Numerical Methods II
Assignment 2
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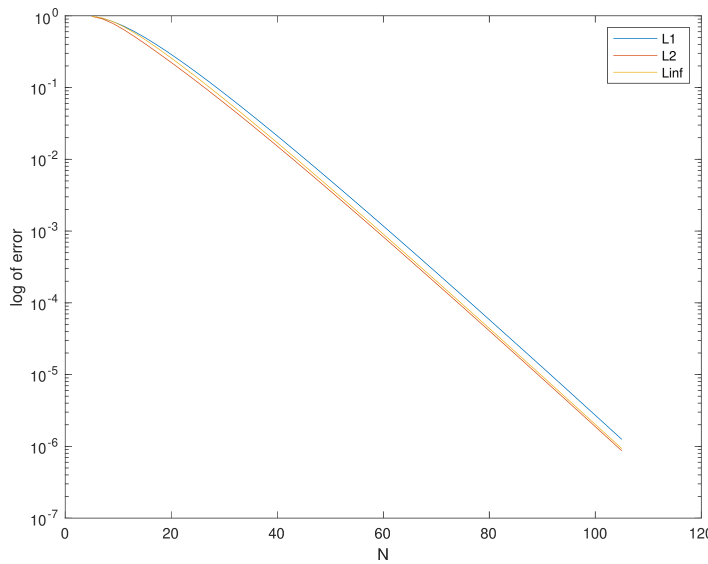
1.1 (a) The semilogy plot in L_1[1], L_2[2], L_{inf}[3] norm of the relative error is:

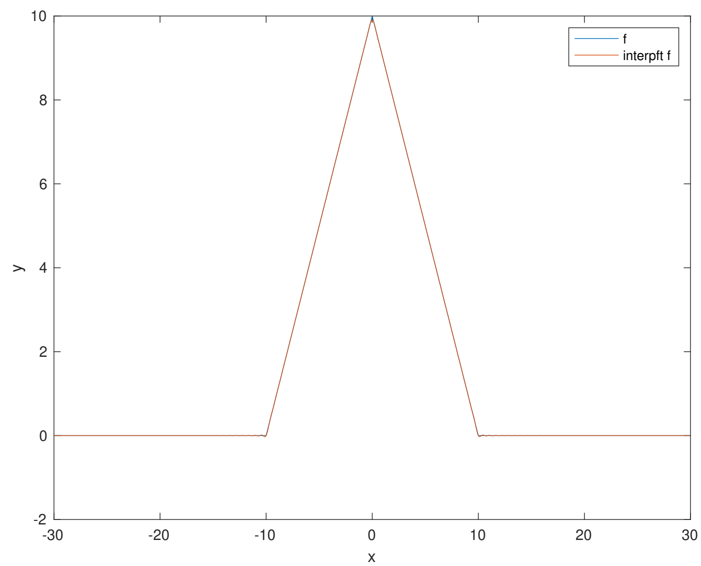


We can tell from the graph that the relative error decays exponentially as N grows larger, and $L_1 > L_{inf} > L_2$.

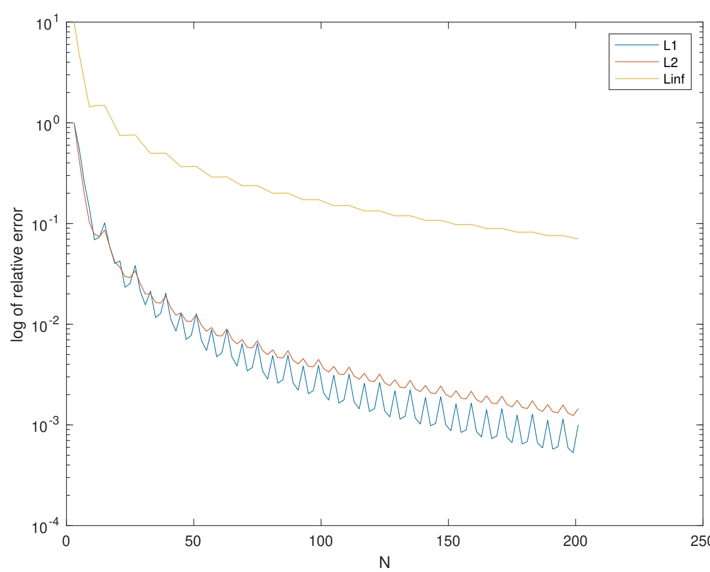
[1]function f L_1 norm: $\int |f| du$, we use trapz() to do the integration
[2]function f L_2 norm: $\sqrt{\int |f|^2 du}$
[3]function f L_{inf} norm: $inf\{C \geq 0 : |f(x)| \leq C, \forall x \in D\}$

