Find the eigenvalues for:

$$A = \begin{pmatrix} 4 & 8 & -1 \\ -2 & -9 & -2 \\ 0 & 10 & 5 \end{pmatrix}$$

* Eigenvalues : $\det(A - \lambda I) = 0$

$$A - \lambda I = \begin{bmatrix} 4-\lambda & 8 & -1 \\ -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \end{bmatrix}$$

* Expanding along the 1st row

$$(4-\lambda)\begin{vmatrix} -9-\lambda & -2 \\ 10 & 5-\lambda \end{vmatrix} - 8\begin{vmatrix} -2 & -2 \\ 0 & 5-\lambda \end{vmatrix} + (-1)\begin{vmatrix} -2 & -9-\lambda \\ 0 & 10 \end{vmatrix} = 0$$

* 1st: $(-9-\lambda)(5-\lambda) - (-2)(10) = (-45 - 5\lambda + 9\lambda + \lambda^2) + 20 = \lambda^2 + 4\lambda - 25$
* 2nd: $(-2)(5-\lambda) - (-2)(0) = -10 + 2\lambda$
* 3rd: $(-2)(10) - (-9-\lambda)(0) = -20$

* Substitute the minors back into the determinant expansion

$$(4-\lambda)(\lambda^2 + 4\lambda - 25) - 8(-10 + 2\lambda) - (-1)(-20) = 0$$

* 1st: $(-9-\lambda)(5-\lambda) - (-2)(10) = (-45 - 5\lambda + 9\lambda + \lambda^2) + 20 = \lambda^2 + 4\lambda - 25$
* 2nd: $(-2)(5-\lambda) - (-2)(0) = -10 + 2\lambda$
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* Substitute the minors back into the determinant expansion

$$(4-\lambda)(\lambda^2 + 4\lambda - 25) - 8(-10 + 2\lambda) - (-1)(-20) = 0$$

$$(4\lambda^2 + 16\lambda - 100) - (\lambda^3 + 4\lambda^2 - 25\lambda) + 60 - 16\lambda = 0$$

$$\lambda^3 + 4\lambda^2 - 25\lambda + 4\lambda^2 + 16\lambda - 100 + 6 = 0$$

$$\lambda^3 + 8\lambda^2 - 9\lambda - 40 = 0$$

* Solve for λ

factoring: $\lambda^3 + 8\lambda^2 - 9\lambda = 0$

$$(\lambda + 5)(\lambda - 5)(\lambda - 0) = 0$$

Thus the first eigenvalue is: $\lambda_1 = -5$

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⇒ Continuation

Now $\lambda = -5$ ⇒ $\lambda - 5 = 0$

We divide Using synthetic division

$\lambda^3 - 18\lambda^2 + 137\lambda - 320$ (coefficients of this are: 1, -18, 13, -320)

by : $(\lambda + 5)$

Perform synthetic devision

-5	1	-18	137	-320
		-5	115	-640
	1	-23	252	0

The result is $\lambda^2 - 23\lambda + 252$

$P(\lambda) = (\lambda+5)(\lambda^2 - 23\lambda + 252)$

We now going to discriminant method

$\lambda^2 - 23\lambda + 252 = 0$

$\Delta = b^2 - 4ac$

$\Delta = (-23)^2 - 4(1)(252)$

$\Delta = 529 - 1008$

$\Delta = -479$ / this means it have two complex roots to let use another easy way

We need two numbers that Multiply to give 252

Add up to -23

This numbers are -9 and -14

$\boxed{\lambda^2 - 23\lambda + 252 = (\lambda - 9)(\lambda - 14)}$

$P(\lambda) = (\lambda+5)(\lambda-9)(\lambda-14)$

eigenvalue are the roots of the polynomial

$\lambda_1 = -5$ $\lambda_2 = 9$ $\lambda_3 = 14$

For clarity rewrite polynomial in its factored form

$P(\lambda) = (\lambda+5)(\lambda)(\lambda-5)$

$\lambda_1 = -5$ $\lambda_2 = 0$ $\lambda_3 = 5$

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=> Continuation

Second minor : $\det\begin{bmatrix} -2 & -2 \\ 0 & 5-\lambda \end{bmatrix} = (-2)(5-\lambda) - (-2)(0) = -10+2\lambda$

$= -10+2\lambda$

Third minor : $\det\begin{bmatrix} -2 & -9-\lambda \\ 0 & 10 \end{bmatrix} = (-2)(10) - (-9-\lambda)(0) = -20$

Now we are going to substitute minors back into the determinant expansion:

$\det(A-\lambda I) = (4-\lambda)(\lambda^2-14\lambda+65) - 8(-10+2\lambda) - 1(-20)$

$(4-\lambda)(\lambda^2-14\lambda+65) = 4\lambda^2 - 56\lambda + 260 - \lambda^3 + 14\lambda^2 - 65\lambda$

$= -\lambda^3 + 18\lambda^2 - 121\lambda + 260$

$\det(A-\lambda I) = (-\lambda^3 + 18\lambda^2 - 121\lambda + 260) + (80 - 16\lambda) + (-20)$

$\det(A-\lambda I) = -\lambda^3 + 18\lambda^2 - 137\lambda + 320$

$\det(A-\lambda I) = 0$

Hence $-\lambda^3 + 18\lambda^2 - 137\lambda + 320 = 0$

$\lambda^3 - 18\lambda + 137\lambda = 320$

$\lambda^3 - 18\lambda + 137\lambda - 320 = 0$

We are going to perform synthetic division

let us start with polynomial i mean cubic polynomial

$P(\lambda) = \lambda^3 - 18\lambda^2 + 137\lambda - 320$

let us test $\lambda = -5$

Substitute $\lambda = -5$ into $P(\lambda)$

$P(\lambda) = \lambda^3 - 18\lambda^2 + 137\lambda - 320$

$P(-5) = (-5)^3 - 18(-5)^2 + 137(-5) - 320$

$P(-5) = 125 - 18(25) + 137(-5) - 320$

$P(-5) = -125 - 450 - 685 - 320$

$P(-5) = 0$

Thus $\lambda = -5$ is a root