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Insider Trading and Nonlinear Equilibria: Single Auction Case

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ABSTRACT. – A nonlinear version of the KYLE [1985] model is studied. If linear structure might work for small orders, it would hardly be the case for large orders. No restriction is made neither on the form of equilibrium nor on the probability distribution of the *ex-ante* asset value. Equilibrium is characterized as a fixed point of an operator, which depends only on the distribution of the asset value; that is, the equilibrium is fully determined by the probability distribution of the asset value. A necessary and sufficient condition for the existence of the equilibrium is established. Furthermore, some explicit examples of equilibria are explored. In the simple case of Bernoulli distribution (just good news and bad news), it is shown that there is a unique equilibrium in which the price is strongly nonlinear and has plausible empirical counterparts. The problem is more complex if the *ex-ante* asset value is a continuous random variable. In this case, we restrict ourselves to a class of equilibria in which the price can be obtained explicitly. The existence of a unique equilibrium is then characterized in this class, and this provides KYLE's linear equilibrium as an example. The paper moves to the question on how risk aversion affects the equilibrium. Specifically, we assume that the insider has negative exponential utility, and prove that there exists a unique linear equilibrium, in which both quality of the signal and the initial position play important role. It is not surprising that the price pressure is lower than that in the risk neutral case. As far as the insider's strategy is concerned, insiders taking long-position (such as corporate insiders) want more to sell when the asset price is to go downwards than to buy when the price is to go upwards, and in this case, the price becomes higher than the risk neutral price as long as the aggregated market order is smaller than some sufficiently large number. Naturally, the risk averse equilibrium converges uniformly to the risk neutral equilibrium as the risk aversion rate tends to zero.

Délit d'initié et équilibre non-linéaire : le modèle à une période

RÉSUMÉ. – Nous étudions une version non-linéaire du modèle de KYLE [1985]. Aucune restriction n'a été imposée ni sur la forme de l'équilibre, ni sur la loi de probabilité associée à la valeur de l'actif. On caractérise l'équilibre comme un point fixe d'un opérateur, qui dépend uniquement de la distribution de la valeur de l'actif ; c'est-à-dire, l'équilibre est entièrement déterminé par la loi de probabilité associée à la valeur de l'actif. Nous donnons une condition nécessaire et suffisante pour l'existence d'un équilibre et analysons deux exemples en détail. Nous étudions ensuite les conséquences sur l'équilibre d'un caractère risquophobe de l'initié. Dans le cas où l'utilité de l'initié peut être décrite par une fonction exponentielle négative, nous montrons qu'il existe un unique équilibre dans lequel à la fois la qualité du signal et la position initiale sur le marché de l'initié importent. La quantité échangée dans cette situation est plus importante en cas de mauvaise nouvelle qu'en cas de bonne nouvelle. Le prix est donc plus grand que dans la situation où l'initié est neutre au risque (tant que les ordres passés sur le marché gardent un volume mesuré). Naturellement l'équilibre (avec initié risquophobe) converge uniformément vers celui avec initié risque neutre quand l'aversion pour le risque tend vers zéro.

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1 Introduction

In his seminal paper, KYLE [1985] proposed a model of asset pricing in the presence of new private information, and obtained a unique linear equilibrium in the Gaussian-linear framework: by assuming that the *ex-ante* asset value is Normally distributed and the price is a linear function of the aggregated market order (or equivalently,[1] the insider's demand is a linear function of the asset value). This model is not only economically meaningful, but also mathematically tractable.

