VE281

Data Structures and Algorithms

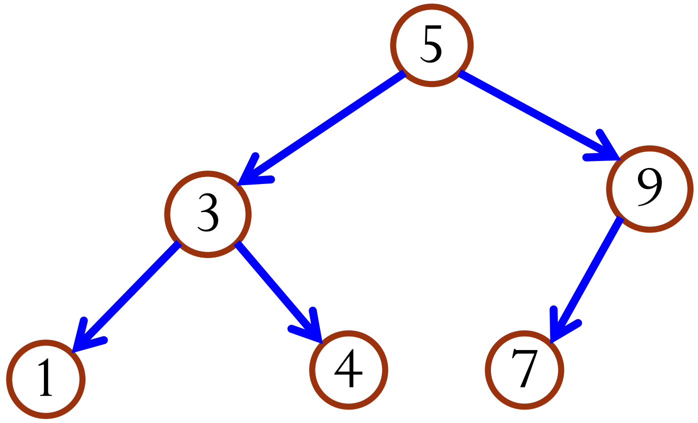
Binary Search Trees

Binary Search Tree

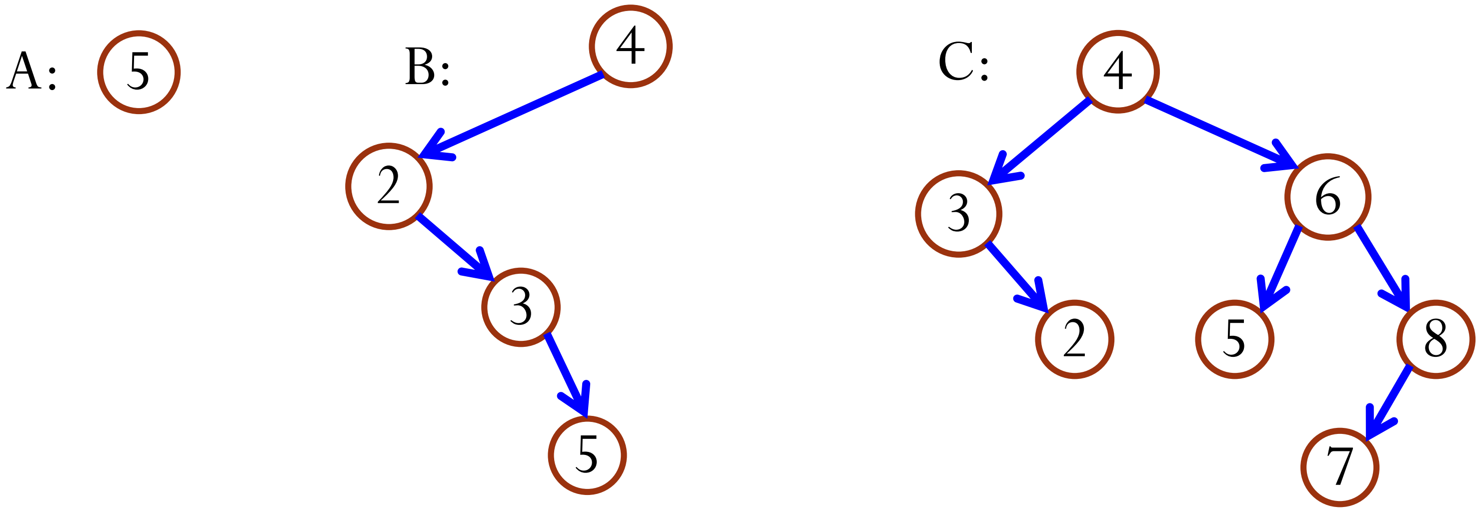
- A **binary search tree (BST)** is a binary tree with the following properties:
  - Each node is associated with a **key**.
    - A key is a value that can be compared.
    - **Assume**: all the keys are **distinct**.
  - The key of **any** node is greater than the keys of all nodes in its left subtree and smaller than the keys of all nodes in its right tree.

Binary Search Tree

Example



Exercise: which of the following trees are BST?



Basic Binary Search Tree Operations

- A BST allows search, insertion, and removal by key.
  - The **average case** time complexities for these operations are $O(\log n)$.
  - Average over all possible BSTs.