(b) i. Triangle wave f:

The plot of original triangle wave f versus the interpft f plot when N = 201 is shown as below:

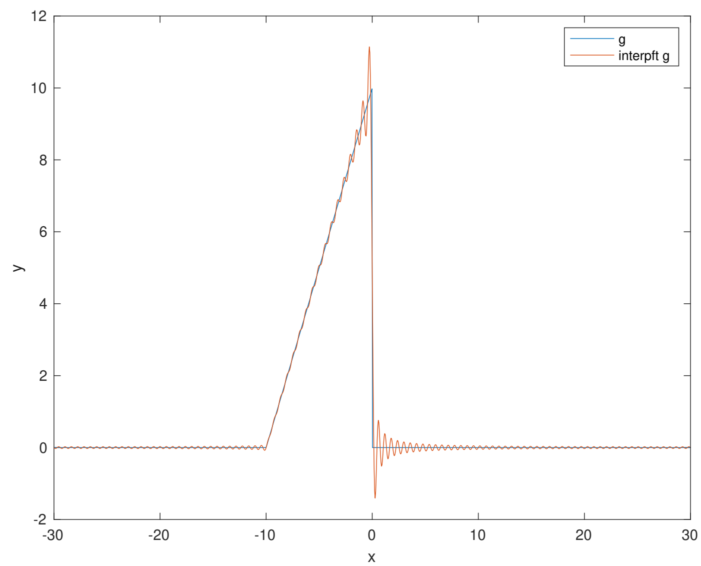


The semilogy plot of relative error norm $\|\phi - \hat{\phi}\|/\|\phi\|$ of triangle wave f in L_1, L_2, L_{inf} norm is shown as below:

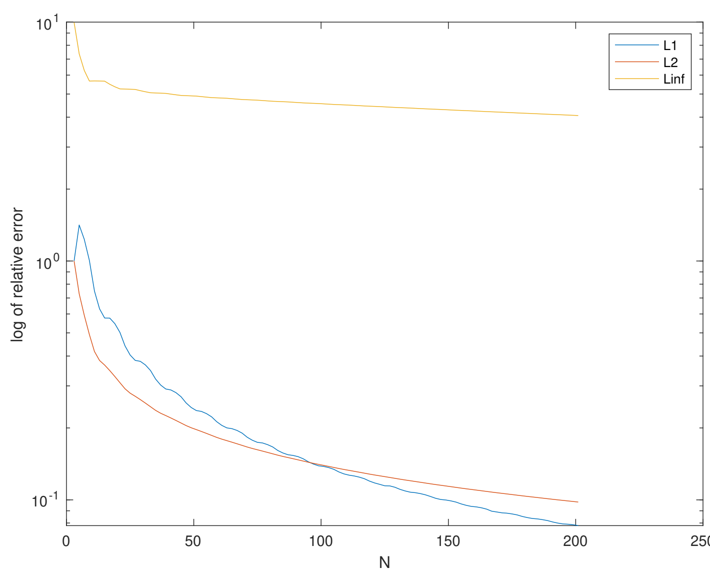


From the plot of the wave graph nd the interpft result we can see some obvious errors at the nonsmooth points, i.e., when $x = -10, x = 0, x = 10$, and these points cause the relative error plot of the three norm forms decays very slow even as N grows larger.

ii. Sawtooth wave g:
The plot of original sawtooth wave g versus the interpft g plot when N = 201 is shown as below:

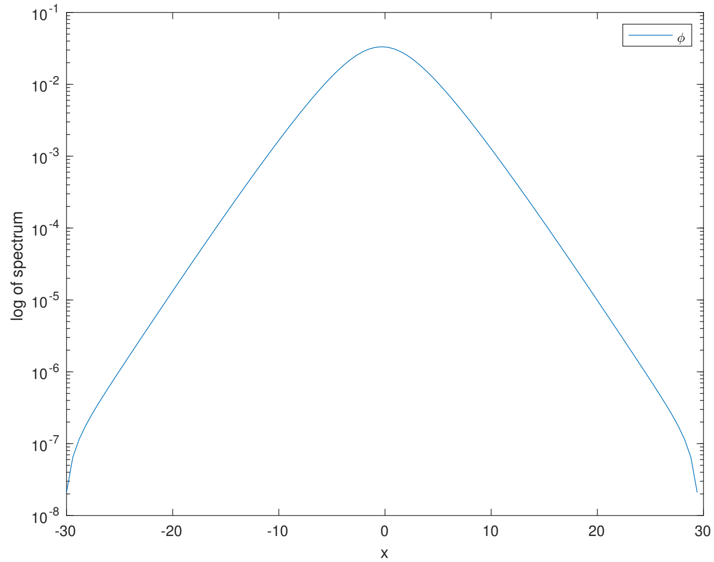


The semilogy plot of relative error norm $\|\phi - \hat{\phi}\|/\|\phi\|$ of sawtooth wave g in L_1, L_2, L_{inf} norm is shown as below:



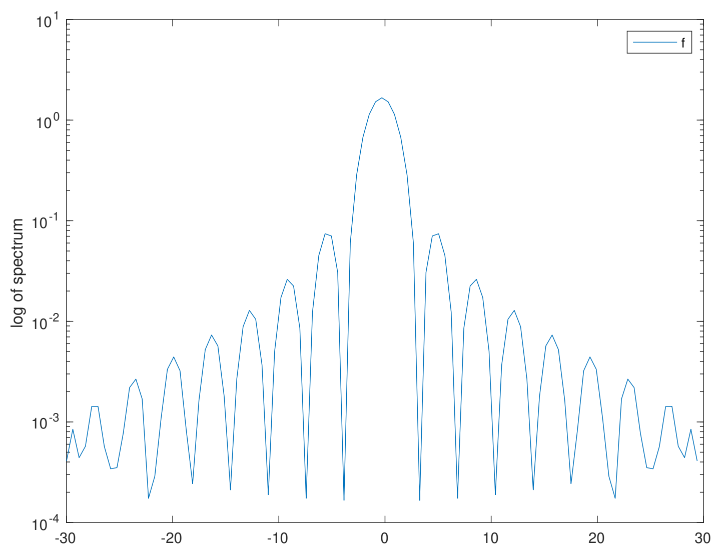
From the plot of the sawtooth graph and the interpft result we can see some obvious errors at the nonsmooth points, i.e., when $x = -10, x = 0$, and these points cause the absolute error plot decays very slow even as N grows larger, the relative error of L_1, L_2 approaches to 10^{-1} while in L_{inf} it approaches to 10^1.

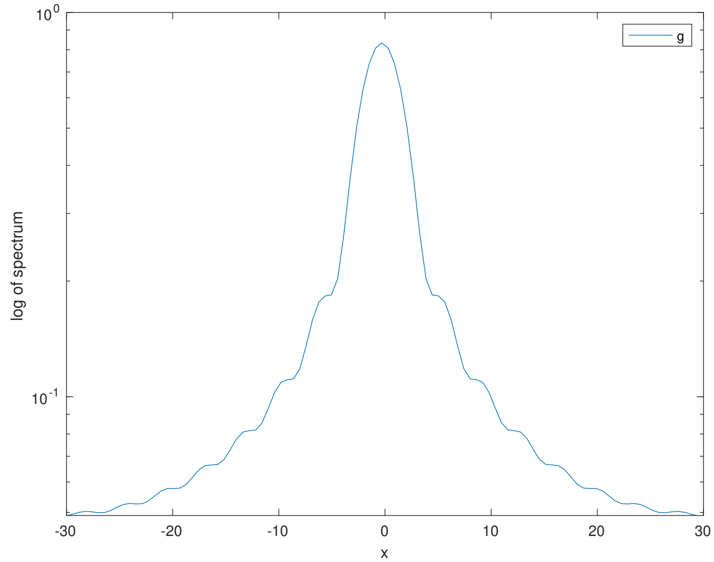
If we analyze from the spectrum side, let's first look at the semilogy plot of the spectrum of smooth initial function, ϕ_{soln} (N = 135 for all three spectrum graphs below):



We can see the spectrum decays very smoothly as $|x|$ grows larger, and that's because ϕ_{soln} itself is smooth.

