FOS Project
First-Order Systems

Project and Course Goals	280 points

This project supports your learning to:

- Apply basic programming concepts to the solution of engineering problems,
- Represent and interpret data in multiple formats
- Develop, select, modify, and justify mathematical models to solve an engineering problem,
- Function effectively as a member of a team, and
- Demonstrate habits of a professional engineer

Problem Background and Scope

Parameter Identification

Engineers in all disciplines commonly work with models (physical, virtual, mathematical) that represent real-world systems, structures, and processes. One of the most important functions that engineers perform is *parameter identification*: the process of identifying a best estimate of the value of one or more characteristic features of a model.

<u>An example: biomechanics materials testing</u>

Materials testing often takes the form of a uni-axial stress test in which material is deformed along a single axis. The figure shows biomechanics data indicating that for small forces applied to the specimen (in this case, forearm soft tissue), the relationship between force and deformation is linear. This relationship is called Hooke's Law, and the slope of the line *E* is called Young's modulus.

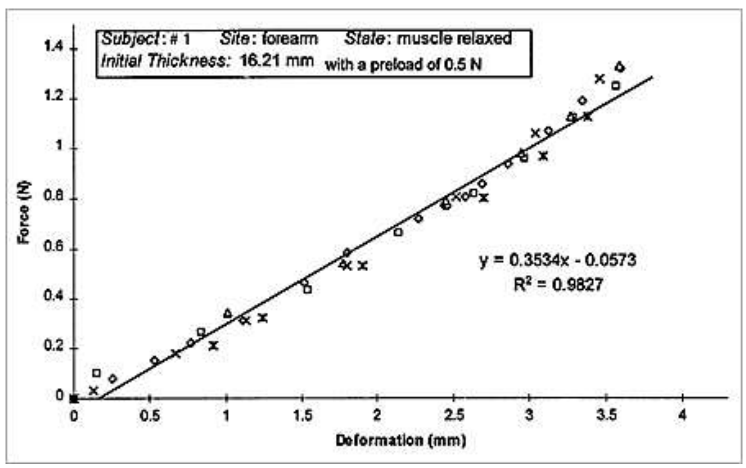

Image source: Zheng, Mak, and Lue, Journal of Rehabilitation Research & Development*, **36**(2), 1999.*

On the figure, the symbols are experimental data points, while the line is the best-fit linear regression for the data points. The slope of the line E is the *parameter* that has been *identified* from the experimental data. For this data, *E* = 0.3534 N/mm. Young's modulus is an important material property to understand, because any structural model for material response to loading within the elastic range (that is, a 'small' load) will use this parameter.

First-Order Systems

A first-order system is a dynamic system characterized by a time history that looks like Figure 1(a). Here, the word 'dynamic' means that the system performance *varies with time*. This time history of system response is called a *step response*, because it represents the response of the system due to a step (i.e., 'abrupt') change in the external stimulus [as shown in Figure 1(b)]. A good example of a step function can be shown with an experiment in which a thermocouple is used to measure temperature as it is moved from ice water to boiling water (or boiling water to ice water); the temperature change is an abrupt ('step') change in the operating condition of the system. Figure 1 is expressed in terms of that thermocouple experiment for a heating condition: the step change in 'reference' temperature [i.e., the temperature to be measured, Figure 1(b)] results in a gradual change in the 'measured' temperature [Figure 1(a)].

