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Adaptive Finite-time Preassigned Performance Control for Hypersonic Vehicles with Lumped Perturbations

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Abstract— **This paper proposes an adaptive finite-time preassigned performance control (PPC) approach to solve the attitude control problem of hypersonic vehicles (HSVs) suffering from lumped disturbances. A specific preassigned performance function guaranteeing finite-time stability and small overshoot is presented to constrain the attitude tracking errors. Moreover, a novel synchronous disturbance estimation is presented to build a disturbance observer (DO). Based on the devised preassigned performance function and the information from the DO, an adaptive finite-time preassigned performance controller is devised to solve the attitude control problem. Finally, numerical examples demonstrate the validity of the presented algorithm.**

I. INTRODUCTION

As HSVs play a significant role in the military domain, more and more researchers focus on the attitude controller design [1]. Meanwhile, the attitude control for HSVs is more challenging than traditional aircraft due to their differences in flight envelope, aerodynamic shape, and kinematics characteristics [2].

Preassigned performance control can regulate the control system with both desired transient performance and steady-state performance [3]. The core idea of PPC is to construct a constraint envelope called performance function, which is applied to constrain the system states and achieve the desired prescribed performance [4]. In recent years, many kinds of performance functions have been devised, including performance functions independent from initial errors [5], readjusting performance functions [6], finite-time performance functions [7], and so on. In [8], an attitude angle control algorithm is proposed utilizing PPC, which describes the convergence rate and overshoot of the system state. A fault-tolerant control method is investigated and applied, achieving the prescribed performance by adaptive control law in [9]. However, the PPC method has not been applied to HSV systems due to the strong uncertainty, heavy coupling, and constraint features.

The existing attitude control algorithms for HSVs still have deficiencies. The perturbation caused by aerodynamic parameter uncertainties is one of the challenges to be solved, which makes the anti-disturbance control technique extremely important. The basic idea of disturbance observer is to estimate the uncertainties and disturbance by output signals and compensate for them. In [10], a DO-based attitude control algorithm is proposed, which estimates the unmeasured states and constrains the system states. The problem of adaptive control for HSVs is studied in [11], which introduces a high-order DO to estimate the disturbances and their derivatives. A disturbance rejection controller is designed to maintain a good tracking effect and satisfy the system performance requirements [12]. Based on the existing results, this paper aims to solve the disturbance and guarantee satisfactory control performance for HSV's attitude control system by combining the PPC method and DO technique.

This article is organized as follows. First, the attitude control system model of HSVs is introduced in Section II. Then, a preassigned performance controller is designed in Section III, including preassigned performance inner loop control and adaptive outer loop control. To illustrate the validity of the proposed method, the system stability is analyzed in Section IV, and the simulations are conducted in Section V. Finally, the summary of this paper is given in Section VI.

II. PROBLEM FORMULATION

Consider the simplified attitude system of HSV which can be described by

$$\begin{aligned} \dot{\sigma} &= A_{11}\sigma + A_{12}\omega + \Delta_{\sigma} \\ \dot{\omega} &= A_{21}\sigma + A_{22}\omega + Bu + f(\omega) + \Delta_{\omega 0} \end{aligned} \tag{1}$$

where $\sigma = [\alpha, \beta, \gamma_v]^T \in \mathbb{R}^3$ represents the state vector composed of three attitude angles. $\omega = [\omega_x, \omega_y, \omega_z]^T \in \mathbb{R}^3$ denotes the state vector consisting of the angular velocities. $u = [\delta_{\varphi}, \delta_{\psi}, \delta_{\gamma}]^T \in \mathbb{R}^3$ stands for the input vector consisting of the deflection angles. $A_{11}, A_{12}, A_{21}, A_{22}, B \in \mathbb{R}^{3\times 3}$ are structure matrices. $\Delta_{\sigma}, \Delta_{\omega 0} \in \mathbb{R}^{3\times 1}$ denotes the lumped disturbances caused by model simplification as well as aerodynamic torques. The specific expressions of the aforementioned matrices and vectors can be seen in [13].

Assumption 1: The attitude angles and angular velocities can be measured and calculated by the gyroscope and sensors equipped by the HSV.

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Assumption 2: The lumped disturbances and their derivatives are assumed to be bounded, i.e., $\|\Delta_\sigma\| \le \bar{\Delta}_\sigma$, $\left\|\dot{\Delta}_\sigma\right\| \le \bar{\dot{\Delta}}_\sigma$, $\|\Delta_{\omega 0}\| \le \bar{\Delta}_{\omega 0}$, $\left\|\dot{\Delta}_{\omega 0}\right\| \le \bar{\dot{\Delta}}_{\omega 0}$.

