HPC23
Homework 2
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1. Please see the comments in the code

2. (a) The running time when row iteration is the outer for loop gives a better running time result. This is caused by the storage of matrix entries. Matrices are stored in column major order,therefore when we read one column we read that exact column in cache, but if we want to read one row (of one block) we need to read the whole block. Hence, column iteration in the inner loop, row iteration in the outer loop saves more time.

   (b) The function I implentmented blocks both row and column. I try BLOCKSIZE $= 4, \cdots 64$ and BLOCKSIZE $= 8$ gives the best result.
   Here are some running time when BLOCKSIZE $= 4, 8, 16, 24, 32, 64$ running on CIMS (on the left) versus using OpenMP (on the right).

Dimension	Time	Gflop/s	GB/s	Error
4	1.227775	1.628963	0.054087	0.000000e+00
52	1.295128	1.544255	0.048490	0.000000e+00
100	1.236495	1.619093	0.050723	0.000000e+00
148	1.234483	1.622888	0.050801	0.000000e+00
196	1.228027	1.630954	0.051032	0.000000e+00
244	1.314020	1.525621	0.047725	0.000000e+00
292	1.259209	1.621305	0.050709	0.000000e+00
340	1.258449	1.624069	0.050789	0.000000e+00
388	1.453834	1.446381	0.045229	0.000000e+00
436	1.383758	1.557302	0.048694	0.000000e+00
484	1.277202	1.597898	0.049960	0.000000e+00
532	1.320664	1.596139	0.049903	0.000000e+00
580	1.456524	1.607488	0.050256	0.000000e+00
628	1.601317	1.546684	0.048353	0.000000e+00
676	1.528092	1.617262	0.050558	0.000000e+00
724	1.415524	1.608606	0.050286	0.000000e+00
772	1.705603	1.618547	0.050596	0.000000e+00
820	1.479081	1.491110	0.046611	0.000000e+00
868	1.617148	1.617594	0.050564	0.000000e+00
916	2.042010	1.505527	0.047061	0.000000e+00
964	2.219153	1.614745	0.050474	0.000000e+00
1012	1.313983	1.577546	0.049310	0.000000e+00
1060	1.473702	1.616359	0.050523	0.000000e+00
1108	1.687601	1.612054	0.050388	0.000000e+00
1156	2.017375	1.531499	0.047870	0.000000e+00
1204	2.162158	1.614440	0.050462	0.000000e+00
1252	2.435376	1.611673	0.050375	0.000000e+00
1300	2.734928	1.606624	0.050217	0.000000e+00
1348	3.046255	1.608176	0.050265	0.000000e+00
1396	3.578443	1.520520	0.047525	0.000000e+00
1444	3.875924	1.553661	0.048560	0.000000e+00
1492	4.135461	1.606248	0.050204	0.000000e+00
1540	4.631987	1.576975	0.049288	0.000000e+00
1588	5.056571	1.583891	0.049504	0.000000e+00
1636	5.481100	1.597762	0.049938	0.000000e+00
1684	5.973062	1.599040	0.049977	0.000000e+00
1732	6.669350	1.558081	0.048697	0.000000e+00
1780	7.069045	1.595619	0.049870	0.000000e+00
1828	7.831826	1.559896	0.048753	0.000000e+00
1876	8.581714	1.538702	0.048091	0.000000e+00
1924	9.028486	1.577721	0.049310	0.000000e+00
1972	9.683784	1.583819	0.049501	0.000000e+00

