Exercise 3.1: (a) Design a $3*3$ kernel that detects vertical lines in a black and white image, and returns the value 8 when applied to the upper-left-hand side of the image in Figure 3.2. It should return zero if all the pixels in the patch are of equal intensity. (b) Design another such kernel.

0.0	0.0	0.0	0.0	0.0	0.0
0.0	2.0	2.0	2..0	0.0	0.0
0.0	2.0	2.0	2..0	0.0	0.0
0.0	2.0	2.0	2..0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0

Figure 3.2: Image of a small square

a)
$$\begin{matrix} -2 & 1 & 1 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{matrix}$$

b)
$$\begin{matrix} 0 & 2 & 0 \\ 0 & 2 & 0 \\ 0 & 2 & 0 \end{matrix}$$

Exercise 3.2: In our discussion of Equation 3.2 we said in an off-hand comment that the size of the convolution filter had no impact on the number of applications when using Same padding. Explain how this can be.

Same padding is a scheme where you pad the image with enough zeros on the borders to ensure that the output image after convolution has the same spatial dimensions as the input image. Thus, no matter the size of the kernel, the padding adjust compensate for the kernel's size, allowing for the output image to maintain the same width and height as the input. So, this means the number of times the convolution operation is applied (number of applications) across the spatial dimensions of the image remains same, regardless of kernel size.

Exercise 3.4: Suppose the input to a convolution NN is a $32 * 32$ color image. We want to apply eight convolution filters to it, all with shape $5 * 5$. We are using Valid padding and a stride of two both vertically and horizontally. (a) What is the shape of the variable in which we store the filters' values? (b) What is the shape of the output of `tf.nn.conv2d`?

a) since it is a color image, we will have 3 channels (RGB)
so each filter will have 3 5x5 kenels. And we have 8 filters. So the shape of variable in which we store the filter values is [5, 5, 3, 8]

kernel_height kernel_width in_channel out_channel.

b)

$$\text{output\_dimension} = \frac{\text{input\_dimension} - \text{filter\_dimension}}{\text{stride}} + 1$$

$$\text{output\_height} = \frac{32 - 5}{2} + 1 = 14$$

$$\text{Output\_width} = \frac{32 - 5}{2} + 1 = 14$$

And we have 8 filters. Thus the shape of output is [14, 14, 8]

A.4 Same padding when combined with stride of one has the property that the size of the output is the same as that of the original image. Let's consider a convolutional operation on an image with dimensions (H, W), where H represents the height and W represents the width of the image. For simplicity assume that W=H=I, and I is the input size of the image. In same padding with the stride of 1, the number of rows and columns of padding are added to the input image so that the output feature map has the same spatial dimensions. If P is the padding size, then 2P columns (or rows) are padded to the input image. This means that P columns (or rows) are added to both the left and right (or top and bottom) sides of the input image.

Let us assume,

- I is the input size (H or W) of the image
- F is the filter/kernel size
- P is the padding size
- O output size

please calculate the padding size in "same padding" (20 points).

$$P = \frac{F-1}{2}$$