Lemma 1: Consider the following nonlinear system

$$\dot{x} = f(x) \tag{2}$$

with the initial state $f(0) = 0$, $x(0) = x_0$. If there exists a Lyapunov function $\mathcal{V}(x)$ satisfying

$$\dot{\mathcal{V}}(x) \le -k_1 \mathcal{V}(x) - k_2 \mathcal{V}^\delta(x) \tag{3}$$

with positive constants $k_1, k_2 > 0$ and $0 < \delta < 1$, the system is globally fast finite-time stable with the settling time

$$t_f \le \frac{1}{k_1(1-\delta)} \ln\left(\frac{k_1 \mathcal{V}^{1-\delta}(x_0) + k_2}{k_2}\right). \tag{4}$$

Lemma 2: For arbitrary positive constant ε and real number x, the following inequality holds

$$0 \le |x| - x \tanh\left(\frac{x}{\varepsilon}\right) \le \kappa\varepsilon \tag{5}$$

where $\kappa = 0.2785$.

Lemma 3: For an arbitrary symmetric matrix $\mathcal{Z} \in \mathbb{R}^{n\times n}$ and a vector $z \in \mathbb{R}^n$, the subsequent inequality holds

$$\lambda_{\min}(\mathcal{Z}) z^T z \le z^T \mathcal{Z} z \le \lambda_{\max}(\mathcal{Z}) z^T z \tag{6}$$

where $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ represent the minimum eigenvalue and maximum eigenvalue, separately.

III. Preassigned Performance Controller Design

A. Preassigned control performance and state transformation

Denote the reference command vector as $\sigma_d = [\alpha_d, \beta_d, \gamma_{vd}]^T$ where α_d, β_d, γ_{vd} are the desired trajectory of attitude angles. Then, the error dynamics can be formulated as

$$\begin{aligned} \dot{e}_\sigma &= A_{11}\sigma + A_{12}\omega + \Delta_\sigma - \dot{\sigma}_d \\ \dot{\omega} &= A_{21}\sigma + A_{22}\omega + Bu + f(\omega) + \Delta_{\omega 0} \end{aligned} \tag{7}$$

where $e_\sigma = \sigma - \sigma_d$ denotes the attitude angle error.

Aiming at the error dynamic system, a performance function is devised to regulate the state error to converge to the origin in finite time

$$p = \begin{cases} \varpi_0 \cdot e^{\kappa\left(1-\frac{t_f}{t_f - t}\right)} + P_{t_f}, & 0 \le t < t_f \\ P_{t_f}, & t \ge t_f \end{cases} \tag{8}$$

where $\varpi_0, \kappa, t_f, P_{t_f} > 0$ are control parameters. It can be obtained that

$$\dot{p} = \begin{cases} -\frac{\varpi_0 \kappa t_f e^{\kappa - \frac{\kappa t_f}{t_f - t}}}{(t_f - t)^2}, & 0 \le t < t_f \\ 0, & t \ge t_f \end{cases}. \tag{9}$$

Thus, for arbitrary $t \ge 0$, $\dot{p} \le 0$ holds. Moreover, although the performance function p is a piecewise function, $\lim\limits_{t\to t_f} p^{(n)}(t) = 0, n \in \mathbb{N}_+$. So p is a smooth function.

To meet the requirement of fast and precise control of HSV, the upper and lower boundaries of the preassigned performance are introduced below

$$\begin{aligned} & p_{l,k} < e_k < p_{u,k} \\ & p_{u,k} = (sign(e_k(0)) + \mu_u)\, p - P_{t_f} sign(e_k(0)) \\ & p_{l,k} = (sign(e_k(0)) - \mu_l)\, p - P_{t_f} sign(e_k(0)) \\ & k = \alpha, \beta, \gamma_v \end{aligned} \tag{10}$$

with parameters $\mu_u, \mu_l \in [0, 1]$. However, the additional performance constraints regulated by the performance function make it difficult to regulate the system states directly. As a result, a transformed error is proposed whose formulation is

$$z_k = \ln\left(\frac{\lambda_k}{1-\lambda_k}\right) \tag{11}$$

where $\lambda_k = (e_k - p_l)/(p_u - p_l)$. Note that $0 < \lambda_k < 1$ holds if $p_{l,k} < e_k < p_{u,k}$.

Proposition 1: On condition that z_k is bounded, the tracking error of the attitude angle e_k can be restricted to the prescribed limited area generated by the performance functions (8), (10) and converge to the preassigned neighborhood of the origin in t_f.