Dimension	Time	Gflop/s	GB/s	Error
4	36.485907	0.054816	0.001820	0.000000e+00
52	0.254889	7.846590	0.246385	0.000000e+00
100	0.215036	9.310073	0.291667	0.000000e+00
148	0.105470	18.995238	0.594604	0.000000e+00
196	0.108625	18.438245	0.576930	0.000000e+00
244	0.154173	13.002924	0.406758	0.000000e+00
292	0.201885	10.112512	0.316287	0.000000e+00
340	0.105990	19.283004	0.603037	0.000000e+00
388	0.117506	17.895254	0.559587	0.000000e+00
436	0.115340	18.683308	0.584188	0.000000e+00
484	0.207863	9.818204	0.306977	0.000000e+00
532	0.113139	18.631661	0.582513	0.000000e+00
580	0.229166	10.216782	0.319412	0.000000e+00
628	0.143873	17.214726	0.538174	0.000000e+00
676	0.174343	14.175088	0.443135	0.000000e+00
724	0.120798	18.849804	0.589260	0.000000e+00
772	0.156200	17.673431	0.552474	0.000000e+00
820	0.126307	17.461253	0.545831	0.000000e+00
868	0.164990	15.854802	0.495605	0.000000e+00
916	0.177309	17.338664	0.541981	0.000000e+00
964	0.254444	14.083125	0.440212	0.000000e+00
1012	0.134755	15.382499	0.480822	0.000000e+00
1060	0.155679	15.300886	0.478265	0.000000e+00
1108	0.261777	10.392434	0.324837	0.000000e+00
1156	0.223430	13.828088	0.432221	0.000000e+00
1204	0.259106	13.472011	0.421088	0.000000e+00
1252	0.257214	15.259803	0.476964	0.000000e+00
1300	0.349240	12.581599	0.393251	0.000000e+00
1348	0.465285	10.528832	0.329087	0.000000e+00
1396	0.495056	10.990868	0.343526	0.000000e+00
1444	0.531409	11.331898	0.354183	0.000000e+00
1492	0.583760	11.378957	0.355652	0.000000e+00
1540	0.651830	11.206182	0.350250	0.000000e+00
1588	0.757280	10.576091	0.330555	0.000000e+00
1636	0.959855	9.123772	0.285161	0.000000e+00
1684	1.077600	8.863362	0.277021	0.000000e+00
1732	1.048712	9.908721	0.309692	0.000000e+00
1780	1.209116	9.328717	0.291563	0.000000e+00
1828	1.259164	9.702335	0.303239	0.000000e+00
1876	1.500934	8.797657	0.274963	0.000000e+00
1924	1.688591	8.435693	0.263650	0.000000e+00
1972	1.692193	9.063603	0.283273	0.000000e+00

Dimension $= 4$, 10X speed up with OpenMP

Dimension	Time	Gflop/s	GB/s	Error
8	0.222245	8.999093	0.142808	0.000000e+00
56	0.230390	8.682102	0.135961	0.000000e+00
104	0.228795	8.741497	0.136750	0.000000e+00
152	0.235153	8.512464	0.133117	0.000000e+00
200	0.236754	8.515183	0.133133	0.000000e+00
248	0.240334	8.377476	0.130964	0.000000e+00
296	0.235101	8.604286	0.134499	0.000000e+00
344	0.235738	8.634074	0.134956	0.000000e+00
392	0.237100	8.637843	0.135009	0.000000e+00
440	0.279065	7.325959	0.114501	0.000000e+00
488	0.244965	8.539397	0.133462	0.000000e+00
536	0.253013	8.520777	0.133168	0.000000e+00
584	0.283563	8.428898	0.131730	0.000000e+00
632	0.242265	8.335866	0.130274	0.000000e+00
680	0.297725	8.448932	0.132039	0.000000e+00
728	0.276170	8.382417	0.130998	0.000000e+00
776	0.532046	5.269719	0.082353	0.000000e+00
824	0.271941	8.229367	0.128603	0.000000e+00
872	0.321531	8.248726	0.128905	0.000000e+00
920	0.393366	7.918200	0.123739	0.000000e+00
968	0.546087	6.643912	0.103825	0.000000e+00
1016	0.255351	8.214347	0.128365	0.000000e+00
1064	0.392064	6.144661	0.096022	0.000000e+00
1112	0.346034	7.947412	0.124192	0.000000e+00
1160	0.391810	7.967624	0.124508	0.000000e+00
1208	0.447010	7.887026	0.123248	0.000000e+00
1256	0.511548	7.746632	0.121053	0.000000e+00
1304	0.576337	7.694609	0.120240	0.000000e+00
1352	0.652019	7.580535	0.118457	0.000000e+00
1400	0.827110	6.635149	0.103683	0.000000e+00
1448	0.820649	7.399093	0.115621	0.000000e+00
1496	0.952275	7.031736	0.109880	0.000000e+00
1544	1.065803	6.907088	0.107932	0.000000e+00
1592	1.170414	6.894769	0.107739	0.000000e+00
1640	1.360316	6.485175	0.101339	0.000000e+00
1688	1.531355	6.281618	0.098158	0.000000e+00
1736	1.868867	5.598875	0.087489	0.000000e+00
1784	1.914294	5.932065	0.092695	0.000000e+00
1832	2.065965	5.952281	0.093011	0.000000e+00
1880	2.284900	5.816160	0.090884	0.000000e+00
1928	2.837109	5.052136	0.078945	0.000000e+00
1976	2.703776	5.707161	0.089180	0.000000e+00