Binary Search Tree
Search

```
node *search(node *root, Key k)
// EFFECTS: return the node whose key is k.
// If no matching node, return NULL.
```

- Procedure: Compare the search key with the key of the root
  - If they are equal, return the root.
  - If search key < root key, search the left subtree.
  - If search key > root key, search the right subtree.
  - Recursively applying the above procedure.

Binary Search Tree

Search

```
struct node {
  Item item;
  node *left;
  node *right;
};
```

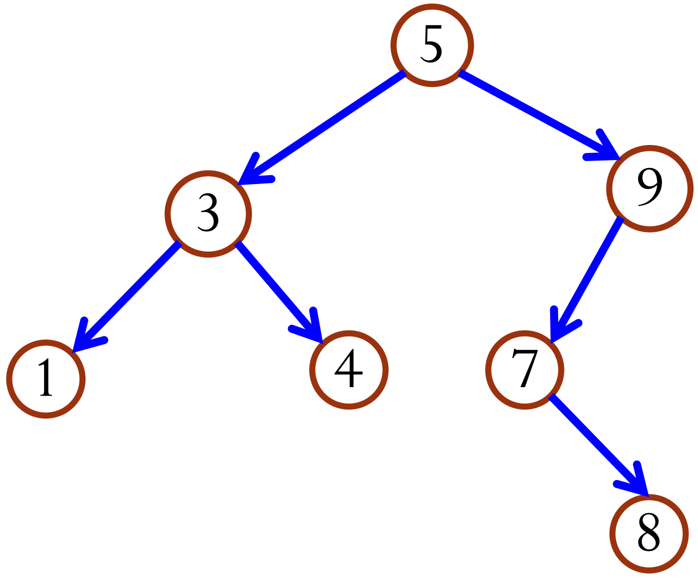
```
struct Item {
  Key key;
  Val val;
};
```

```
node *search(node *root, Key k) {
  if(root == NULL) return NULL;
  if(k == root->item.key) return root;
  if(k < root->item.key)
    return search(root->left, k);
  else return search(root->right, k);
}
```

Binary Search Tree

Insertion

- Insertion inserts the item **as a leaf** of the BST.
- It inserts at a proper location in the BST, maintaining the BST properties.
  - **Pretend** we are searching the key.



Insert a node with key = 8

Binary Search Tree

Insertion

```
void insert(node *&root, Item item)
// EFFECTS: insert the item as a leaf,
// maintaining the BST property.
{
  if(root == NULL) {
    root = new node(item);
    return;
  }
  if(item.key < root->item.key)
    insert(root->left, item);
  else if(item.key > root->item.key)
    insert(root->right, item);
}
```

Question: why define root as the reference-to-pointer?

Question: what happens if the key is already in the BST?

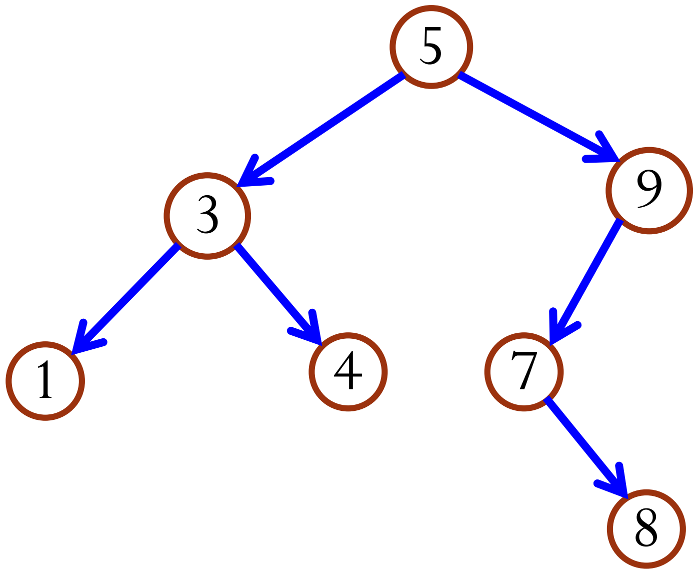
Binary Search Tree

Removal

reference to pointer

```
void remove(node *&root, Key k) {
  if(root == NULL) return;
  if(k < root->item.key) remove(root->left, k);
  else if(k > root->item.key)
    remove(root->right, k);
  else { // root->item.key == k
    // What to do when root->item.key == k?
  }
}
```