Proof 1: According to (11), we can get that

$$\lambda_k = \frac{e^{z_k}}{e^{z_k} + 1}. \tag{12}$$

Define the boundary of z_k as $\bar{z}_k \ge \|z_k\| > 0$. Furthermore, (12) can be expanded as

$$0 < \frac{e^{-\bar{z}_k}}{e^{-\bar{z}_k} + 1} \le \lambda_k \le \frac{e^{\bar{z}_k}}{e^{\bar{z}_k} + 1} < 1. \tag{13}$$

Substitute $\lambda_k = (e_k - p_{l,k})/(p_{u,k} - p_{l,k})$ into (13), we can get that

$$0 < \frac{e_k - P_l}{P_u - P_l} < 1. \tag{14}$$

Furthermore,

$$p_{l,k} < e_k < p_{u,k}. \tag{15}$$

Combining (10) and (15), when $t \ge t_f$, we have $p_{u,k} = \mu_u P_{t_f}$, $p_{l,k} = -\mu_l P_{t_f}$. We can finally get that

$$-\mu_l P_{t_f} < e_k < \mu_u P_{t_f},\ t \ge t_f. \tag{16}$$

The proof is completed.

B. DO-based preassigned performance inner loop control

Combining (11) and $\lambda_k = (e_k - P_{l,k})/(P_{u,k} - P_{l,k})$, the time derivative of $z(t)$ can be formulated as

$$\dot{z}_k = \Upsilon_k \cdot \left[\dot{e}_k + \frac{\dot{P}_{l,k} - \dot{P}_{u,k}}{P_{u,k} - P_{l,k}} \cdot e_k - \frac{P_{l,k}\left(\dot{P}_{l,k} - \dot{P}_{u,k}\right)}{P_{u,k} - P_{l,k}}\right] \tag{17}$$

where

$$\begin{aligned}
\Upsilon_k =& \frac{1}{\lambda_k \left(1-\lambda_k\right)\left(P_{u,k}-P_{l,k}\right)} \\
\dot{P}_{u,k} =& \left(sign\left(e_k\left(0\right)\right)+\mu_u\right)\dot{P}_k \\
\dot{P}_{l,k} =& \left(sign\left(e_k\left(0\right)\right)-\mu_l\right)\dot{P}_k \\
\dot{P}_k =& \left\{ \begin{array}{cc} -\frac{\varpi_0 \kappa t_f e^{\kappa-\frac{\kappa t_f}{t_f-t}}}{\left(t_f-t\right)^2} & 0 \le t < t_f \\ 0 & \mathrm{t} \ge t_f \end{array} \right. .
\end{aligned} \tag{18}$$

To extend the conclusion of Proposition 1, the subsequent definitions are proposed:

$$\begin{aligned}
z =& \left[z_\alpha, z_\beta, z_{\gamma_v}\right]^T \\
\Upsilon =& diag\left\{\Upsilon_\alpha, \Upsilon_\beta, \Upsilon_{\gamma_v}\right\} \\
\mathrm{K}_1 =& diag\left\{\frac{\dot{P}_{l,\alpha}-\dot{P}_{u,\alpha}}{P_{u,\alpha}-P_{l,\alpha}}, \frac{\dot{P}_{l,\beta}-\dot{P}_{u,\beta}}{P_{u,\beta}-P_{l,\beta}}, \frac{\dot{P}_{l,\gamma_v}-\dot{P}_{u,\gamma_v}}{P_{u,\gamma_v}-P_{l,\gamma_v}}\right\} \\
\mathrm{K}_2 =& \left[\frac{P_{l,\alpha}\left(\dot{P}_{l,\alpha}-\dot{P}_{u,\alpha}\right)}{P_{u,\alpha}-P_{l,\alpha}}, \frac{P_{l,\beta}\left(\dot{P}_{l,\beta}-\dot{P}_{u,\beta}\right)}{P_{u,\beta}-P_{l,\beta}}, \right. \\
& \left. \frac{P_{l,\gamma_v}\left(\dot{P}_{l,\gamma_v}-\dot{P}_{u,\gamma_v}\right)}{P_{u,\gamma_v}-P_{l,\gamma_v}}\right]^T .
\end{aligned} \tag{19}$$

Then, equation (17) can be extended to

$$\dot{z} = \Upsilon \cdot \left[\dot{e}_\sigma + \mathrm{K}_1 \cdot e_\sigma - \mathrm{K}_2\right]. \tag{20}$$

Substitute the first equation of (7) into (20), we can get that

$$\dot{z} = \Upsilon \cdot \left[A_{11}\sigma + A_{12}\omega + \Delta_\sigma - \dot{\sigma}_d + \mathrm{K}_1 \cdot e_\sigma - \mathrm{K}_2\right]. \tag{21}$$