Dimension	Time	Gflop/s	GB/s	Error
8	3.903517	0.512359	0.008131	0.000000e+00
56	0.166161	12.038142	0.188516	0.000000e+00
104	0.136570	14.644589	0.229097	0.000000e+00
152	0.123187	16.249491	0.254107	0.000000e+00
200	0.122880	16.406268	0.256508	0.000000e+00
248	0.105778	19.034187	0.297559	0.000000e+00
296	0.103103	19.620016	0.306692	0.000000e+00
344	0.100039	20.345915	0.318020	0.000000e+00
392	0.167027	12.261721	0.191650	0.000000e+00
440	0.123123	16.604679	0.259522	0.000000e+00
488	0.109489	19.105620	0.298602	0.000000e+00
536	0.134259	16.057538	0.250958	0.000000e+00
584	0.149958	15.938566	0.249093	0.000000e+00
632	0.104142	19.391704	0.303055	0.000000e+00
680	0.207872	12.100996	0.189113	0.000000e+00
728	0.161297	14.352226	0.224292	0.000000e+00
776	0.155067	18.080718	0.282557	0.000000e+00
824	0.206060	10.860479	0.169721	0.000000e+00
872	0.215803	12.290021	0.192059	0.000000e+00
920	0.155032	20.091048	0.313965	0.000000e+00
968	0.229813	15.787400	0.246710	0.000000e+00
1016	0.128860	16.277703	0.254370	0.000000e+00
1064	0.164445	14.649908	0.228932	0.000000e+00
1112	0.165759	16.590837	0.259261	0.000000e+00
1160	0.250228	12.475773	0.194955	0.000000e+00
1208	0.272997	12.914371	0.201808	0.000000e+00
1256	0.277452	14.282743	0.223190	0.000000e+00
1304	0.290157	15.283763	0.238832	0.000000e+00
1352	0.369762	13.367115	0.208880	0.000000e+00
1400	0.461062	11.902947	0.186000	0.000000e+00
1448	0.434867	13.963023	0.218191	0.000000e+00
1496	0.464644	14.411349	0.225196	0.000000e+00
1544	0.521980	14.103207	0.220380	0.000000e+00
1592	0.560922	14.386554	0.224808	0.000000e+00
1640	0.708245	12.455988	0.194640	0.000000e+00
1688	0.661830	14.534518	0.227119	0.000000e+00
1736	0.692789	15.103519	0.236009	0.000000e+00
1784	0.791428	14.348387	0.224209	0.000000e+00
1832	0.865527	14.207761	0.222011	0.000000e+00
1880	0.935707	14.202456	0.221928	0.000000e+00
1928	1.134491	12.634269	0.197423	0.000000e+00
1976	1.064796	14.491871	0.226450	0.000000e+00

Dimension = 8, 2-3X speed up with OpenMP

Dimension	Time	Gflop/s	GB/s	Error
16	0.675287	2.961708	0.023229	0.000000e+00
64	0.674407	2.965803	0.023193	0.000000e+00
112	0.673867	2.968863	0.023207	0.000000e+00
160	0.685598	2.927432	0.022879	0.000000e+00
208	0.688817	2.926404	0.022869	0.000000e+00
256	1.890362	1.065016	0.008322	0.000000e+00
304	0.689585	2.933359	0.022922	0.000000e+00
352	0.691371	2.901848	0.022675	0.000000e+00
400	0.703620	2.910662	0.022743	0.000000e+00
448	0.753543	2.863766	0.022376	0.000000e+00
496	0.758348	2.896336	0.022630	0.000000e+00
544	0.883106	2.552184	0.019941	0.000000e+00
592	0.816368	2.541437	0.019857	0.000000e+00
640	0.783873	2.675373	0.020903	0.000000e+00
688	0.918932	2.835124	0.022151	0.000000e+00
736	0.842252	2.840161	0.022191	0.000000e+00
784	1.010232	2.862056	0.022362	0.000000e+00
832	0.853035	2.700618	0.021100	0.000000e+00
880	0.961637	2.834632	0.022147	0.000000e+00
928	1.268941	2.519200	0.019683	0.000000e+00
976	1.318440	2.820649	0.022038	0.000000e+00
1024	6.077878	0.353328	0.002761	0.000000e+00
1072	1.037308	2.375234	0.018558	0.000000e+00
1120	1.032731	2.720802	0.021257	0.000000e+00
1168	1.161818	2.742967	0.021431	0.000000e+00
1216	1.379909	2.606036	0.020361	0.000000e+00
1264	1.634329	2.471336	0.019308	0.000000e+00
1312	2.041209	2.212809	0.017288	0.000000e+00
1360	2.161172	2.327863	0.018187	0.000000e+00
1408	2.400715	2.325398	0.018168	0.000000e+00
1456	2.686827	2.297600	0.017951	0.000000e+00
1504	2.680689	2.538207	0.019831	0.000000e+00
1552	2.931487	2.550452	0.019926	0.000000e+00
1600	3.866160	2.118898	0.016555	0.000000e+00
1648	3.524675	2.539701	0.019842	0.000000e+00
1696	4.195636	2.325465	0.018168	0.000000e+00
1744	4.571910	2.320448	0.018129	0.000000e+00
1792	10.329428	1.114212	0.008705	0.000000e+00
1840	5.840228	2.133308	0.016667	0.000000e+00
1888	5.529629	2.434109	0.019017	0.000000e+00
1936	5.786580	2.507980	0.019594	0.000000e+00
1984	7.769315	2.010353	0.015706	0.000000e+00

