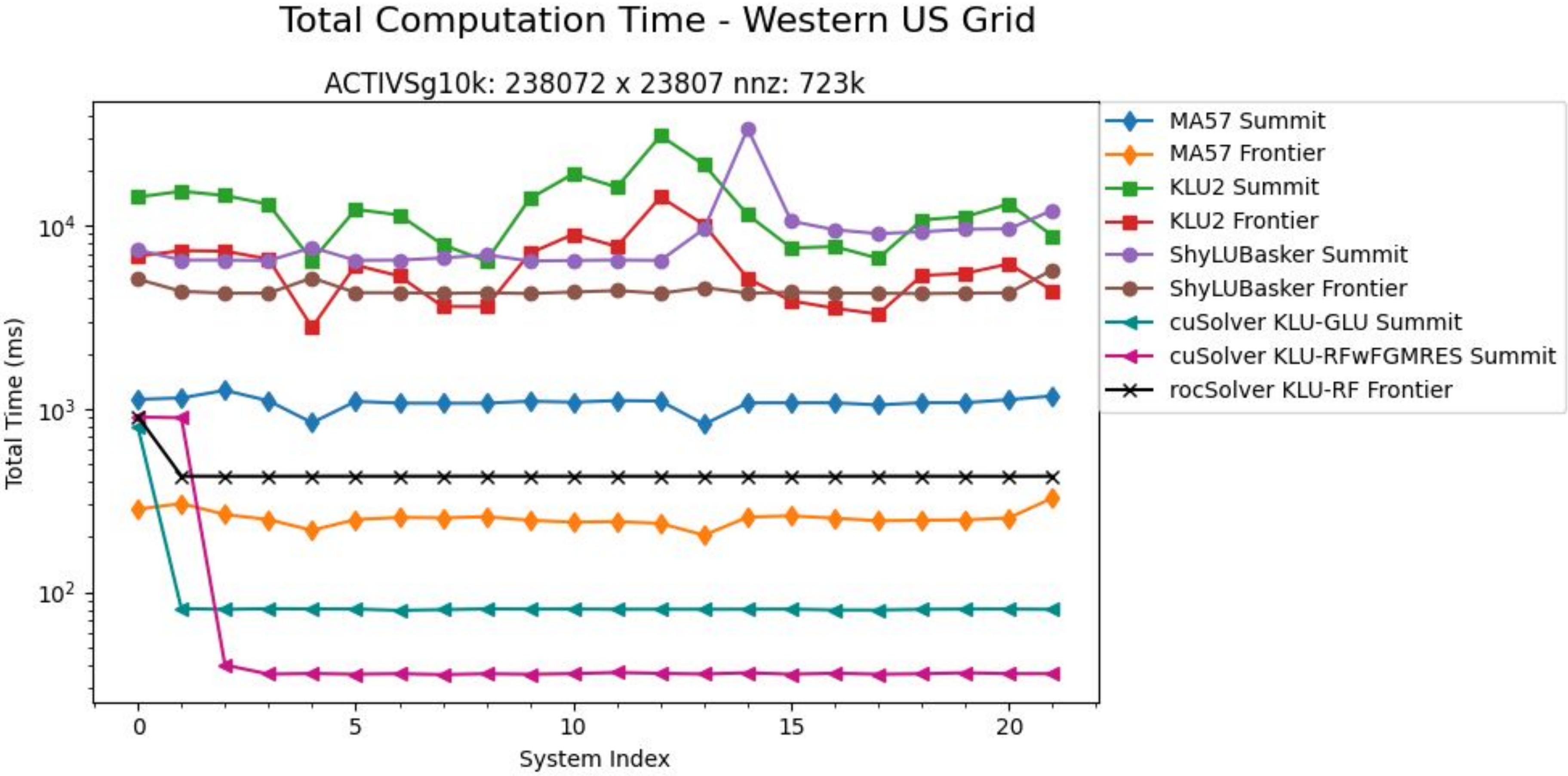


Figure 3. Total runtimes for the Western US (10k-bus) test cases. KLU2 and ShyLUBasker perform worse on Summit and slightly improve on Frontier. CUDA cuSolvers have the best runtimes that outperform the MA57 baseline.