Thus, the virtual control law is devised as

$$\begin{aligned}
\omega_c =& A_{12}^{-1}\Big[-k_z z - k_{z0}\int_0^t z\left(\tau\right)d\tau - A_{11}\sigma \\
& - \hat{\Delta}_\sigma + \dot{\sigma}_d - \mathrm{K}_1 \cdot e_\sigma + \mathrm{K}_2\Big]
\end{aligned} \tag{22}$$

where $k_z, k_{z0} > 0$ are the control gains. $\hat{\Delta}_\sigma$ represents the output of the observation of the disturbance observer, which has the following formulation

$$\begin{aligned}
\hat{\Delta}_\sigma =& q + k_{\Delta_\sigma} e_\sigma \\
\dot{q} =& -k_{\Delta_\sigma}\hat{\Delta}_\sigma - k_{\Delta_\sigma}\left(A_{11}\sigma + A_{12}\omega - \dot{\sigma}_d\right).
\end{aligned} \tag{23}$$

where $k_{\Delta_\sigma} > 0$ is the observer gain. Define $\tilde{\Delta}_\sigma = \hat{\Delta}_\sigma - \Delta_\sigma$, we can get that

$$\dot{\tilde{\Delta}}_\sigma = -k_{\Delta_\sigma}\tilde{\Delta}_\sigma - \dot{\Delta}_\sigma. \tag{24}$$

Substituting the virtual control law (22) into the transformed inner loop system results in the closed-loop system

$$\dot{z} = \Upsilon \cdot \left[-k_z z - k_{z0}\int_0^t z\left(\tau\right)d\tau - \tilde{\Delta}_\sigma\right]. \tag{25}$$

Based on the definition of λ_k and the expression of Υ_k in (18), we can know that $\lambda_k \in (-1, 1)$ and $\Upsilon_k > 0$. Therefore, it can be inferred that Υ is an invertible matrix with positive eigenvalues. The following Lyapunov function candidate is selected

$$V_1 = \frac{1}{2}z^T\Upsilon^{-1}z + \frac{1}{2}\left(\int_0^t z\left(\tau\right)d\tau\right)^T \Upsilon^{-1}\int_0^t z\left(\tau\right)d\tau. \tag{26}$$

Utilizing (22), the time derivative of V_1 is taken as

$$\begin{aligned}
\dot{V}_1 =& -k_z z^T z - k_{z0} z^T\int_0^t z\left(\tau\right)d\tau \\
& - z^T\tilde{\Delta}_\sigma + \left[\int_0^t z\left(\tau\right)d\tau\right]^T \Upsilon^{-1}z.
\end{aligned} \tag{27}$$

Taking use of Young's inequality, we can obtain that

$$-z^T\tilde{\Delta}_\sigma \le \frac{1}{2}z^T z + \frac{1}{2}\tilde{\Delta}_\sigma^T\tilde{\Delta}_\sigma. \tag{28}$$

Ultimately, we can get that

$$\begin{aligned}
\dot{V}_1 \le& \left(-k_z + \frac{1}{2}\right)z^T z + \frac{1}{2}\tilde{\Delta}_\sigma^T\tilde{\Delta}_\sigma \\
& - k_{z0}z^T\int_0^t z\left(\tau\right)d\tau + \left[\int_0^t z\left(\tau\right)d\tau\right]^T \Upsilon^{-1}z.
\end{aligned} \tag{29}$$

Define $z_0 = \int_0^t z\left(\tau\right)d\tau$, $Z = \left[z_0, z\right]^T$. (36) can be transformed as

$$\dot{V}_1 \le -Z^T P Z + \frac{1}{2}\tilde{\Delta}_\sigma^T\tilde{\Delta}_\sigma \tag{30}$$

where

$$P = \begin{bmatrix} 0_3 & -\Upsilon^{-1} \\ k_{z0}I_3 & \left(k_z - \frac{1}{2}\right)I_3 \end{bmatrix}. \tag{31}$$

C. Adaptive outer loop control

Define the tracking error of the outer loop as $e_\omega = \omega - \omega_c$. Then the error dynamic of the outer loop can be formulated as

$$\dot{e}_\omega = A_{21}\sigma + A_{22}\omega + Bu + f\left(\omega\right) + \Delta_{\omega 0} - \dot{\omega}_c. \tag{32}$$

Aiming at the transformed outer loop system (32), the controller u is devised as

$$\begin{aligned}
u =& B^{-1}\Big[-k_2 e_\omega - A_{21}\sigma - A_{22}\omega \\
& - f\left(\omega\right) - \hat{\bar{\Delta}}_\omega \mathrm{Tanh}\left(\frac{e_\omega}{\varepsilon_\omega}\right) + \dot{\omega}_d\Big]
\end{aligned} \tag{33}$$

where k_2 is a positive constant. ω_d can be obtained by a first-order filter with the following formulation

$$c\dot{\omega}_d = -\omega_d + \omega_c, \ \omega_d\left(0\right) = \omega_c\left(0\right) \tag{34}$$

where c is a given positive scalar. The filter error is defined as $e_3 = \omega_d - \omega_c$ and its derivative is evaluated as