Dimension	Time	Gflop/s	GB/s	Error
16	1.270115	1.574663	0.012350	0.000000e+00
64	0.338293	5.912501	0.046237	0.000000e+00
112	0.271561	7.367110	0.057588	0.000000e+00
160	0.526615	3.811210	0.029787	0.000000e+00
208	0.275204	7.324591	0.057241	0.000000e+00
256	0.419248	4.802094	0.037526	0.000000e+00
304	0.241864	8.363384	0.065352	0.000000e+00
352	0.335300	5.983453	0.046754	0.000000e+00
400	0.265835	7.704030	0.060197	0.000000e+00
448	0.515761	4.184050	0.032692	0.000000e+00
496	0.345620	6.355055	0.049655	0.000000e+00
544	0.336329	6.701315	0.052360	0.000000e+00
592	0.265757	7.806934	0.060998	0.000000e+00
640	0.335549	6.249912	0.048832	0.000000e+00
688	0.316640	8.227908	0.064286	0.000000e+00
736	0.320676	7.459645	0.058283	0.000000e+00
784	0.420093	6.882624	0.053775	0.000000e+00
832	0.358576	6.424636	0.050196	0.000000e+00
880	0.464202	5.872207	0.045880	0.000000e+00
928	0.536003	5.963987	0.046597	0.000000e+00
976	0.489372	7.599240	0.059373	0.000000e+00
1024	2.124247	1.010939	0.007898	0.000000e+00
1072	0.383259	6.428681	0.050227	0.000000e+00
1120	0.432578	6.495610	0.050750	0.000000e+00
1168	0.413069	7.715004	0.060277	0.000000e+00
1216	0.535309	6.717784	0.052485	0.000000e+00
1264	0.573136	7.047148	0.055059	0.000000e+00
1312	0.641858	7.037086	0.054980	0.000000e+00
1360	0.677492	7.425788	0.058017	0.000000e+00
1408	0.812765	6.868674	0.053664	0.000000e+00
1456	0.888827	6.945391	0.054263	0.000000e+00
1504	1.017436	6.687543	0.052249	0.000000e+00
1552	1.080665	6.918534	0.054053	0.000000e+00
1600	1.135880	7.212032	0.056346	0.000000e+00
1648	1.262057	7.092880	0.055415	0.000000e+00
1696	1.415247	6.894064	0.053862	0.000000e+00
1744	1.432559	7.405541	0.057858	0.000000e+00
1792	2.170155	5.303387	0.041434	0.000000e+00
1840	1.630540	7.641032	0.059698	0.000000e+00
1888	1.891960	7.114166	0.055581	0.000000e+00
1936	1.828915	7.935100	0.061995	0.000000e+00
1984	1.987230	7.859715	0.061406	0.000000e+00

