Report for Project 2

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I. INTRODUCTION

In this project, the main task is to decode an image of a suspect with the encoding key received from SpyPhone™. In order to accomplish the task, we need to design an equalizer and a detector that takes the received transmission on the SpyPhone™ and reveals the decoding key to decode the image.

II. PROBLEM FORMULATION AND SOLUTION

A. Design the equalizer

In order to determine appropriate values for the equalizer filter taps w(l), for l = 0, . . . , L, the transmission needs to be started with a training sequence that is known to the receiver, so that the receiver can estimate the filter coefficients w. First, we can start by using the training sequence of 32 symbols that are known to the receiver, to make sure that:

$$\sum_{l=0}^{L} w(l)\, r(k-l) \approx b(k), \quad \text{for } k = L+1, \ldots, 32. \tag{1}$$

In this equation, r(k) represents the received sequence, and b(k) represents the original sequence. Then, we can choose an appropriate measure of the equalization performance and try to find a good filter order L that minimizes it.
To accomplish this, we choose the FIR Wiener filter as the equalizer because it is very effective in addressing signal distortion and noise problems, has high stability and is easy to implement. An estimator with a FIR is given by the FIR structure

$$\hat{X}(n) = \mathbf{h}^{\mathbf{T}}\mathbf{Y}(n) \tag{2}$$

The linear MMSE estimator is given by (2) with the weights given by the solution to the normal equations, that is,

$$\mathbf{h_{lopt}} = \mathbf{R_Y}^{-1}\mathbf{r_{XY}} \tag{3}$$

In this problem, the optimal filter coefficients are given by the equation below

$$\mathbf{w} = \mathbf{R_r^{-1} r_{br}}, \quad \text{for k=L+1, ... , 32.} \tag{4}$$

Where the vectors $\mathbf{R}_{r(k)}$ and $\mathbf{r}_{r(k)b(k)}$ are given by

$$\mathbf{R_r} = E\left[\begin{pmatrix} r(k) \\ \vdots \\ r(k-L) \end{pmatrix}\big(r(k)\cdots r(k-L)\big)\right] = \begin{pmatrix} r_r(0) & \cdots & r_r(L) \\ & \ddots & \\ r_r(L) & \cdots & r_r(0) \end{pmatrix} \tag{5}$$

And

$$\mathbf{r}_{\mathbf{b(k)r(k)}} = E\left[b(k) \begin{pmatrix} r(k) \\ \vdots \\ r(k-L) \end{pmatrix} \right] = \begin{pmatrix} r_{br}(0) \\ \vdots \\ r_{br}(L) \end{pmatrix} \tag{6}$$

Since we already know the original sequence b(k) and the received sequence r(k) (k=L+1, …, 32), **w** can be calculated. Then we use it to estimate z(k), that is the symbols of the key from L+1 to 32 by taking convolution between **w** and r(k).

To measure the equalizer performance for k = L + 1, . . . , 32, we can use the mean of MSEs between the estimated sequence and the original sequence, that is

$$mse = \left(\hat{z}(k) - b(k)\right)^2 / (32 - L) \tag{7}$$

When taking different values of order L, we can get different MSEs. Based on both numerical results and visual results (decoded images), the filter order L can be determined.

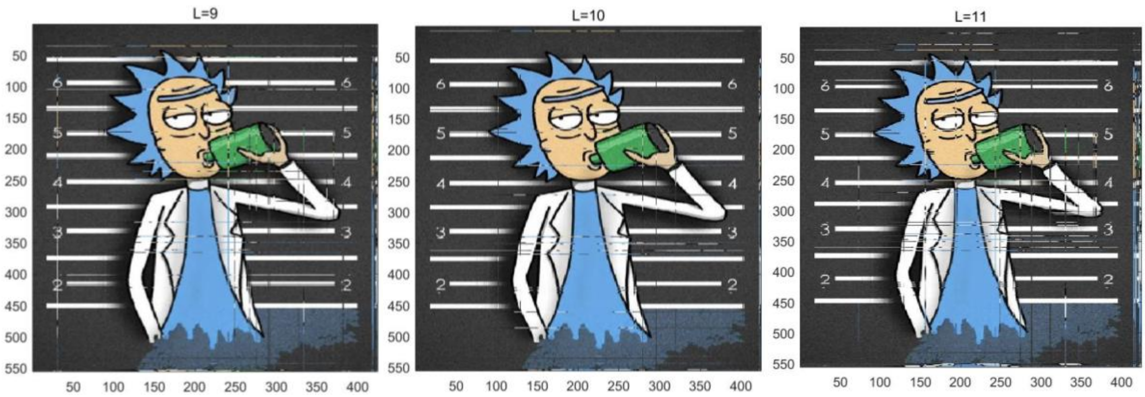
After knowing L, we can convolve the received signal r(k) with the estimated $\hat{w}$. Then use the detector below:

$$output(k) = \operatorname{sign}\left\{ \sum_{l=0}^{k} w(l) r(k-l) \right\} \tag{8}$$

to get the final key sequence output(k), i.e., the reconstructed key.

Finally, we can put the reconstructed key and the original picture (cPic) into the decoder to decode the image.

Implementing the above theoretical ideas in MATLAB, we can get visual results below (based on different filter order L):



(a) (b) (c)

Figure 1: Decoded images with different L.

We also get numerical results. The influence of L on MSE is shown in figure 2.

Based on both visual and numerical results, we finally choose L=10 as the filter order and use it to get the completed estimated key sequence output(k).

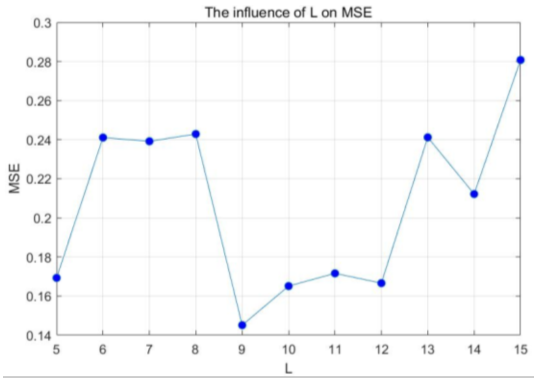


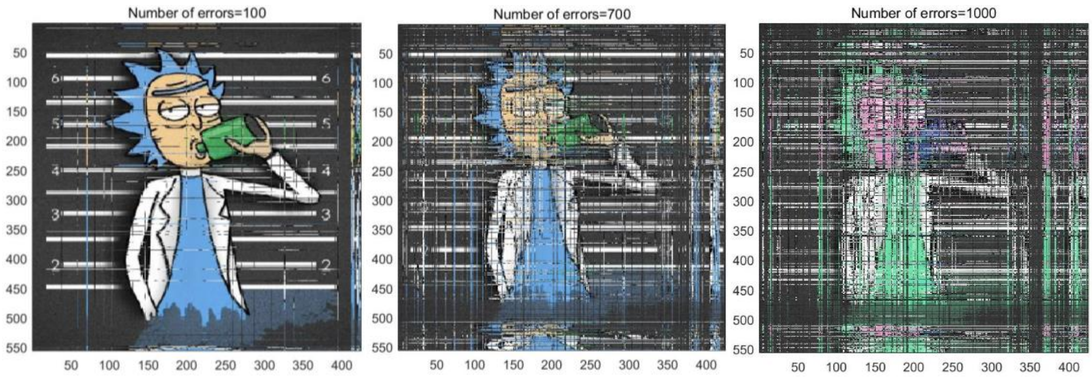
Figure 2: The influence L on MSE.

B. Decode the image

The decoded image is Figure 1 (b). It seems like the enemy is Rick Sanchez, a crazy scientist.

C. Introduce errors in the reconstructed key

To introduce random bit errors in the reconstructed key, we can use the randperm function to obtain a number of random error locations in $b(k)$, and then invert the values at the error locations to obtain bit errors. As the number of errors increases, the image becomes harder to recognize. When the number of errors reaches 700, the image "cannot" be decoded, i.e., it is not possible to recognize the person in the image.



(a) **(b)** **(c)**

Figure 2: Decoded images with different error numbers.

III. CONCLUSIONS

In this project, our main work is to reconstruct the decoding key. We designed an equalizer, an FIR Wiener filter, to achieve the reconstruction of the distorted signal and selected the appropriate filter order (L=10) based on the visual and data results. We also investigate the number of bit errors that the key can contain, which is less than 700.