$$\dot{e}_3 = -\frac{e_3}{c} - \dot{\omega}_c \tag{35}$$

$\hat{\bar{\Delta}}_\omega$ represents the devised adaptive parameter to evaluate the amplitude of the outer loop lumped disturbance $\Delta_{\omega 0}$. The adaptive law is given by

$$\dot{\hat{\bar{\Delta}}}_\omega = \eta_{\Delta_\omega} e_\omega^T \mathrm{Tanh}\left(\frac{e_\omega}{\varepsilon_\omega}\right) - \eta_{\Delta_\omega}\sigma_{\Delta_\omega}\hat{\bar{\Delta}}_\omega \tag{36}$$

where $\eta_{\Delta_\omega}, \sigma_{\Delta_\omega} > 0$ are the gain of the adaptive law and the parameter of σ-modification, separately. Define the estimation error as $\tilde{\bar{\Delta}}_\omega = \hat{\bar{\Delta}}_\omega - \bar{\Delta}_\omega$.

Substitute (33), (36) into (32), the closed-loop system equation is obtained

$$\dot{e}_\omega = -k_2 e_\omega - \hat{\bar{\Delta}}_\omega \mathrm{Tanh}\left(\frac{e_\omega}{\varepsilon_\omega}\right) + \Delta_{\omega 0} + \dot{e}_3. \tag{37}$$

The following Lyapunov function is selected

$$V_2 = \frac{1}{2}e_\omega^T e_\omega + \frac{1}{2}e_3^T e_3 + \frac{1}{2\eta_{\Delta_\omega}}\tilde{\bar{\Delta}}_\omega^T\tilde{\bar{\Delta}}_\omega. \tag{38}$$

Taking the time derivative of (38) and substituting (35), (36), (37) into it yields

$$\begin{aligned}\dot{V}_2 =& -k_2 e_\omega^T e_\omega - \hat{\bar{\Delta}}_\omega e_\omega^T \mathrm{Tanh}\left(\frac{e_\omega}{\varepsilon_\omega}\right) + e_\omega^T\Delta_{\omega 0}\\ &- \frac{e_\omega^T e_3}{c} - e_\omega^T\dot{\omega}_c - \frac{e_3^T e_3}{c} - e_3^T\dot{\omega}_c\\ &+ \tilde{\bar{\Delta}}_\omega e_\omega^T \mathrm{Tanh}\left(\frac{e_\omega}{\varepsilon_\omega}\right) - \sigma_{\Delta_\omega}\tilde{\bar{\Delta}}_\omega\hat{\bar{\Delta}}_\omega.\end{aligned} \tag{39}$$

According to Lemma 2, it can be checked that

$$e_\omega^T\Delta_{\omega 0} \le \bar{\Delta}_\omega\left[e_\omega^T \mathrm{Tanh}\left(\frac{e_\omega}{\varepsilon_\omega}\right) + 3\kappa\varepsilon_\omega\right]. \tag{40}$$

Utilizing Young's inequality, we have

$$\begin{aligned}-\frac{1}{c}e_\omega^T e_3 \le& \frac{1}{c}e_\omega^T e_\omega + \frac{1}{4c}e_3^T e_3\\ -e_\omega^T\dot{\omega}_c \le& e_\omega^T e_\omega + \frac{1}{4}\|\dot{\omega}_c\|^2\\ -e_3^T\dot{\omega}_c \le& e_3^T e_3 + \frac{1}{4}\|\dot{\omega}_c\|^2.\end{aligned} \tag{41}$$

Combining the definition of $\tilde{\Delta}_\omega$, it can be obtained that

$$-\sigma_{\Delta_\omega}\tilde{\bar{\Delta}}_\omega\hat{\bar{\Delta}}_\omega \le -\frac{1}{2}\sigma_{\Delta_\omega}\tilde{\bar{\Delta}}_\omega^2 + \frac{1}{2}\sigma_{\Delta_\omega}\bar{\Delta}_\omega^2. \tag{42}$$

Substituting (40), (41) and (42) into (39) yields that

$$\begin{aligned}\dot{V}_2 \le& \left(-k_2 + 1 + \frac{1}{c}\right)e_\omega^T e_\omega + \left(-\frac{3}{4c} + 1\right)e_3^T e_3\\ &+ \frac{1}{2}\|\dot{\omega}_c\|^2 - \frac{1}{2}\sigma_{\Delta_\omega}\tilde{\bar{\Delta}}_\omega^2 + \frac{1}{2}\sigma_{\Delta_\omega}\bar{\Delta}_\omega^2 + 3\kappa\varepsilon_\omega\bar{\Delta}_\omega.\end{aligned} \tag{43}$$

The block of the devised control scheme is provided in Figure 1. Aiming at the tracking error of the attitude angle e_σ, the preassigned control performance with appointed convergence time is devised. Moreover, the DO receives the angle and angular velocity signals to evaluate the lumped disturbance Δ_σ. Then, the preassigned performance inner loop controller can be given and the desired angular velocity signals are obtained. Based on the tracking error of the angular velocity signals e_ω, the adaptive parameter $\hat{\bar{\Delta}}_\omega$ as