Dimension = 16, 3-4X speed up with OpenMP

Dimension	Time	Gflop/s	GB/s	Error
24	0.813396	2.458829	0.012829	0.000000e+00
72	0.710758	2.814753	0.014669	0.000000e+00
120	0.718212	2.786119	0.014516	0.000000e+00
168	0.699970	2.858649	0.014892	0.000000e+00
216	0.698163	2.886916	0.015039	0.000000e+00
264	0.881138	2.296998	0.011965	0.000000e+00
312	0.736176	2.722864	0.014183	0.000000e+00
360	0.717773	2.860046	0.014898	0.000000e+00
408	0.720334	2.828574	0.014734	0.000000e+00
456	0.760445	2.743147	0.014289	0.000000e+00
504	0.855890	2.393282	0.012466	0.000000e+00
552	0.825347	2.445466	0.012738	0.000000e+00
600	0.799353	2.702184	0.014075	0.000000e+00
648	0.801259	2.716703	0.014150	0.000000e+00
696	0.905430	2.234210	0.011637	0.000000e+00
744	1.009872	2.446830	0.012745	0.000000e+00
792	1.270062	2.346940	0.012224	0.000000e+00
840	0.885952	2.676008	0.013938	0.000000e+00
888	1.049039	2.669975	0.013907	0.000000e+00
936	1.250375	2.623296	0.013664	0.000000e+00
984	1.560699	2.441891	0.012719	0.000000e+00
1032	0.924233	2.378414	0.012388	0.000000e+00
1080	1.150178	2.190464	0.011409	0.000000e+00
1128	1.277386	2.247167	0.011704	0.000000e+00
1176	1.296323	2.509221	0.013069	0.000000e+00
1224	1.450898	2.527769	0.013166	0.000000e+00
1272	1.626818	2.530185	0.013178	0.000000e+00
1320	2.053992	2.239510	0.011664	0.000000e+00
1368	2.077431	2.464686	0.012837	0.000000e+00
1416	2.343204	2.423314	0.012622	0.000000e+00
1464	2.542481	2.468286	0.012856	0.000000e+00
1512	3.019925	2.289228	0.011923	0.000000e+00
1560	3.492286	2.174173	0.011324	0.000000e+00
1608	3.963601	2.097965	0.010927	0.000000e+00
1656	4.106735	2.211639	0.011519	0.000000e+00
1704	4.798105	2.062382	0.010742	0.000000e+00
1752	4.442935	2.420819	0.012609	0.000000e+00
1800	4.809028	2.425438	0.012633	0.000000e+00
1848	5.777779	2.184615	0.011378	0.000000e+00
1896	6.040223	2.256795	0.011754	0.000000e+00
1944	6.005290	2.446723	0.012744	0.000000e+00
1992	6.763231	2.337458	0.012175	0.000000e+00

Dimension	Time	Gflop/s	GB/s	Error
24	0.882566	2.266120	0.011823	0.000000e+00
72	0.285509	7.007168	0.036517	0.000000e+00
120	0.348159	5.747442	0.029945	0.000000e+00
168	0.233665	8.563407	0.044612	0.000000e+00
216	0.293930	6.857214	0.035722	0.000000e+00
264	0.214760	9.424338	0.049093	0.000000e+00
312	0.256104	7.826915	0.040771	0.000000e+00
360	0.217746	9.427807	0.049109	0.000000e+00
408	0.249147	8.177990	0.042598	0.000000e+00
456	0.291291	7.161283	0.037302	0.000000e+00
504	0.369590	5.542315	0.028869	0.000000e+00
552	0.330842	6.100673	0.031777	0.000000e+00
600	0.274100	7.880328	0.041046	0.000000e+00
648	0.294496	7.391555	0.038500	0.000000e+00
696	0.270500	7.478464	0.038953	0.000000e+00
744	0.286605	8.621557	0.044906	0.000000e+00
792	0.371714	8.018965	0.041768	0.000000e+00
840	0.254465	9.316855	0.048528	0.000000e+00
888	0.303491	9.228961	0.048070	0.000000e+00
936	0.347087	9.450367	0.049223	0.000000e+00
984	0.444673	8.570477	0.044640	0.000000e+00
1032	0.255549	8.601902	0.044803	0.000000e+00
1080	0.341998	7.366776	0.038370	0.000000e+00
1128	0.351759	8.160420	0.042504	0.000000e+00
1176	0.433578	7.502138	0.039075	0.000000e+00
1224	0.436560	8.400982	0.043757	0.000000e+00
1272	0.568907	7.235186	0.037684	0.000000e+00
1320	0.555347	8.282998	0.043142	0.000000e+00
1368	0.659023	7.769407	0.040467	0.000000e+00
1416	0.713736	7.955766	0.041438	0.000000e+00
1464	0.745716	8.415501	0.043832	0.000000e+00
1512	0.798455	8.658347	0.045097	0.000000e+00
1560	0.879214	8.635929	0.044980	0.000000e+00
1608	0.940816	8.838597	0.046036	0.000000e+00
1656	1.235467	7.351567	0.038290	0.000000e+00
1704	1.156196	8.558694	0.044578	0.000000e+00
1752	1.298426	8.283522	0.043144	0.000000e+00
1800	1.374715	8.484665	0.044192	0.000000e+00
1848	1.435238	8.794515	0.045806	0.000000e+00
1896	1.488590	9.157349	0.047696	0.000000e+00
1944	1.619400	9.073285	0.047258	0.000000e+00
1992	1.762717	8.968409	0.046711	0.000000e+00