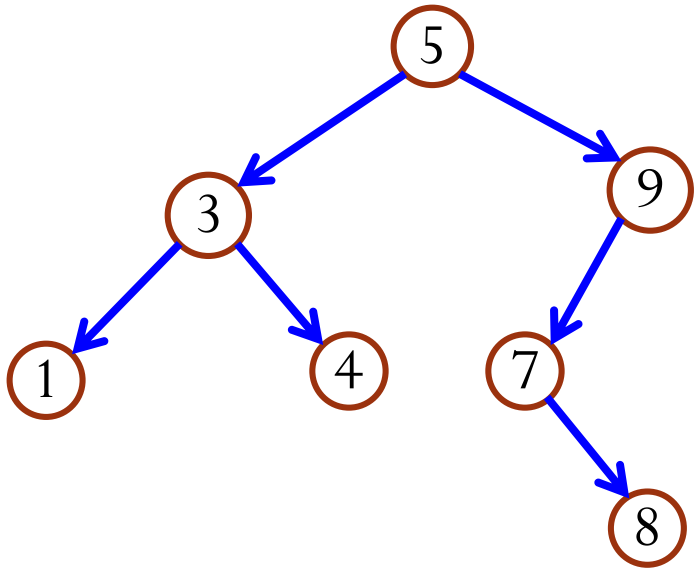
- How will you remove 8?
- How will you remove 9?
- How will you remove 5?



Binary Search Tree

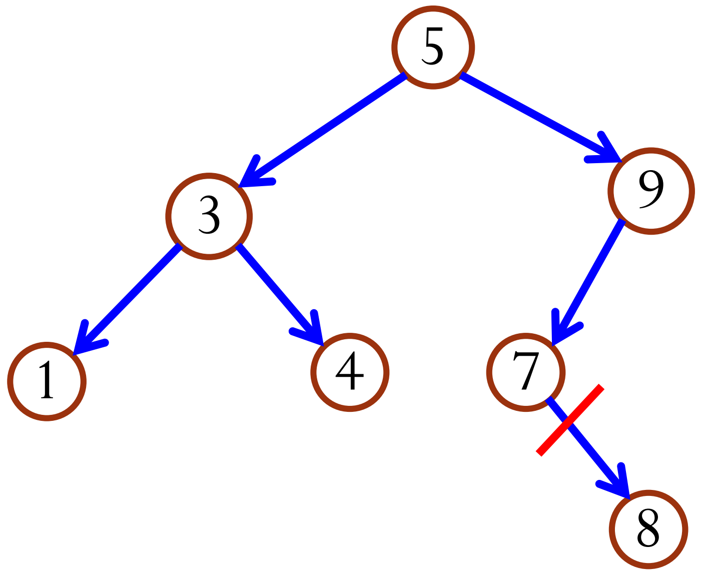
Removal

- We distinguish three cases:
  - Node to be removed is a leaf.
  - Node to be removed is a degree-one node.
  - Node to be removed is a degree-two node.