If the Gaussian-linear equilibrium might work for small orders, however, it would hardly be the case for large orders. Having realized that the tractability becomes unclear outside Gaussian-linear restriction, extensions have been made so far (to our knowledge) mainly in two ways: first, the family of probability distributions is characterized consistent with linear equilibrium, and second, both linear and Gaussian assumptions are relaxed by introducing new (but reasonable) assumptions. More precisely, we have been naturally asking, after KYLE, the questions such as

(i) Is the Normality assumption necessary for the existence of linear equilibria? (ii) Is the linear equilibrium the only possible one in Normal distribution setting? (iii) Do there exist some nonlinear equilibria which would have plausible empirical counterparts? (iv) Could we relaxe the unrealistic assumption that the informed agent is risk neutral? (v) What are the consequences if the informed agent observes the asset value with some noise? (vi) What happens if some non-cooperative informed agents compete?

Questions (i), (ii) and (iii) are very closely related.The distributional role of the uncertainty is addressed in FOSTER and VISWANATHAN [1993] and BAGNOLI, VISWANATHAN and HOLDEN [1994] in the study of question (i), while ROCHET and VILA [1994] exploit a game theoretic aspect of the problem and show the existence of a unique equilibrium regardless of the probability distribution of the asset value and that of noise trading volume. Their setting, however, is different from KYLE's initial model in that the informed agent can observe the market order before her own order submission.[2]

The major concern of this paper is to answer some of the above questions, especially (ii) and (iii), in a somewhat different manner. Firstly, this paper relaxes both linear and Gaussian assumption in order to study question under KYLE's initial setting: in particular, the informed agent cannot observe the market order. Secondly, the problem is thought of a Stackelberg game (see section 3) between the market-maker and the informed agent. The equilibrium is then understood as a fixed point of an operator and the resulting equilibrium is a perfect Baysian equilibrium. Thirdly, it provides an explicit example of nonlinear equilibrium. Question (iv) is considered with a negative exponential utility function, and an independent Gaussian noise is introduced to answer question (v). Finally question (vi) is the subject of recent papers by

1. The one implies the other.
2. It was shown to be equivalent to assuming that the informed agent is allowed to submit limit orders.

FOSTER and VISWANATHAN [1996] and BACK, CAO and WILLARD [1998], among others.

This paper begins with a brief description of the model in section 2. Section 3 focuses on the characterization of equilibrium and then some explicit example of *nonlinear* equilibria, in a simple context that every agent is risk neutral. Finally, section 4 studies how *risk aversion* affects the equilibrium, and section 5 summarizes the main contributions of the paper.

2 The Model

The market is waiting for a public release of new information. This information reveals the value of a financial asset. The asset value is the price at which the asset will trade after the announcement. We consider, in this paper, the simple situation where there is only one trading opportunity before the announcement. The announcement day is called time 1, and the asset value is denoted by V.

Some financial agents, called market-markers, organize the market and all transactions go through them. They liquidate the orders with their own accounts. There are at least two market-markers in the market, and they are in competition. Therefore, it will be as if there were only one (Bertrand) *representative* market-marker, called simply the market-marker, who is *reasonable* in the sense that he sets the price as the expected asset value given his observation (see the definition of the rational price). We assume that the market-marker is *risk neutral.*[3]

There is a single agent who receives the signal S at time 0. We assume that

$$S = V + \Xi,$$

where Ξ is a Gaussian $\mathcal{N}(0,\eta^2)$ noise, which is independent of all other variables. This agent is referred to as an informed agent or an insider, and is not one of the market-markers. She is aware of her monopolistic informational power and seeks to maximize her wealth using her private observation.

The market-marker knows that there is an insider in the market.[4] He observes the market, especially the order flow because it partially reveals the signal. Hence the insider, who anticipates the adjustment of the price according to the trading quantity, has to be careful to decide on her trading.

A serious game is produced between these two strategic agents. The game, however, is not simple because there exists some agents of another (third) type. They might participate in the market for hedging or liquidity, or other reasons. In any case, the important aspect is that their trading is *independent*

3. As noted in BACK [1994], this assumption is generally accepted: market making firms hold a diversified portfolio and the risk about which there is private information should be idiosyncratic.