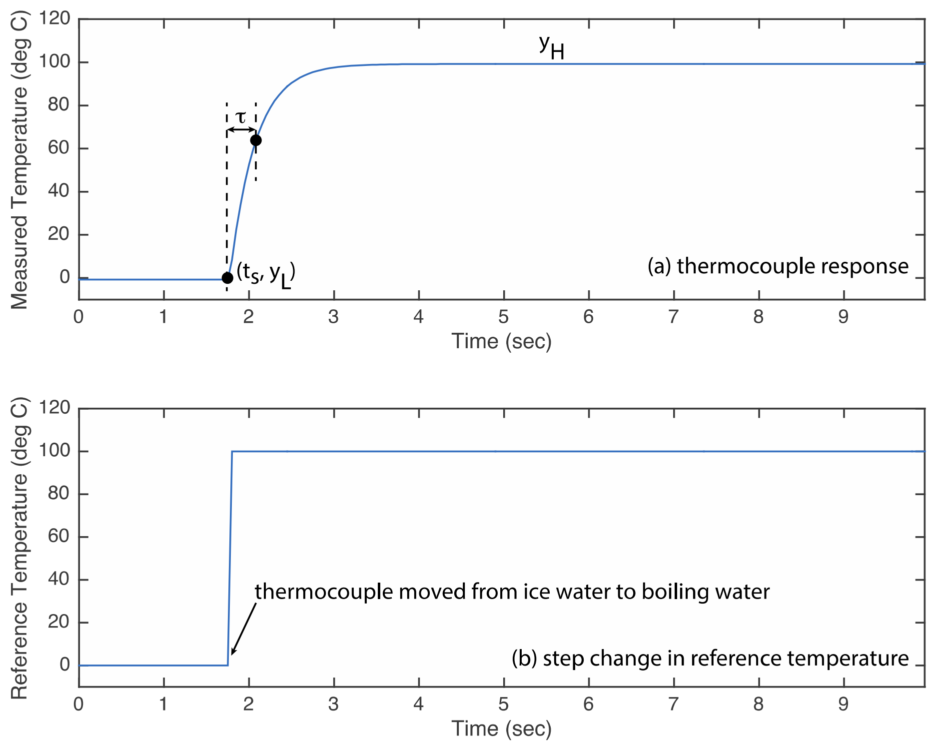

Figure 1. Thermocouple system showing first-order response in a heating condition: (a) measured temperature, (b) reference temperature.

The important parameters characterizing this step response are labeled on the figure, and these parameters are all defined in terms of the heating curve:

- The starting point (t_s, y_L):
    - t_s is the time at which the step input is applied
    - y_L is the initial 'low' value for the dependent variable. From time 0 (i.e., t_0) to time t_s, the value for y_L is relatively constant. It is important to note, that this parameter is NOT always equal to 0.
- y_H is the 'high' value for the dependent variable. As $t \to \infty$, the value for the dependent variable approaches this asymptotic value.
- τ is the time constant for the system, and it characterizes how fast the dependent variable responds to changes in external stimuli.

Physically, τ (which has units of seconds) represents the time it takes for the dependent variable to achieve a value of $y_\tau = y_L + 0.632(y_H - y_L)$. The 0.632 multiplier in front of the $y_{step} = (y_H - y_L)$ term is mathematically related to the exponential form of the equation shown below in equation (1), which is the solution to the first-order differential equation describing this dynamic system in a heating condition.

$$y(t) = \begin{cases} y_L & ;\ t < t_s \\ y_L + (y_H - y_L)\left[1 - \exp\left(-\dfrac{t - t_s}{\tau}\right)\right] & ;\ t \ge t_s \end{cases} \tag{1}$$

where $exp(\cdots)$ is the exponential function, and this equation is written *piecewise*. A piecewise equation is defined over intervals ('ranges') of the independent variable, in this case the intervals $(0 \le t < t_s)$ and $(t \ge t_s)$. Note that y(t) is *continuous*, meaning that at $t = t_s$, the two equations describing y(t) over the two intervals have the *same value*.

We can also write a first-order response equation for cooling, and again it is a piecewise equation as shown here and in Figure 2:

$$y(t) = \begin{cases} y_H & ;\ t < t_s \\ y_L + (y_H - y_L)\left[\exp\left(-\dfrac{t - t_s}{\tau}\right)\right] & ;\ t \ge t_s \end{cases} \tag{2}$$