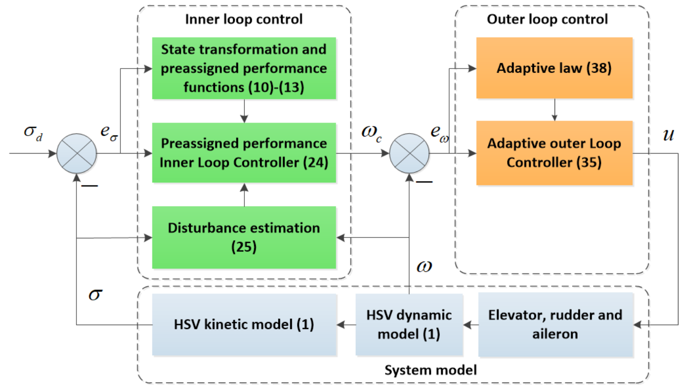

Fig. 1. Main framework of the proposed controller

well as its updating law are devised to approximate and suppress the lumped disturbance $\Delta_{\omega 0}$, which is essential for the adaptive outer loop controller. As a result, the kinetic and dynamic systems of HSV are regulated by the actuators and the system states σ, ω are applied to the feedback controller.

IV. Stability analysis

Theorem 1: Consider HSV's attitude system (1) with Assumption 1-Assumption 2 satisfied. Design the controller (22), (33) employing the DO (23) and the adaptive law (36). Then, all the system signals are bounded and the system can achieve the preassigned performance.

Proof 2: From (26) and (38), we can pick up the following Lyapunov function

$$\begin{aligned}V =& V_1 + V_2\\ =& \frac{1}{2}Z^T diag\left\{\Upsilon^{-1}, \Upsilon^{-1}\right\}Z\\ &+ \frac{1}{2}e_\omega^T e_\omega + \frac{1}{2}e_3^T e_3 + \frac{1}{2\eta_{\Delta_\omega}}\tilde{\bar{\Delta}}_\omega^2.\end{aligned} \tag{44}$$

Along with (30) and (43), we have

$$\begin{aligned}\dot{V} \le& -Z^T P Z + \frac{1}{2}\tilde{\Delta}_\sigma^T\tilde{\Delta}_\sigma + \left(-k_2 + 1 + \frac{1}{c}\right)e_\omega^T e_\omega\\ &+ \left(-\frac{3}{4c} + 1\right)e_3^T e_3 + \frac{1}{2}\|\dot{\omega}_c\|^2 - \frac{1}{2}\sigma_{\Delta_\omega}\tilde{\bar{\Delta}}_\omega^2\\ &+ \frac{1}{2}\sigma_{\Delta_\omega}\bar{\Delta}_\omega^2 + 3\kappa\varepsilon_\omega\bar{\Delta}_\omega.\end{aligned} \tag{45}$$

Taking into consideration that Υ is a diagonal matrix with positive eigenvalues, the following inequality holds

$$\begin{aligned}-Z^T P Z \le& -\lambda_{\min}(P)Z^T Z\\ \le& -\frac{\lambda_{\min}(P)Z^T diag\left\{\Upsilon^{-1}, \Upsilon^{-1}\right\}Z}{\lambda_{\max}\left(diag\left\{\Upsilon^{-1}, \Upsilon^{-1}\right\}\right)}\\ =& -\lambda_{\min}(P)\lambda_{\max}(\Upsilon)V_1.\end{aligned} \tag{46}$$

To facilitate the subsequent analysis, the following parame-

TABLE I

THE CONTROLLER PARAMETERS.

The control gains	$k_{z0}=2$, $k_z=5$, $k_2=8$
The preassigned performance parameters	$\varpi_0=2$, $\kappa=1$, $t_f=2$ $P_{t_f}=0.7$, $\mu_u=0.3$, $\mu_l=0.7$
The adaptive parameters	$\eta_{\Delta_\omega}=5$, $\sigma_{\Delta_\omega}=0.1$
The DO parameter	$k_d=3$

ters are introduced

$$\begin{aligned}\lambda_1 =& \min\left\{2\cdot\lambda_{\min}\left(P\right)\cdot\lambda_{\max}\left(\Upsilon\right), 2\left(k_2-1-\frac{1}{c}\right)\right.\\ &\left., 2\left(-\frac{3}{4c}+1\right), \eta_{\Delta_\omega}\sigma_{\Delta_\omega}\right\} \\ \lambda_2 =& \frac{1}{2}\tilde{\bar{\Delta}}_\sigma^2+\frac{1}{2}\left\|\dot{\omega}_c\right\|^2+\frac{1}{2}\sigma_{\Delta_\omega}\bar{\Delta}_\omega^2+3\kappa\varepsilon_\omega\bar{\Delta}_\omega.\end{aligned} \tag{47}$$