Remove A Leaf

- Remove node 8

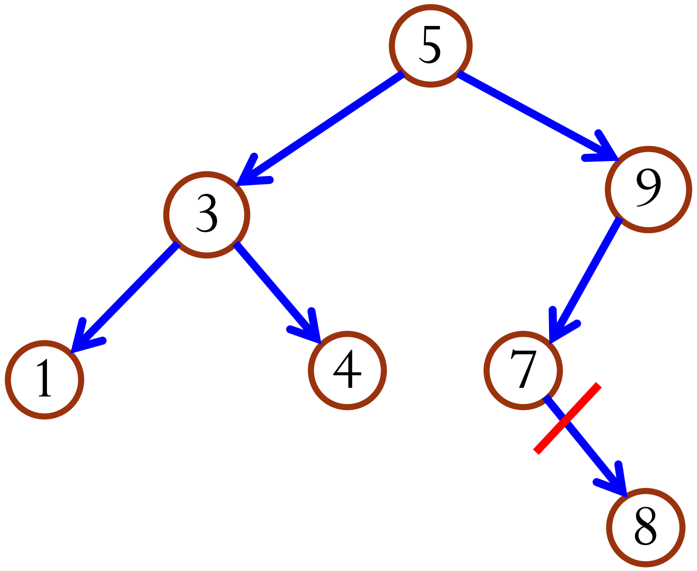


Remove A Leaf

Code

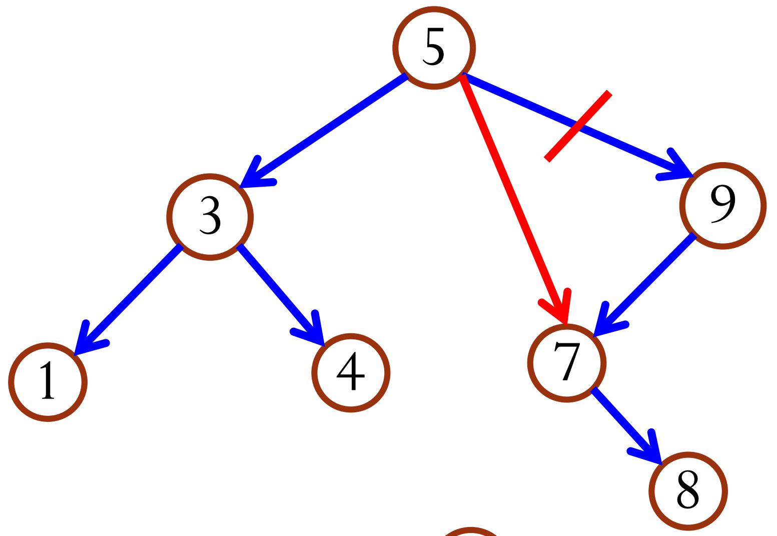
```
else { // root->item.key == k
  if(isLeaf(root)) {
    delete root;
    root = NULL;
  }
  else { // remove degree-one or two node
    ...
  }
}
```

Note: **`root`** is a **reference to a pointer**, which could be its parent's **`left`** pointer or **`right`** pointer. Our code effectively changes that pointer to NULL.

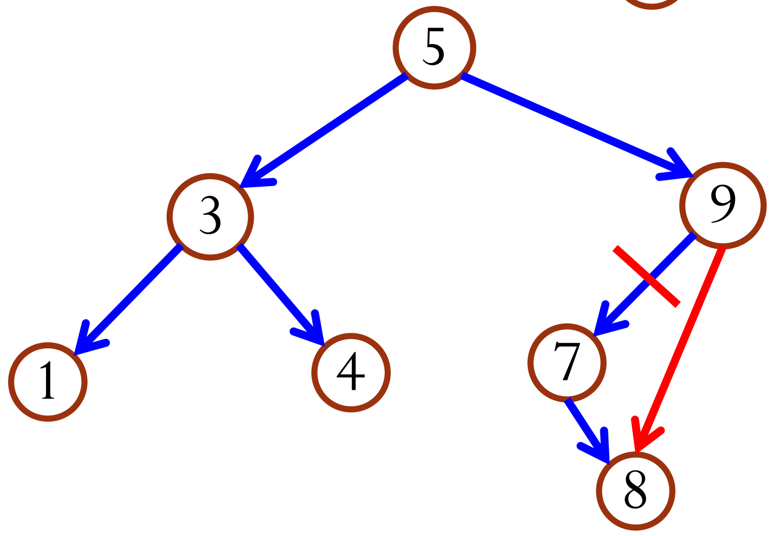


Remove A Degree-One Node

- Remove node 9



- Remove node 7



Remove A Degree-One Node

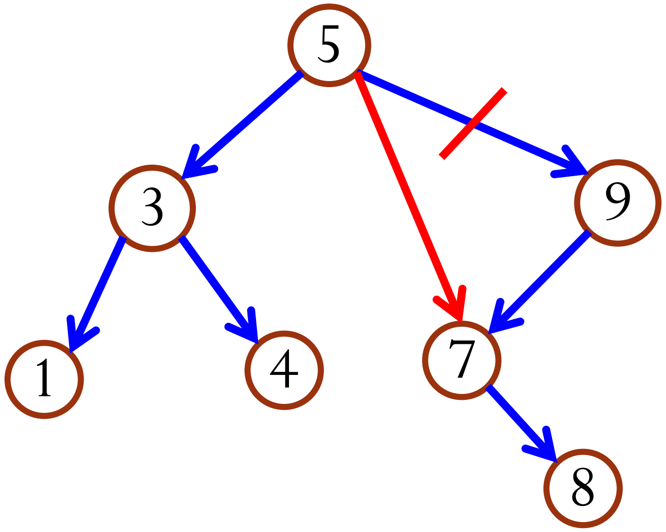
Code

```
else { // remove degree-one or two node
  if(root->right == NULL) { // no right child
    node *tmp = root;
    root = root->left;
    delete tmp;
  }



}
```