Dimension = 24, 3-5X speed up with OpenMP

Dimension	Time	Gflop/s	GB/s	Error
32	0.956074	2.091917	0.008180	0.000000e+00
64	0.960710	2.081960	0.008137	0.000000e+00
96	0.952737	2.100552	0.008208	0.000000e+00
128	1.009499	1.981858	0.007744	0.000000e+00
160	1.167019	1.719800	0.006719	0.000000e+00
192	0.971785	2.068483	0.008081	0.000000e+00
224	0.960267	2.083396	0.008139	0.000000e+00
256	2.707867	0.743488	0.002905	0.000000e+00
288	1.020651	1.965983	0.007680	0.000000e+00
320	0.999175	2.033294	0.007943	0.000000e+00
352	1.010967	1.984489	0.007753	0.000000e+00
384	1.836977	1.109666	0.004335	0.000000e+00
416	0.997708	2.020388	0.007893	0.000000e+00
448	1.469431	1.468575	0.005737	0.000000e+00
480	1.169823	1.890748	0.007386	0.000000e+00
512	5.632166	0.381289	0.001490	0.000000e+00
544	1.124516	2.004283	0.007830	0.000000e+00
576	1.430076	1.603577	0.006264	0.000000e+00
608	1.137348	1.976138	0.007720	0.000000e+00
640	1.739688	1.205476	0.004709	0.000000e+00
672	1.233588	1.968012	0.007688	0.000000e+00
704	1.053518	1.987135	0.007763	0.000000e+00
736	1.196617	1.999077	0.007809	0.000000e+00
768	3.643825	0.745895	0.002914	0.000000e+00
800	1.025505	1.997066	0.007801	0.000000e+00
832	1.258347	1.830752	0.007152	0.000000e+00
864	1.555849	1.658188	0.006478	0.000000e+00
896	2.380836	1.208522	0.004721	0.000000e+00
928	1.597850	2.000635	0.007815	0.000000e+00
960	1.790594	1.976408	0.007721	0.000000e+00
992	1.958537	1.993715	0.007788	0.000000e+00
1024	5.582990	0.384648	0.001503	0.000000e+00
1056	1.204330	1.955583	0.007639	0.000000e+00
1088	1.369169	1.881306	0.007349	0.000000e+00
1120	1.596739	1.759747	0.006874	0.000000e+00
1152	2.368304	1.291071	0.005043	0.000000e+00
1184	1.696245	1.957026	0.007645	0.000000e+00
1216	2.002805	1.795527	0.007014	0.000000e+00
1248	2.021935	1.922678	0.007511	0.000000e+00
1280	4.729049	0.886923	0.003465	0.000000e+00
1312	2.363791	1.910832	0.007464	0.000000e+00
1344	2.580276	1.881749	0.007351	0.000000e+00
1376	2.869170	1.816055	0.007094	0.000000e+00
1408	4.647836	1.201122	0.004692	0.000000e+00
1440	3.070531	1.944930	0.007598	0.000000e+00
1472	3.699202	1.724429	0.006736	0.000000e+00
1504	3.539385	1.922409	0.007510	0.000000e+00
1536	17.642749	0.410807	0.001605	0.000000e+00
1568	4.145232	1.860027	0.007266	0.000000e+00
1600	4.353016	1.881914	0.007351	0.000000e+00
1632	4.665667	1.863274	0.007279	0.000000e+00
1664	7.120199	1.294189	0.005056	0.000000e+00
1696	5.072206	1.923582	0.007514	0.000000e+00
1728	5.667436	1.820852	0.007113	0.000000e+00
1760	6.437030	1.693879	0.006617	0.000000e+00
1792	12.938544	0.889526	0.003475	0.000000e+00
1824	6.442298	1.883925	0.007359	0.000000e+00
1856	6.946434	1.840781	0.007191	0.000000e+00
1888	6.968640	1.931470	0.007545	0.000000e+00
1920	10.898134	1.298917	0.005074	0.000000e+00
1952	7.731416	1.924024	0.007516	0.000000e+00
1984	8.427673	1.853307	0.007240	0.000000e+00