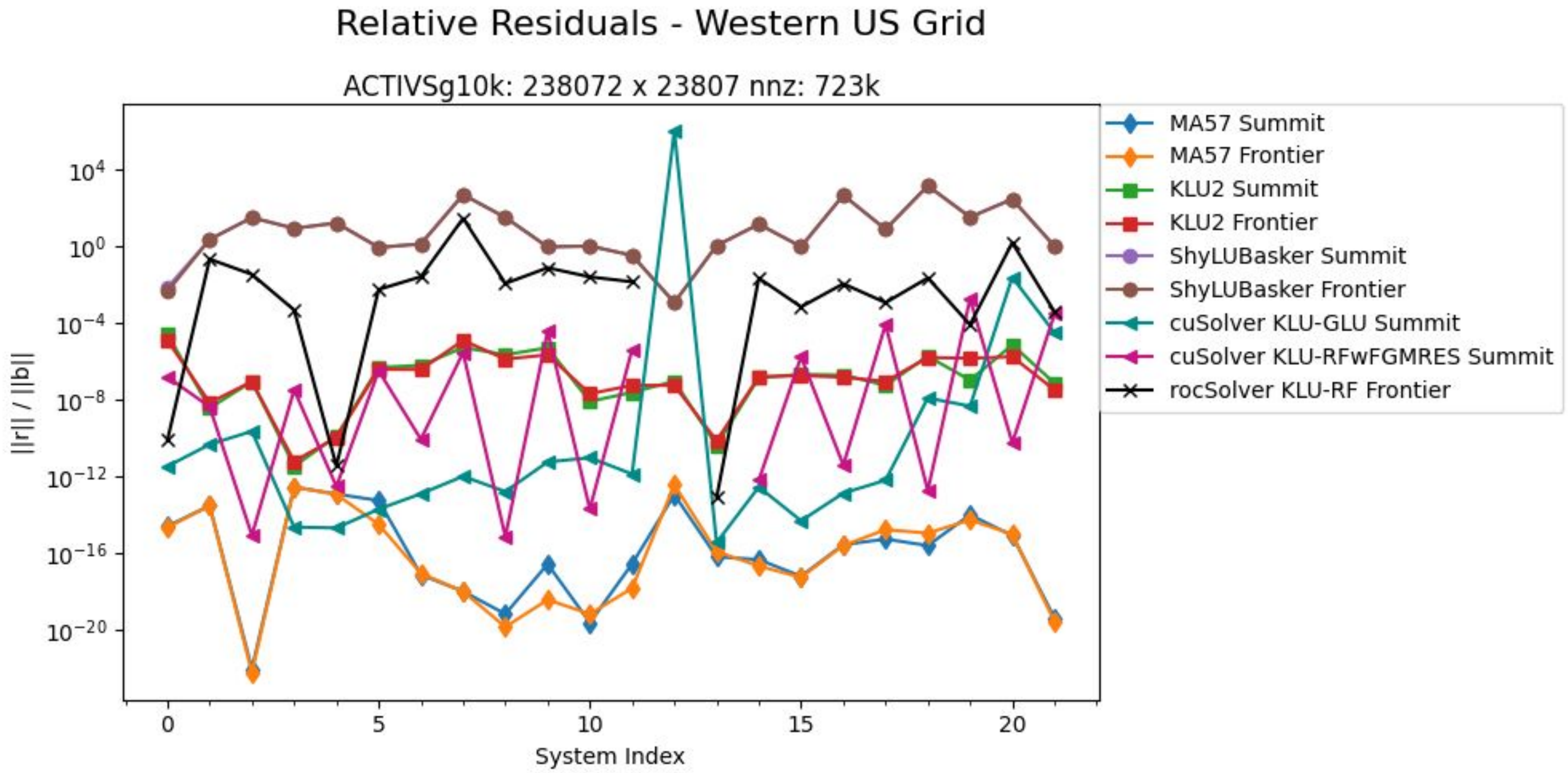


Figure 4. Relative residuals for the Western US (10k-bus) matrix market files. A data point for rocSolver at index 12 was omitted where the solution failed to converge. MA57 is the the most stable, while ShyLUBasker and rocSolver have very poor accuracy for the Western US grid.

