QDD model Algorithm

XXY(USTC)

July 8, 2024

Input: $\left\{\varphi^{(0)}, n^{(0)}, \varphi_n^{(0)}, G_n^{(0)}, p^{(0)}, \varphi_p^{(0)}, G_p^{(0)}\right.$, toll , kmax, jmax$\}$.
set $\gamma_n^{(0)} = \ln\left(G_n\right)^{(0)}, \gamma_p^{(0)} = \ln\left(G_p^{(0)}\right)^p$.
For $k = 1, \ldots,$ kmax (k is the outer iteration counter)

1. For $j = 1, \ldots, j\max$(j is the inner iteration counter)

   (a) Solve for φ (using a damped Newton method):

   $$-\operatorname{div}\left(\lambda^2 \nabla \varphi\right) + \gamma_n^{(k)}{}_{j+1} \exp\left(\varphi - \varphi_n^{(k)}\right) - \gamma_p^{(k)}{}_{j+1} \exp\left(-\varphi + \varphi_p^{(k)}\right) - D = 0$$

   (b) set:

   $$\varphi_{j+1}^{(k)} = \varphi.$$

   (c) Solve for w (using the modified Newton method):

   $$-\operatorname{div}\left(\delta_n^2 \nabla w\right) + w\left(\varphi_n^{(k)} - \varphi_{j+1}^{(k)} + 2\ln(w)\right) = 0$$

   (d) Solve for v (using the modified Newton method):

   $$-\operatorname{div}\left(\delta_p^2 \nabla v\right) + v\left(-\varphi_p^{(k)} + \varphi_{j+1}^{(k)} + 2\ln(v)\right) = 0.$$

   (e) Set:

   $$\begin{array}{ll}
   n_q = w^2, & p_q = v^2 \\
   n_{\mathrm{cl}} = \exp\left(\varphi_{j+1}^{(k)} - \varphi_n^{(k)}\right), & p_{\mathrm{cl}} = \exp\left(\varphi_p^{(k)} - \varphi_{j+1}^{(k)}\right) \\
   G_n^{(k)}{}_{j+1} = \varphi_n^{(k)} - \varphi_{j+1}^{(k)} + 2\ln(w), & G_p^{(k)}{}_{j+1} = \varphi_p^{(k)} - \varphi_{j+1}^{(k)} - 2\ln(w) \\
   \gamma_n^{(k)}{}_{j+1} = \frac{n_q}{n_{\mathrm{cl}}} = \exp\left(G_n^{(k)}{}_{j+1}\right), & \gamma_p^{(k)}{}_{j+1} = \frac{p_q}{p_{\mathrm{cl}}} = \exp\left(G_p^{(k)}\right)_{j+1} \\
   n_{j+1}^{(k)} = \gamma_n^{(k)}{}_{j+1} n_{\mathrm{cl}}, & p_{j+1}^{(k)} = \gamma_p^{(k)}{}_{j+1} p_{\mathrm{cl}}
   \end{array}$$

   (f) if $\left\|\varphi_{j+1}^{(k)} - \varphi_j^{(k)}\right\|_{\infty,\Omega}$ toll set:

   $$\begin{array}{ll}
   \varphi^{(k+1)} = \varphi_{j+1}^{(k)} & \\
   n^{\left(k+\frac{1}{2}\right)} = n_{j+1}^{(k)}, & p^{\left(k+\frac{1}{2}\right)} = p_{j+1}^{(k)} \\
   G_n^{(k+1)} = G_n^{(k)}{}_{j+1}, & G_p^{(k+1)} = G_p^{(k)}{}_{j+1} \\
   \gamma_n^{(k+1)} = \gamma_n^{(k)}{}_{j+1}, & \gamma_p^{(k+1)} = \gamma_p^{(k)}{}_{j+1}
   \end{array}$$

and proceed to step 2.

(g) else : repeat steps a) $\to$ e).

2. Solve for n:

$$-\operatorname{div}\left(\mu_n \nabla n - \mu_n n \nabla\left(\varphi^{(k+1)} + G_n^{(k+1)}\right)\right) = 0$$

3. Set:

$$n_{k+1} = n, \quad \varphi_n^{(k+1)} = \varphi^{(k+1)} - \ln\left(\frac{n^{(k+1)}}{\gamma_n^{(k+1)}}\right)$$

4. Solve for p:

$$-\operatorname{div}\left(\mu_p \nabla p + \mu_p p \nabla\left(\varphi^{(k+1)} + G_p^{(k+1)}\right)\right) = 0$$

5. Set:

$$p^{(k+1)} = p, \quad u_p^{(k+1)} = u^{(k+1)} + \ln(\frac{p^{(k+1)}}{\gamma_p^{(k+1)}})$$

6. If $\|\varphi^{(k+1)} - \varphi^{(k)}\|_{\infty,\Omega} \leq$ toll and $\|\varphi_p^{(k+1)} - \varphi_p^{(k)}\|_{\infty,\Omega_{Si}} \leq$ toll and $\|\varphi_p^{(k+1)} - \varphi_p^{(k)}\|_{\infty,\Omega_{Si}} \leq$ toll exit

7. Else: go back to step 1.