Dimension	Time	Gflop/s	GB/s	Error
32	0.787206	2.540666	0.009934	0.000000e+00
64	0.399338	5.008686	0.019575	0.000000e+00
96	0.262971	7.610236	0.029737	0.000000e+00
128	0.257243	7.777419	0.030388	0.000000e+00
160	0.333384	6.020205	0.023521	0.000000e+00
192	0.330261	6.086459	0.023779	0.000000e+00
224	0.222161	9.005278	0.035182	0.000000e+00
256	0.707299	2.846412	0.011120	0.000000e+00
288	0.255138	7.864686	0.030725	0.000000e+00
320	0.234120	8.677658	0.033900	0.000000e+00
352	0.303215	6.616600	0.025848	0.000000e+00
384	0.352598	5.781171	0.022585	0.000000e+00
416	0.295927	6.811669	0.026610	0.000000e+00
448	0.352419	6.123313	0.023921	0.000000e+00
480	0.335168	6.599189	0.025780	0.000000e+00
512	2.279720	0.941994	0.003680	0.000000e+00
544	0.316011	7.132182	0.027862	0.000000e+00
576	0.286121	8.014909	0.031310	0.000000e+00
608	0.246539	9.116442	0.035613	0.000000e+00
640	0.310069	6.763498	0.026421	0.000000e+00
672	0.340406	7.131824	0.027860	0.000000e+00
704	0.288455	7.257566	0.028351	0.000000e+00
736	0.292173	8.187364	0.031983	0.000000e+00
768	1.019132	2.666887	0.010418	0.000000e+00
800	0.290438	7.051430	0.027546	0.000000e+00
832	0.369100	6.241448	0.024382	0.000000e+00
864	0.313157	8.238319	0.032182	0.000000e+00
896	0.483891	5.946156	0.023228	0.000000e+00
928	0.455701	7.014938	0.027403	0.000000e+00
960	0.422038	8.385377	0.032756	0.000000e+00
992	0.486458	8.026934	0.031356	0.000000e+00
1024	2.345954	0.915399	0.003576	0.000000e+00
1056	0.332081	7.092149	0.027705	0.000000e+00
1088	0.381706	6.748196	0.026361	0.000000e+00
1120	0.367342	7.649158	0.029880	0.000000e+00
1152	0.388904	7.862216	0.030713	0.000000e+00
1184	0.421931	7.867628	0.030734	0.000000e+00
1216	0.437326	8.222914	0.032122	0.000000e+00
1248	0.488288	7.961549	0.031101	0.000000e+00
1280	1.352635	3.100839	0.012113	0.000000e+00
1312	0.540953	8.349724	0.032617	0.000000e+00
1344	0.577899	8.401870	0.032821	0.000000e+00
1376	0.714113	7.296562	0.028503	0.000000e+00
1408	0.798039	6.995421	0.027326	0.000000e+00
1440	0.669902	8.914694	0.034824	0.000000e+00
1472	0.734405	8.685958	0.033930	0.000000e+00
1504	0.804907	8.453328	0.033022	0.000000e+00
1536	7.526134	0.963012	0.003762	0.000000e+00
1568	0.946782	8.143636	0.031812	0.000000e+00
1600	0.910693	8.995345	0.035139	0.000000e+00
1632	0.968550	8.975701	0.035062	0.000000e+00
1664	1.201415	7.670029	0.029962	0.000000e+00
1696	1.036585	9.412444	0.036768	0.000000e+00
1728	1.178090	8.759570	0.034218	0.000000e+00
1760	1.194521	9.127967	0.035657	0.000000e+00
1792	3.661919	3.142934	0.012277	0.000000e+00
1824	1.276258	9.509683	0.037148	0.000000e+00
1856	1.400707	9.128862	0.035660	0.000000e+00
1888	1.539736	8.741575	0.034147	0.000000e+00
1920	1.845014	7.672448	0.029971	0.000000e+00
1952	1.552848	9.579445	0.037420	0.000000e+00
1984	1.709188	9.138296	0.035697	0.000000e+00

Dimension = 32, 3-9X speed up with OpenMP

Dimension	Time	Gflop/s	GB/s	Error
64	0.724292	2.761535	0.005395	0.000000e+00
128	1.429886	1.399190	0.002733	0.000000e+00
192	1.070441	1.877843	0.003668	0.000000e+00
256	2.430360	0.828382	0.001618	0.000000e+00
320	0.922764	2.201663	0.004300	0.000000e+00
384	1.386455	1.470247	0.002872	0.000000e+00
448	1.012948	2.130386	0.004161	0.000000e+00
512	5.279284	0.406776	0.000795	0.000000e+00
576	1.017699	2.253353	0.004401	0.000000e+00
640	1.373669	1.526679	0.002982	0.000000e+00
704	0.934162	2.241026	0.004377	0.000000e+00
768	2.936789	0.925470	0.001808	0.000000e+00
832	1.030471	2.235601	0.004366	0.000000e+00
896	1.982294	1.451496	0.002835	0.000000e+00
960	1.713681	2.065113	0.004033	0.000000e+00
1024	5.192044	0.413610	0.000808	0.000000e+00
1088	1.160315	2.219937	0.004336	0.000000e+00
1152	2.158947	1.416268	0.002766	0.000000e+00
1216	1.639163	2.193858	0.004285	0.000000e+00
1280	4.455607	0.941354	0.001839	0.000000e+00
1344	2.210375	2.196655	0.004290	0.000000e+00
1408	3.966175	1.407557	0.002749	0.000000e+00
1472	2.917168	2.186714	0.004271	0.000000e+00
1536	16.931201	0.428071	0.000836	0.000000e+00
1600	3.788309	2.162442	0.004224	0.000000e+00
1664	6.404040	1.438918	0.002810	0.000000e+00
1728	4.670170	2.209676	0.004316	0.000000e+00
1792	12.608668	0.912798	0.001783	0.000000e+00
1856	5.789558	2.208607	0.004314	0.000000e+00
1920	9.328971	1.517399	0.002964	0.000000e+00
1984	7.160976	2.181136	0.004260	0.000000e+00