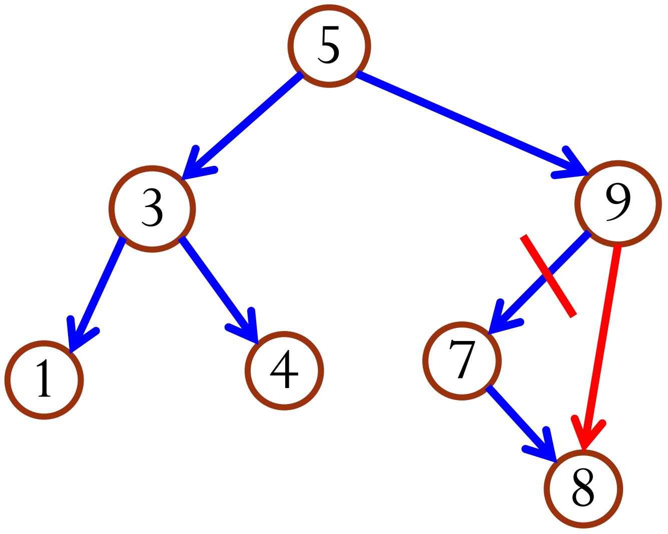
Note the order!



Remove A Degree-One Node

Code

```
else { // remove degree-one or two node
  if(root->right == NULL) { // no right child
    node *tmp = root;
    root = root->left;
    delete tmp;
  }
  else if(root->left == NULL) { // no left child
    node *tmp = root;
    root = root->right;
    delete tmp;
  }
  else {
  // remove degree-two node
  }
}
```



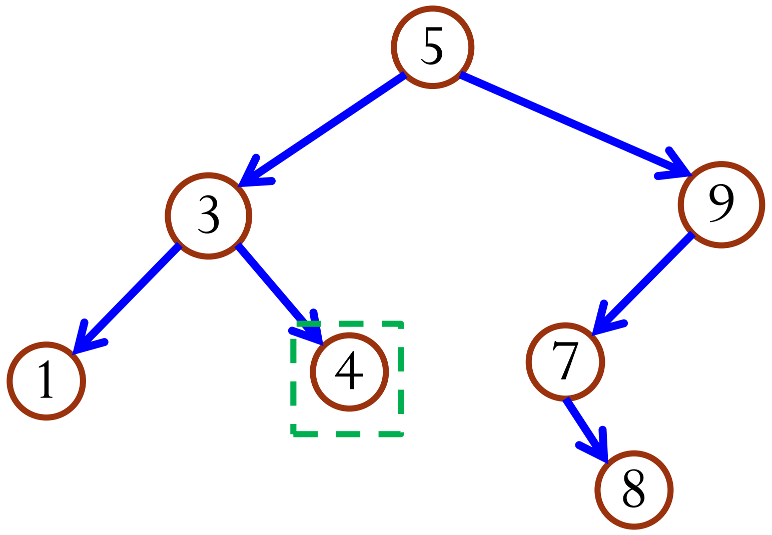
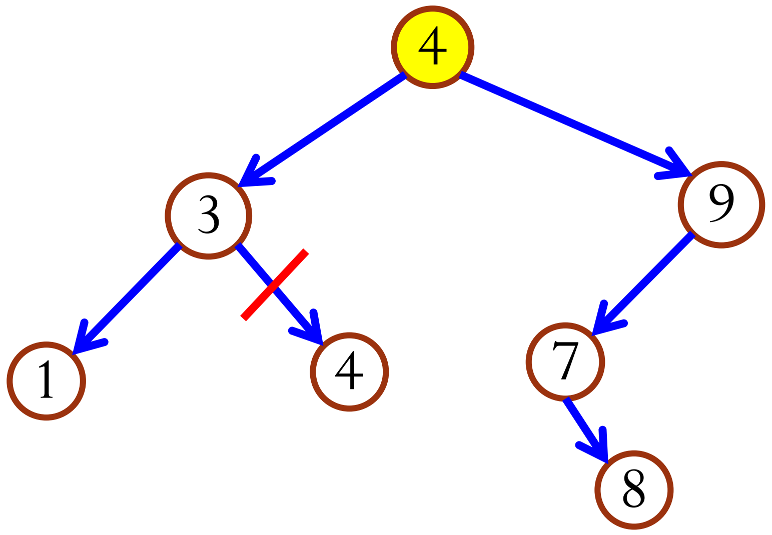
Remove A Degree-Two Node

- Remove node 5

How shall we do this?