Then with the aid of (46) and (47), (45) can be transformed as

$$\dot{V}\le-\lambda_1 V+\lambda_2. \tag{48}$$

It is worth noting that the control parameters k_{z0}, k_z, k_2, c, η_{Δ_ω}, σ_{Δ_ω} should be appropriately selected so that $\lambda_1>0$ holds. Furthermore, the boundness of λ_2 can be guaranteed. Integrating both sides of (48), we have

$$\begin{aligned}V=&\left(V\left(0\right)-\frac{\lambda_2}{\lambda_1}\right)e^{-\lambda_1 t}+\frac{\lambda_2}{\lambda_1}\\ \le& V\left(0\right)e^{-\lambda_1 t}+\frac{\lambda_2}{\lambda_1}.\end{aligned} \tag{49}$$

It can be derived from (49) that V will be eventually restricted by λ_2/λ_1 as $t\to\infty$. Then, the boundness of $z\left(t\right)$, $z_0\left(t\right)$, e_ω, e_3, $\tilde{\Delta}_\omega$ can be achieved. From the definition of $\tilde{\Delta}_\omega=\hat{\Delta}_\omega-\Delta_\omega$, we can tell that $\hat{\Delta}_\omega$ is also bounded. Ultimately, based on the boundness of $z\left(t\right)$ and Proposition 1, we can infer that the tracking error of the attitude angles $e_\sigma\left(t\right)$ can be limited to the prescribed area generated by the performance functions (10), (12) and converge in the settling time $T=\max\left(t_d,t_f\right)$. This is the end of the proof.

V. NUMERICAL SIMULATIONS

Simulations are conducted to validate the practicability of the presented control algorithm. The command signals of the HSV attitude system are set as $\alpha_d=2+3\sin\left(0.2\pi t\right)\deg$, $\beta_d=-1-3\sin\left(0.1\pi t\right)\deg$, $\gamma_{vd}=0\deg$. The initial states of the attitude angles are $\alpha\left(0\right)=-1\deg$, $\beta\left(0\right)=-1\deg$, $\gamma_v\left(0\right)=1\deg$ and the initial angular velocities are zero. The control parameters are provided in Table I.

The adaptive finite-time preassigned performance control approach is implemented with the fixed step $0.01s$ and the total simulation time $15s$. We simulate two working conditions including: 1) the ideal case without the lumped disturbances Δ_σ, Δ_ω and 2) the general case with time-varying disturbances $\Delta_\sigma\left(t\right)$, $\Delta_\omega\left(t\right)$.

First, the simulation is conducted under the ideal case, whose results are demonstrated in Figure 2-Figure 3. Figure 2 demonstrates the attitude angle curves and their reference signals, from which we can see that the proposed approach

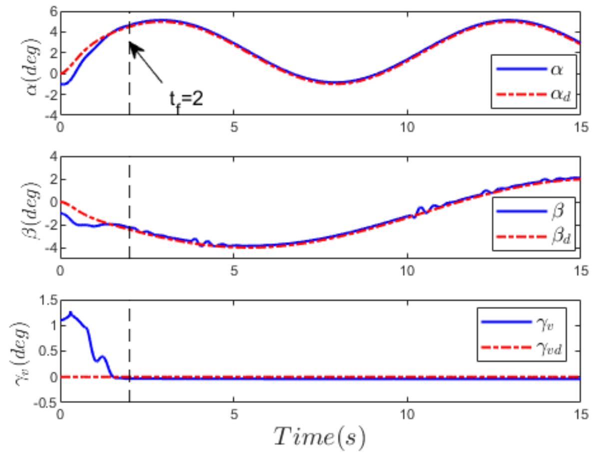

Fig. 2. Attitude angle response

achieves the objective of attitude tracking control with a steady state error of less than 0.1. The tracking errors can be seen explicitly in Figure 3, which converge within the preassigned time t_f.

Now that the stability of the proposed method has been verified, the time-varying lumped disturbances with the following formulation are introduced to test the robustness of the controlled system

$$\Delta_\sigma=\begin{bmatrix}0.05\cos\left(1.5t\right)e^{-0.02t}\\ 0.2\sin\left(10t\right)e^{-0.05t}\\ -0.1\cos\left(2t\right)e^{-0.01t}\end{bmatrix}$$

$$\Delta_\omega=\begin{bmatrix}0.5\sin\left(0.5t\right)e^{-0.1t}\\ 0.1\cos\left(1.5t\right)e^{-0.5t}\\ -0.01\cos\left(2.5t\right)e^{-0.05t}\end{bmatrix}.$$

The simulation results can be seen in Figure 4-Figure 6. Figure 4 demonstrates the tracking errors affected by dumped disturbances, from which we can tell that despite the oscillation trends caused by the lumped disturbances, the tracking errors are strictly restricted within the preassigned boundary. Figure 5 shows the angular velocity generated by the control signal. Figure 6 exhibits the estimation error of the DO. The favorable estimation capability of the disturbance observer as well as the robustness of the control system is demonstrated.