However, if we look at the nonsmooth initial function f and g, the semilogy plot of the spectrum look like:





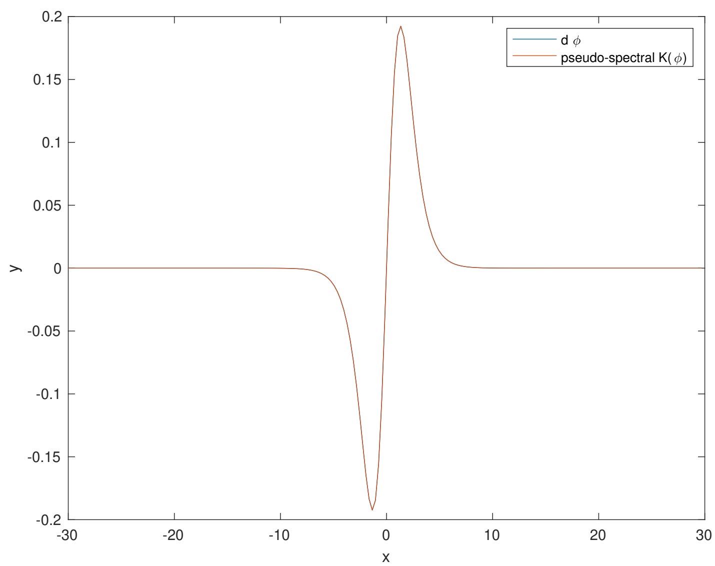
The spectrum has a oscillating decaying shape as $|x|$ grows, and it doesn't decay to 10^{-8} as ϕ_{soln} does. That's because they are not smooth initial conditions.

1.2 (a) The code that evaluate $\mathbf{F}(\hat{\phi})$ is listed below:

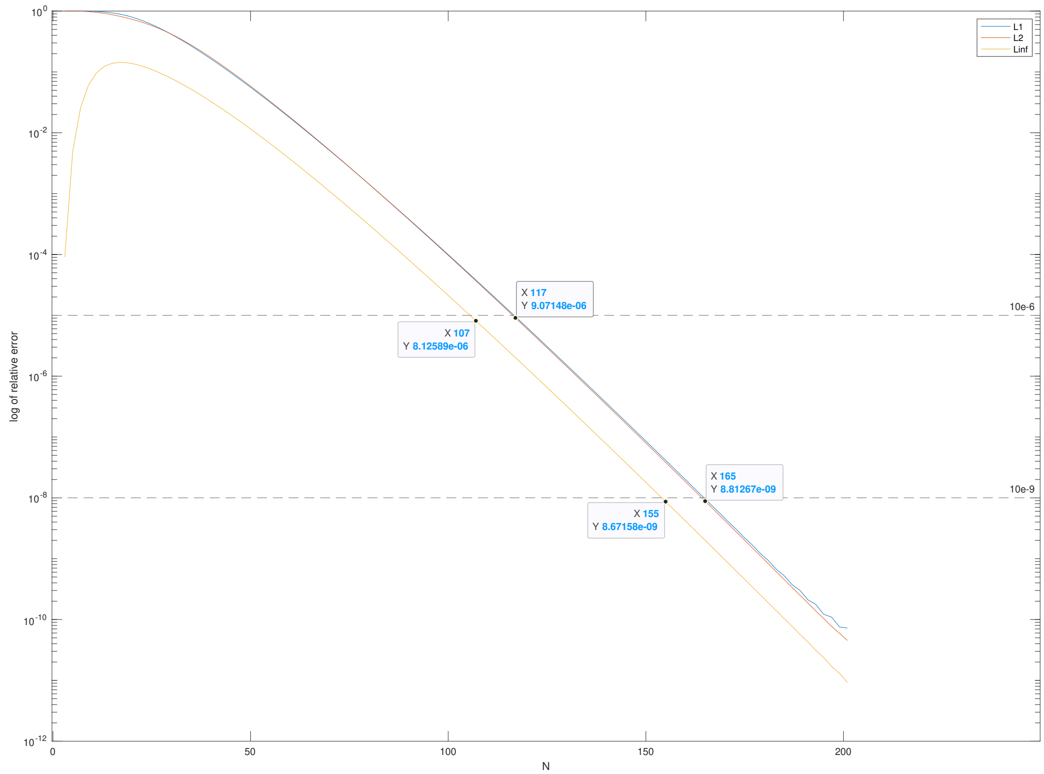
```
function dphidt = pseudospec(L, phi, N)
    x = equalspace(-L/2, L/2,N);
    phi_hat = fft(phi(x));
    K = [0:(N-1)/2, -(N-1)/2:-1].*(2*pi/L);
    dphidt = 1i*K.^3.*phi_hat - 3*1i*K.*fft((ifft(phi_hat).^2));

end
```

function **equalspace(start, end, N)** returns a list of N nodes that includes the first one but discards the last one.