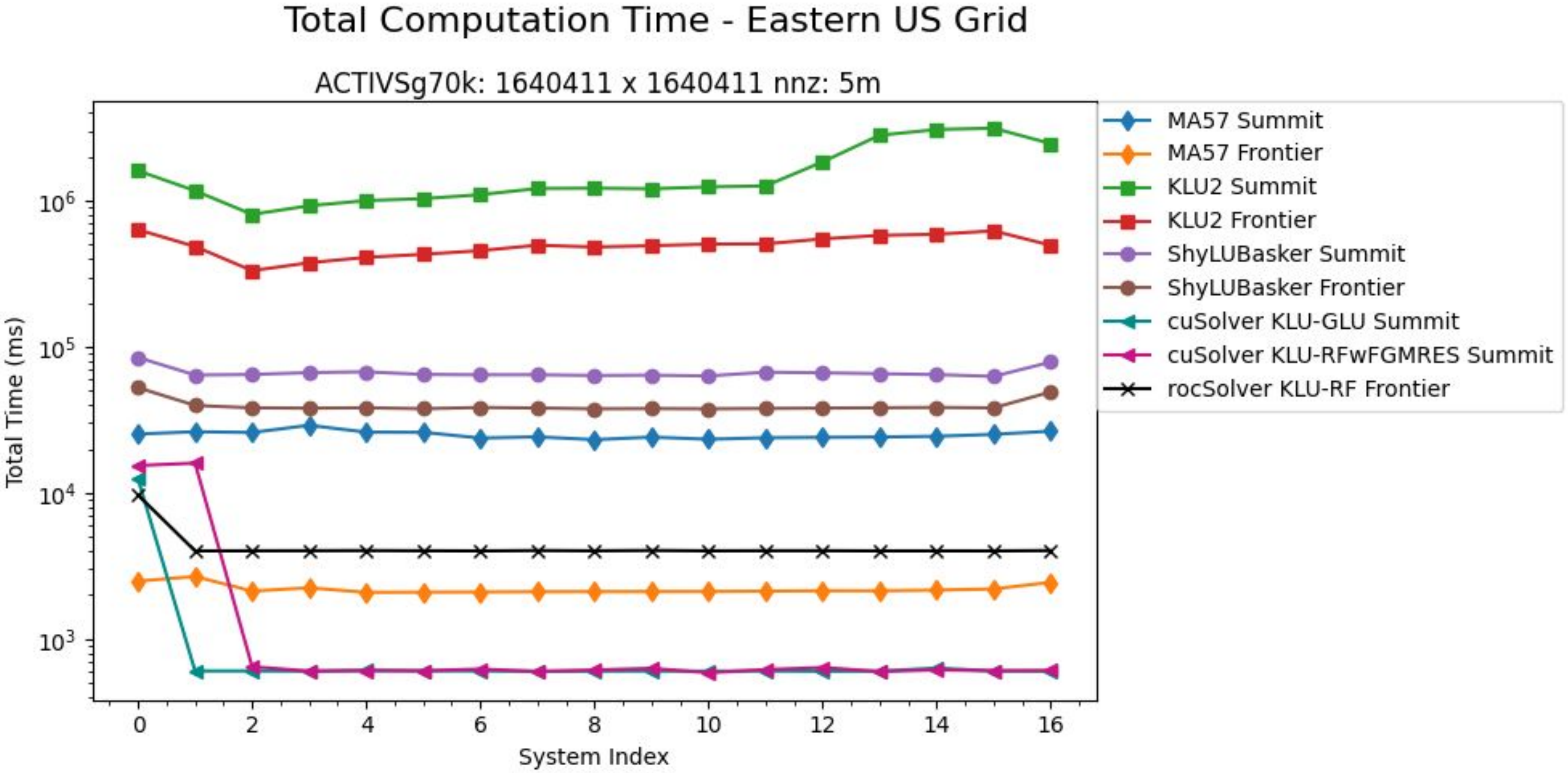


Figure 5. Total runtimes for the Eastern US (70k-bus) matrix market files. KLU2 performs significantly worse that all other solvers for the Eastern US test case and rocSolver performs better than MA57 on Summit.