VI. CONCLUSION

An adaptive finite-time preassigned performance control strategy has been developed for HSVs suffering from lumped disturbances. To achieve finite-time convergence and a small overshoot of the system state, a specific performance function is devised. Furthermore, a DO as well as an adaptive law are employed to estimate the disturbances and compensate them. The convergence of both the system states and the estimation error of the observer is achieved in the preset finite time. Simulation results show the remarkable effectiveness and robustness of the proposed algorithm.

In the future, the research topics would be extended to the attitude control problem for HSVs subject to more complex

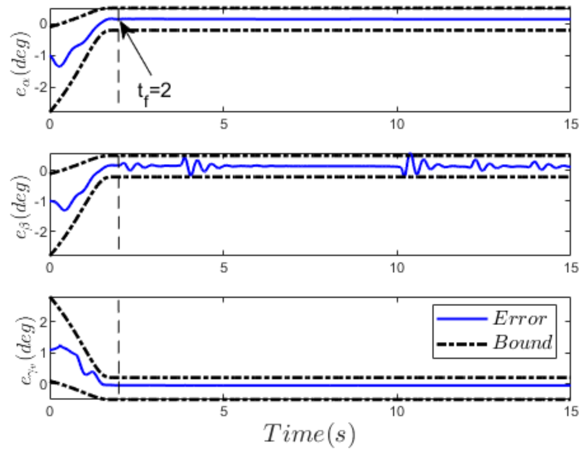

Fig. 3. Tracking error without disturbances

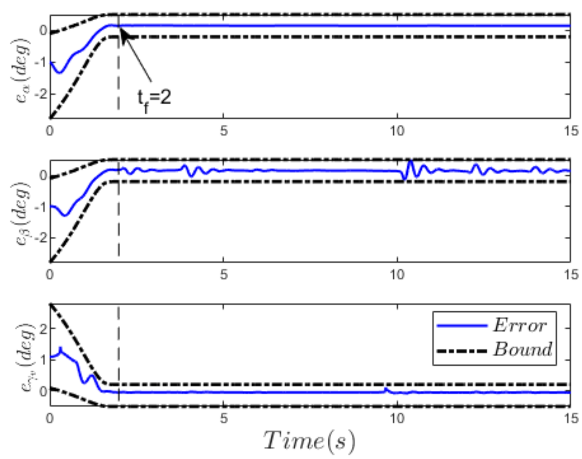

Fig. 4. Tracking error with disturbances

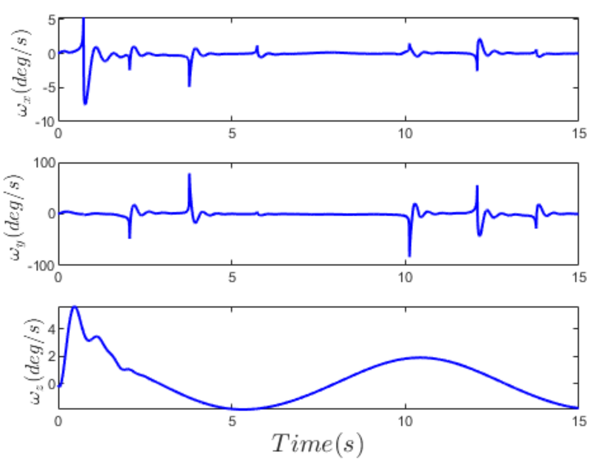

Fig. 5. Angular velocity response

disturbances with specific formulations, for example, measurement error and actuator fault, which brings predictable challenges to the controller design.

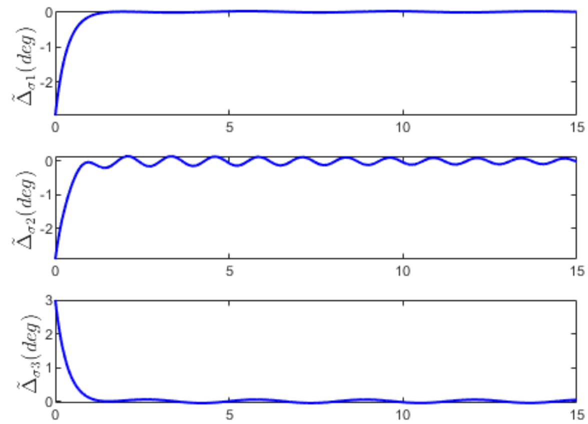

Fig. 6. Disturbance estimation error

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