The graph of $\partial\phi$ versus the result of $K(\phi)$ is shown as below (N $=$ 201):

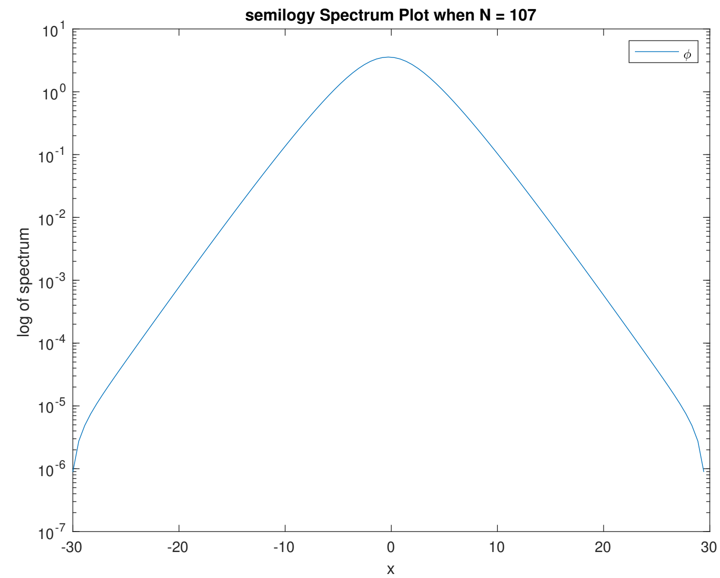


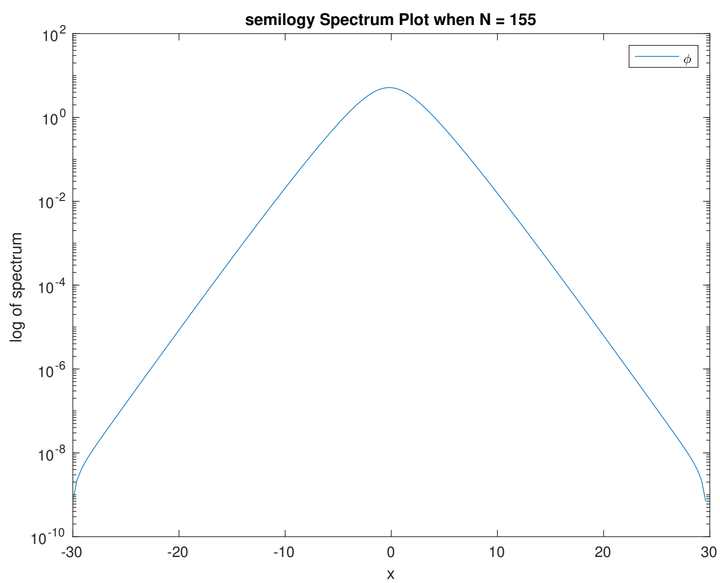
(b) The semilogy plot of relative error $|\partial\phi - K(\phi)|/\|\partial\phi\|$ in L_1, L_2, L_{inf} norms as N increases is shown below:



The error first increases in L_{inf} norm, then starts to decay exponentially as N increases after $N \approx 21$. This discretization gets more and more accurate when we take larger N.

(c) As shown from the graph, at least approximately over 155 grid points are needed if we want 9 digits of accuracy.
We also observe that the number of digits of accuracy is highly related to the value that the spectrum of ϕ decays to w.r.t. different N. From the error graph in (b) we can tell that the relative error $\approx 10^{-6}$ when N reaches 107 and $\approx 10^{-9}$ when N reaches 155.
If we look at the semilogy plot of the spectrum of ϕ with different N sizes, here we take N = 107 and N = 155:





The value the spectrum finally decays to, i.e., when $|x| = L/2$ has the same precision of the errors for corresponding N.