- Idea: Replace with the largest key in the left subtree.
  - or replace with the smallest key in the right subtree.
- Claim: The largest key must be in a leaf node or in a degree-one node.

Great! We know how to remove such a node!

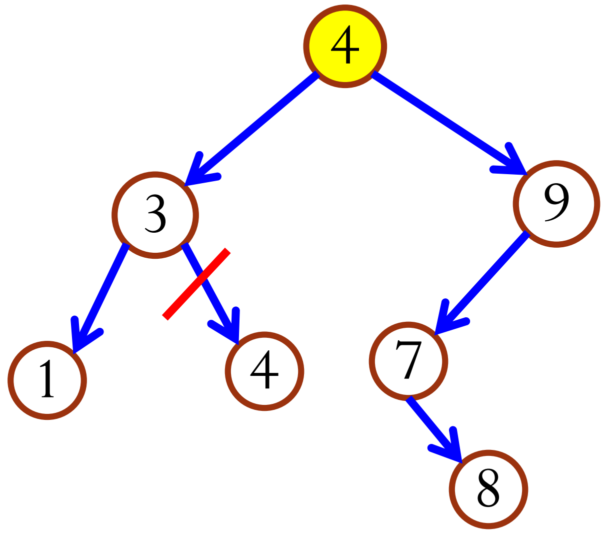



Remove A Degree-Two Node

Code

```
else { // remove degree-two node
  node *&replace = findMax(root->left);
  root->item = replace->item;
  node *tmp = replace;
  replace = replace->left;
  // both leaf and degree-one node are OK
  delete tmp;
}
```

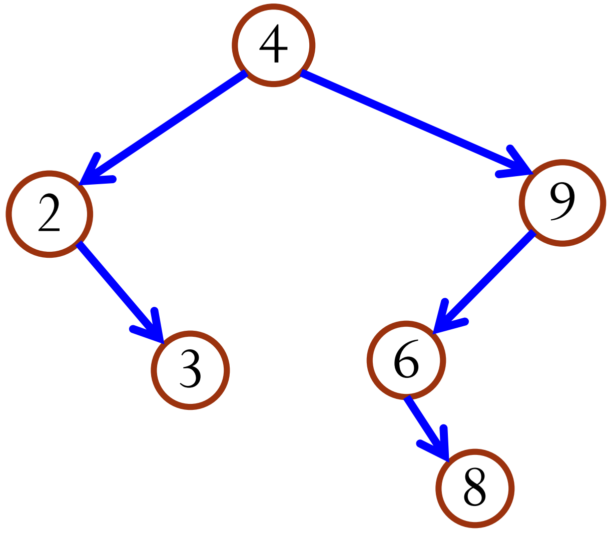
```
node *&findMax(node *&root)
// REQUIRES: tree is non-empty.
// EFFECTS: return the reference
// to the left/right pointer of
// the parent of the node
// that has the largest key in
// the tree rooted at root
```



Remove A Degree-Two Node
Code

- How do you implement the function **`findMax()`**?

```
node *&findMax(node *&root) {
  if(root->right == NULL) return root;
  return findMax(root->right);
}
```



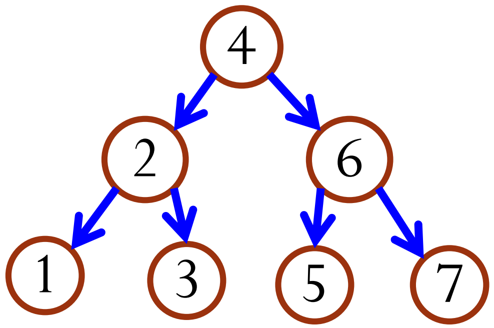
Removal of Binary Search Tree

Summary

- Node to be removed is a leaf.
  - Delete the node.
- Node to be removed is a degree-one node.
  - "Bypass" the node from its parent to its child.
- Node to be removed is a degree-two node.
  - Replace the node key with the largest key in the left subtree and remove the node with the largest key

Exercise

- Insert 4, 2, 6, 3, 7, 1, 5



- Delete 2, insert 9, delete 5, delete 1