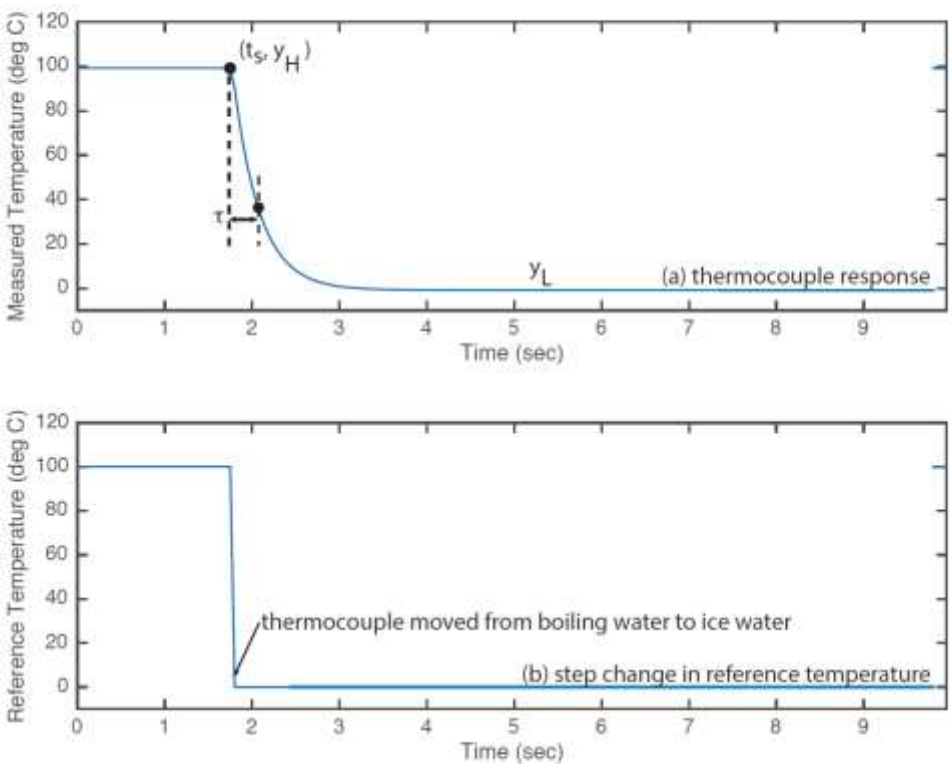

Figure 2. Thermocouple system showing first-order response in a cooling condition: (a) measured temperature, (b) reference temperature.

Similar to the heating first-order system, the important parameters characterizing the cooling step response are labeled on Figure 2 and are all defined as:

- The starting point (t_s, y_H):
  - t_s is the time at which the step input is applied.
  - y_H is the initial 'high' value for the dependent variable. From time 0 (i.e., t_0) to time $t_{s,}$ the value for y_H is relatively constant. It is important to note, that this parameter is NOT always equal to 100.
- y_L is the final 'low' value for the dependent variable. As $t \to \infty$, the value for the dependent variable approaches this asymptotic value.
- τ is the time constant for the system, and it characterizes how fast the dependent variable responds to changes in external stimuli.

> Optional Challenge: Can you analytically prove that this piecewise expression for y(t) is continuous?

First-order systems are very common in engineering, across disciplines, and this project will give you experience working with first-order systems - identifying parameters for such systems from their time histories and using those parameters to predict system requirements for specific performance goals. You probably have personal experience with a variety of first-order systems, perhaps even some of those that will be mentioned in class. Refer back to those slides to review some common engineering examples of first-order systems.

Optional Background Information: First-Order Differential Equations

These systems are called 'first-order' systems because their governing equation is a first-order differential equation. A first-order differential equation contains a single time derivative of a dependent variable. The standard form of a first-order differential equation is:

$$\tau \frac{dy(t)}{dt} + y(t) = G\ u(t) \tag{3}$$

where y(t) is the dependent variable (the "response", in our case, temperature), u(t) is the independent variable (the "stimulus" or "excitation" to the system, in our case, the reference temperatures of the ice water and boiling water), G is a scalar constant (a collection of material, geometric, or other system properties), and τ is the system time constant. The details of this equation's derivation and solution are not important right now; you will learn much more about first-order differential equations in the coming semesters, both in your differential equations course and in courses in your major. What is important right now is for you to understand the characteristics of a first-order response as shown in Figures 1 and 2.