Dimension	Time	Gflop/s	GB/s	Error
64	0.819996	2.439229	0.004765	0.000000e+00
128	0.665788	3.004985	0.005870	0.000000e+00
192	0.329548	6.099619	0.011914	0.000000e+00
256	0.660558	3.047828	0.005953	0.000000e+00
320	0.393540	5.162409	0.010083	0.000000e+00
384	0.460689	4.424741	0.008642	0.000000e+00
448	0.303158	7.118299	0.013903	0.000000e+00
512	2.066655	1.039111	0.002030	0.000000e+00
576	0.358117	6.403594	0.012507	0.000000e+00
640	0.454371	4.615506	0.009015	0.000000e+00
704	0.324438	6.452637	0.012603	0.000000e+00
768	0.873350	3.112050	0.006078	0.000000e+00
832	0.341364	6.748583	0.013181	0.000000e+00
896	0.626081	4.595718	0.008976	0.000000e+00
960	0.468331	7.556506	0.014759	0.000000e+00
1024	2.077933	1.033471	0.002019	0.000000e+00
1088	0.356432	7.226700	0.014115	0.000000e+00
1152	0.586922	5.209629	0.010175	0.000000e+00
1216	0.537024	6.696335	0.013079	0.000000e+00
1280	1.315948	3.187286	0.006225	0.000000e+00
1344	0.651351	7.454404	0.014560	0.000000e+00
1408	1.095961	5.093813	0.009949	0.000000e+00
1472	0.858041	7.434393	0.014520	0.000000e+00
1536	6.754127	1.073086	0.002096	0.000000e+00
1600	1.079119	7.591374	0.014827	0.000000e+00
1664	1.706546	5.399729	0.010546	0.000000e+00
1728	1.399705	7.372671	0.014400	0.000000e+00
1792	3.570467	3.223436	0.006296	0.000000e+00
1856	1.658355	7.710569	0.015060	0.000000e+00
1920	2.489107	5.687091	0.011108	0.000000e+00
1984	2.177625	7.172523	0.014009	0.000000e+00

Dimension = 64, 3-4X speed up with OpenMP
OpenMP speeds up the program compared with just blocking.

3. Improvements are made for both functions

4. Part (b)

- compute.cpp:
  Timing of sqrt, $\sin$ and $\cos$ repeating 1000000000 times with different opt flags (s)

	-O2	-O3
sqrt	6.503751	6.863789
$\sin$	11.672068	11.711119
$\cos$	13.974836	13.927087

  Latency for sqrt, $\sin$ and $\cos$ with different opt flags (cycles)

	-O2	-O3
sqrt	21.462541	22.650973
$\sin$	38.518231	38.647047
$\cos$	46.117384	45.959807

  '-O3' doesn't improve the timing or the latency of the functions compared to '-O2'. In fact, it even makes the latency larger.

- compute-vec.cpp
  The running times of three functions are:
  regular function:1.599532 s
  Explicit vectorization: 1.885009 s
  Implicit vectorization: 2.084650 s
  vectorization doesn't really improve the function, one major reason is that the vector length is too small (VECLEN = 4). Plus, it only allows one AVX vector FMA instruction to execute at once, therefore the processor can;t utilize pipeling and multiple exection units (professor comments from the code).

- compute-vec-pipe.cpp
  The flop rates of three functions when M = 1,4,8,12,16 (Gflop/s)

	M = 1	M = 4	M = 8	M = 12	M = 16
regular	4.493358	5.557254	5.925663	5.918082	8.011525
explicit vec	4.274923	16.268047	31.009102	29.853909	15.605712
implicit vec	4.103285	16.385056	33.832386	31.716805	11.943092

  We can tell that the performance is the best when M ≈ 8. M controls the number of independent FMA instructions that execute at the same time. Therefore when M = 1, we see familiar result in compute-vec.cpp that vectorization doesn't improve the performance. However, as M increases, the number of parallelism is just enough to fill the pipeline so the speed of both vectorization increases compared to regular function. If M is too large, the computation can't fit in registers and the data spills to the L1 cache resulting in slower execution (professor comments from the code).