01 Math for Data Science

Basic Definitions (Not complete notes). See pictures for complete notes, not guaranteed correct, for reference only.

Definition

A ***Vector space*** $(V, +,)$ over $\mathbb{R}$ is a set with two operations:

$V \times V \to V, (x, y) \longmapsto x + y$

$\mathbb{R} \times V \to V, (\lambda, y) \longmapsto \lambda y$

Elements of $V \in \mathbb{R}$ are called ***Vector***

The operations satisfy :

for all $\alpha, \alpha_1, \alpha_2 \in \mathbb{R}$ and $x, y \in V$

- $(V, +)$ is a commutative group
- $1 \cdot y = y$ for all $y \in V$
- $\alpha(x + y) = \alpha x + \alpha y$
- $\alpha_1(\alpha_2 x) = (\alpha_1 \alpha_2)x$
- $(\alpha_1 + \alpha_2)x = \alpha_1 x + \alpha_2 x$

Definition

A ***Linear combination*** of the vector $V_1, V_2 ... V_n \in V$ is an expression of the from

$C_1V_1 + C_2V_2 + ... + C_nV_n$ where $C_1, C_2 ... C_n \in \mathbb{R}$ are called ***Weights or coefficients*** of the linear combination.

Definition

The ***Span*** of $V_1, V_2 ... V_n \in V$ is the set of all the linear combinations of $V_1, V_2 ... V_n$ with coefficients in $\mathbb{R}$, $Span(v_1 ... v_n)$

Definition

A list of vectors $V_1, V_2 ... V_n$ is ***Linear independent*** if none of the vectors can be written as a Linear combination of the others

Definition

A ***Spanning List (or Set)*** of a vector space V is a list $V_1, V_2 ... V_n \in V$ such that $Span(V_1, V_2 ... V_n) = V$

Definition

A Linearly independent spanning list of V is a ***Basis*** of V

All Basis of V have the same number of elements, which is called the ***Dimension*** of V

Definition

Given V, W two vector spaces $L : V \to W$ is a ***Linear transformation***

$L(\alpha_1 v_1 + \alpha_2 v_2) = \alpha_1 L(v_1) + \alpha_2 L(v_2)$

Definition

Given a Linear transformation $L : V \to W$

The Set $L(v) = \{w \in W\}$ there is $v \in V$ so that $L(v) = w$ is the ***Image*** of L, or Image of V under L

The ***Rank*** of L is the dimension of $L(v)$

Definition

The ***Kernel or (null space)*** of a Linear transformation $L : V \to W$ is the set of elements of V which mapped to 0_w by L

$Ker(L) = \{v \in V | L(v) = 0_w\}$

Definition

Let V be a vector space, a non empty subset $U < V$ is a ***Subspace of V***, if V is a vector space with the operations $+, *,$ of V