Context Setting

Background. Your team has been contracted by First-Order Systems, Inc. ("FOS"), a local manufacturer of thermocouples used to measure temperature as a function of time. FOS employees have been testing several new thermocouple designs in their lab, and need to perform their quality assurance (QA) analysis to understand any differences in performance among thermocouples. They tested 100 thermocouples, 20 from each of five different designs. For each set of 20 experiments, 10 use a heating condition, and 10 use a cooling condition. *Regardless of the experimental condition (heating or cooling), the time constant for a given thermocouple will be the same.* These five designs have different performance properties, but within each design all 20 thermocouples should show very similar performance. These five thermocouple designs are intended for use in different markets and different applications; 'fast-responding' thermocouples are more expensive than 'slow-responding' ones, and FOS products are priced accordingly. Note that a 'fast-responding' thermocouple has a *small time constant*.

See the memo from FOS, Inc. president Frank O. Simpson (available as a separate document).

You can learn more about thermocouples by watching these videos:

- A versatile USB probe from National Instruments
- Thermocouple setup for data acquisition (skip forward in this one to about 4:30 and start watching there)

FOS routinely has customers purchase dozens of thermocouples at one time, for use in a single application. As such, their customers expect that if they purchase, say, 20 of the same model of thermocouple, those 20 thermocouples' performance should be essentially identical. *Here, 'performance' means 'time constant'.* Their customers rely on FOS to engineer their products for predictable performance.

The consequences of a customer having poor thermocouple performance can be significant. For instance, the nuclear power industry relies on thermocouples to monitor temperature in the reactor core and the water supplies that cool the reactor core, so poorly-performing or unreliable temperature measurement devices can have profound consequences for public safety as well as economic consequences in the event of a shutdown. Other examples include temperature-sensitive steps in food processing (a great example is chocolate, which is notoriously temperamental and has a narrow working temperature range), combustion efficiency in turbomachinery, climate control in delicate settings (example: the ICU unit in a hospital), and many others. FOS therefore plays an important role in their customers' economic success as well as their protection of public safety and health.

FOS engineers have sent you 100 time histories, one for each of their 100 tests. An example time history for a heating condition is shown in Figure 3, and unfortunately some level of measurement noise is present in the thermocouple output signal. In the experiment, FOS engineers performed a classic 'step response' test: in the heating condition, the thermocouple was initially placed in an ice bath; then, very abruptly, the thermocouple was submerged in boiling water. You will also have data that was generated using a cooling condition, in which the thermocouple was initially in boiling water, and was then moved to ice water. The abruptness of the temperature change can be modeled as a step input. But as you can see, in the presence of measurement noise, the four key parameters for a first-order response (t_s, y_L, τ, and y_H) are not quite as easy to identify as they were in the non-noisy case.

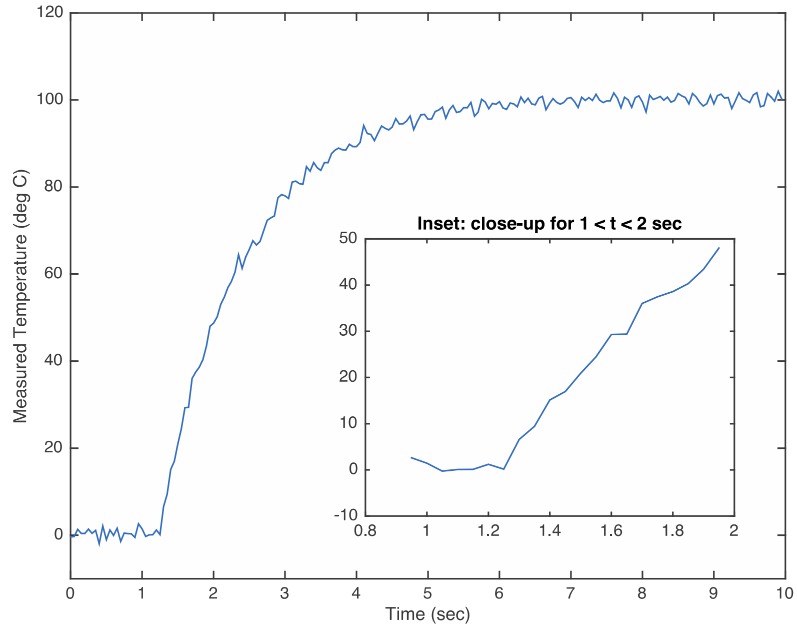

Figure 3. Noisy experimental time history from FOS, Inc. The inset shows a close-up of the time history from 1 < t < 2 seconds.