Relative Residuals - Eastern US Grid

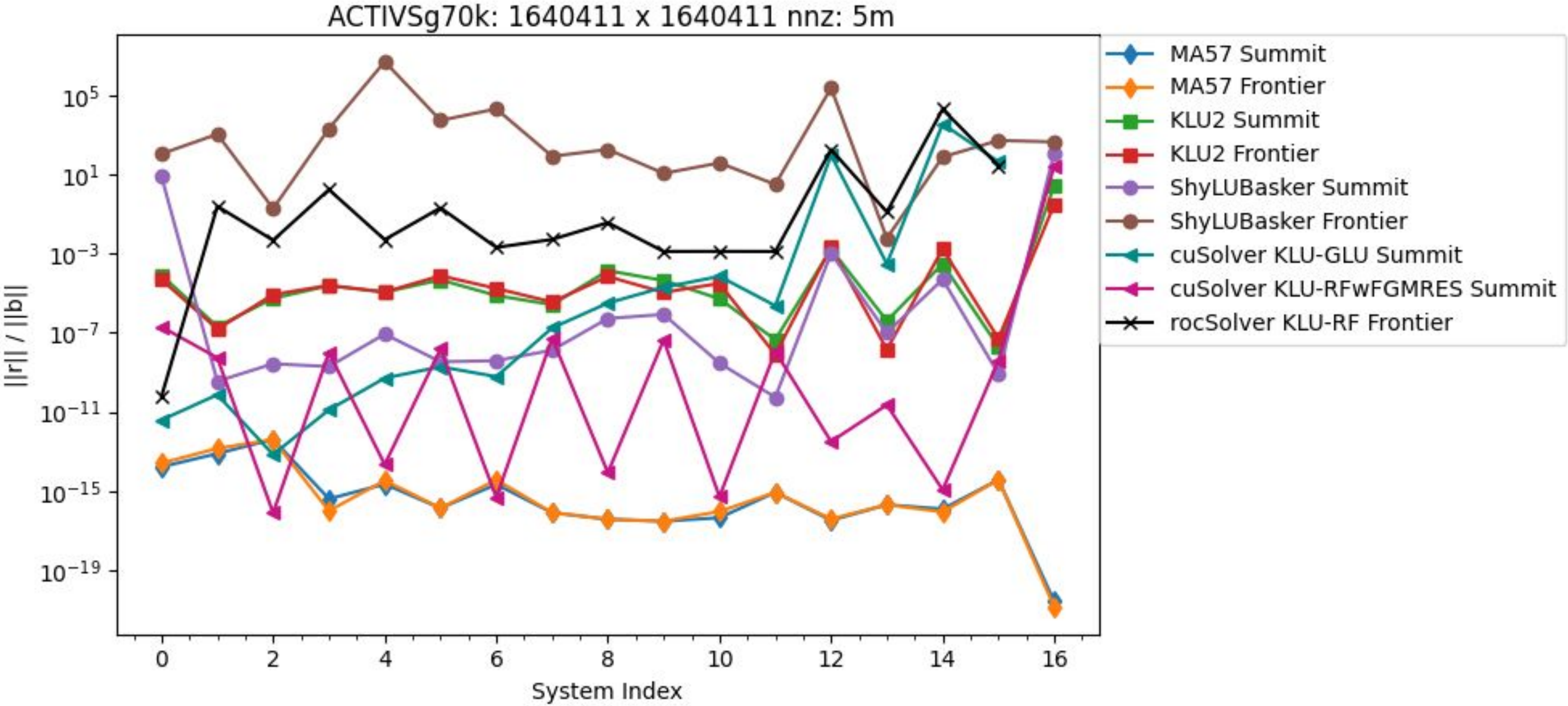


Figure 6. Relative residuals for the Eastern US (70k-bus) matrix market files. MA57 is the most stable, while ShyLUBasker and rocSolver is the least accurate overall for handling Eastern US test cases. Data points for rocSolver and cuSolver KLU-GLU at index 16 were omitted where the solution failed to converge.

Results and Discussion

- CUDA GPU linear solvers significantly outperform CPU based solvers for all test cases provided.
- RocSolver performed closely to the MA57 “gold standard” CPU benchmark, some stability issues observed.
- Amesos2 ShyLUBasker solver performance scales well with the number of threads, however, we observed serious stability issues for some test cases.
- Amesos2 KLU2 solver has limited parameters to adjust compared to vanilla KLU, e.g. it did not support COLAMD ordering.

This report has been authored in part by UT-Battelle, LLC, under contract DE-AC05-00OR22725 with the US Department of Energy (DOE). The US government retains and the publisher, by accepting the work for publication, acknowledges that the US government retains a non-exclusive, paid-up, irrevocable, world-wide license to publish or reproduce the submitted manuscript version of this work, or allow others to do so, for US government purposes. DOE will provide public access to these results of federally sponsored research in accordance with the DOE Public Access Plan (https://energy.gov/doe-public-access-plan).

Questions?

ACKNOWLEDGMENTS

A special thank you to ORNL mentor Slaven Peles and to Kasia Swirydowicz of PNNL for the help and guidance throughout the GRSI program. Thank you Nicholson for answering all of my srun jsrun, and profiling questions and Maksud for quick access to his machine and installing the cuDSS packages. Thank you to the Exascale Computing Project team for additional support and great teamwork.

References

Świrydowicz, K., Koukpaizan, N., Abhyankar, S., & Peleš, S. (2023). Towards efficient alternating current optimal power flow analysis on graphical processing units. 2023 XXIX International Conference on Information, Communication and Automation Technologies (ICAT). 10.1109/icat57854.2023.10171317

Świrydowicz, K., Darve, E., Jones, W., Maack, J., Regev, S., Saunders, M. A., Thomas, S. J., & Peleš, S. (2022). Linear solvers for power grid optimization problems: A review of GPU-accelerated linear solvers. Parallel Computing, 111, 102870. 10.1016/j.parco.2021.102870

Bavier, Eric & Hoemmen, Mark & Rajamanickam, Siva & Thornquist, Heidi. (2012). Amesos2 and Belos: Direct and Iterative Solvers for Large Sparse Linear Systems. Scientific Programming. 20.241-255. 10.1155/2012/243875.

Rajamanickam, Siva. KLU2 Direct Linear Solver Package. Computer software. https://www.osti.gov//servlets/purl/1253280. Vers. 00. USDOE. 4 Jan. 2012. Web.

KLU2 header