FOS wants to use the 100 time histories to guide their characterization of thermocouple performance in the sales literature for their customers. *FOS would like to be able to claim to their customers* that, based upon their in-house testing, *their thermocouples provide highly predictable behavior when making these kinds of measurements*. Right now, FOS is unsure if their data would support that claim, and part of your job is to help them decide what claims they can make based upon the available data.

Deliverables. Your team is to provide FOS with a 2-page technical brief containing both technical analysis and expert recommendations as follows:

- A detailed description of your analysis of the dataset provided, including clear and easy-to-understand graphics that summarize the data.
- An error analysis that characterizes the accuracy of your approach to determining time constant and other performance characteristics of the system.
- A recommendation about what FOS can honestly and ethically claim to its customers about the performance of the new design.

You will work toward these final deliverables according to the following milestone schedule.

Milestone Schedule

M1. Parameter Identification Brainstorming (due prior to Class 24) [20 points]. Using the M1 answer sheet downloaded from Blackboard, each team will generate a list of potential approaches for identifying important first-order system response parameters (t_s, y_L, y_H, and τ) from a single time history. Teams should document their ideas on the M1 answer sheet with sketches, text descriptions, or other ways to explain the idea. You will need to think very carefully about how to handle the measurement noise present in the data. Rate each idea for its anticipated difficulty in fully automating parameter identification in MATLAB and explain your rating. Your response to this milestone will be assessed on how well you have anticipated issues and articulated your understanding of the problem and its potential solution.

M2. Algorithm Development (due prior to Class 26) [50 points]. Within your team, choose two of your approaches for how to identify first-order system response parameters from M1, develop appropriate flowchart/pseudocode for each, and then code them in MATLAB. You should work in pairs, with each pair coding 1 approach. **Each of your algorithms should be programmed as a user-defined function that you can call whenever necessary to make the calculations.** If you are in a team with only 3 members, develop both algorithms together. Test each algorithm with the calibration datasets provided to you (which contain both 'clean' and 'noisy' data for heating and cooling conditions) and evaluate the performance of each. You will submit MATLAB code for each algorithm, along with performance characteristics of each as described on the M2 answer sheet.

M3. Data Analysis and Regression (due prior to Class 28 [100 points]. Choose the better-performing algorithm from M2 and integrate that algorithm into a data analysis code capable of analyzing all the data provided to you—both the heating and the cooling data--in a fully automated way. Write selection and repetition structures as appropriate to process the data. Perform a regression analysis on the time constant data you extracted from the raw dataset. Characterize the regression model according to the various regression model error measures available to you and report the results of your regression analysis using the format defined on the M3 answer sheet.

M4. Algorithm Refinement and Technical Brief Draft (due prior to Class 30) [Algorithm refinement: 50 points; Technical Brief Draft: 30 points]. Based upon your regression results from M3, make at least two significant refinements to your algorithm/data analysis approach (i.e., improvements on Milestones 2 and 3), and document the improvements in your regression model as a result of those refinements. Report the nature of the refinements and the resulting improvements to the regression model using the format provided on the M4 answer sheet. A draft of the technical brief is due with M4. A template is provided for this two-page memo. Your technical brief draft will undergo a formal review by the instructional team. In addition, your peers will provide feedback in Class 30.

M5. Final Technical Brief (due prior to Class 32) [30 points]. Write a two-page memo to FOS summarizing your findings, including: (a) details of your refined, final algorithm and data analysis, (b) your findings for the regression model, and (c) your recommendations to FOS about their marketing of their thermocouple designs based upon the analysis you have completed. For part (c), use detailed evidence-based rationales to defend your recommendations. For this two-page memo, use the outline given on the M5 guidelines document provided to you.