4. We may extend this problem to the case where the market-marker is not aware of the existence of the private information. See, for example, EASLEY and O'HARA [1992].

of the signal. They prevent the market-marker from observing directly the insider's trading: we assume that the market-marker cannot identify the insider. Finally, as usual, we call the traders of this type the noise traders. This situation can be summarized as a game between the market-marker and the insider; the aggregated noise trading is just another source of uncertainty.

Let X_t and Z_t denote the amount of the risky asset held by the insider and the noise traders, respectively, at time $t = 0,1$; the trading volumes are then $X \doteq X_1 - X_0$ and $Z \doteq Z_1 - Z_0$. We assume that Z is Normally $(0,\sigma^2)$ distributed, and is independent of the asset value V.

The market-marker observes the sum $Y \doteq X + Z$, but not X and Z separately. Besides this sum, he might observe some other public information. This information is denoted by $\mathcal{F}_0^Y$, and we assume that it is the common knowledge in the market. For notational convenience, we place ourselves on the conditional probability space based on the information $\mathcal{F}_0^Y$. Now, only Y is the new information for the market-marker. The insider cannot observe Y (or equivalently Z) before submitting her own order. It is important to note that there is no informational hierarchy between these two players, and this makes the game more difficult.

The representative market-marker sets a rational price (or Bertrand price) $P_1 = E[V|Y]$. Note that the conditional expectation $E[V|Y]$ is a measurable function, say H, of Y. This function H is referred to as a pricing rule because it determines the price once Y is realized. Similarly, as justified in KYLE, the insider's order quantity is a measurable function of the signal, write $X = \alpha(S)$, and the function $\alpha(\cdot)$ is termed the strategy of the insider. The realization of X is $x = \alpha(s)$, where s is the realization of the signal S.

Notice that the form of function H depends explicitly on the function α (because $Y = \alpha(S) + Z$) and will from to time be denoted H_α (see equation (2)). We refer to this function as the "*α-rational pricing rule*". Similarly, given a function H, a function α^* is called the "*H-optimal strategy*" if it maximizes the utility $u(\cdot)$ of the insider's post-announcement wealth W_{1^+}:

$$E[\,u(W_{1^+}(H,\alpha))\,|\,\mathcal{I}\,] \leqslant E[\,u(W_{1^+}(H,\alpha^*))\,|\,\mathcal{I}\,],$$

for any measurable function α. Here $\mathcal{I}$ is the information of the insider consisting of $\mathcal{F}_0^Y$ and $\{S, X_0\}$. The post-announcement wealth W_{1^+} is

$$W_{1^+}(H,\alpha) = VX_0 + \{V - H(\alpha(S) + Z)\}\,\alpha(S).$$

Indeed, suppose that the insider's portfolio at time t consists of (B_t, X_t), where B_t is the amount invested in the risk-free asset. Her post-announcement wealth is then $W_{1^+} = B_1 + VX_1$, and $B_1 = B_0 - P_1 X$ because she has to pay $P_1 X$ in order to get the position X_1. Thus

$$W_{1^+} = B_0 - P_1 X + VX_1 = B_0 + VX_0 + (V - P_1)X.$$

Without loss of generality, we assume $B_0 = 0$.

DEFINITION 1: A couple of measurable functions (H^*,α^*) is said to be an equilibrium if it satisfies

E1 The market efficiency condition: H^* is the α^*-rational pricing rule.

E2 The insider's optimality condition: α^* is the H^*-optimal order strategy.

Let a couple (H^*,α^*) be an equilibrium. If the informed agent deviates from this equilibrium and uses some other strategy $\tilde{\alpha}$, then the price might no longer be rational:

$$H^*(\tilde{Y}) \neq E[\, V \mid \tilde{Y}\,], \quad \text{where} \quad \tilde{Y} = \tilde{\alpha}(S) + Z.$$

This means that the market-maker, being unaware of the deviation, would incorrectly compute the conditional expectation of V. The equilibrium concept requires that it cannot be profitable for the informed agent to create such pricing errors – the deviation cannot be preferred to α^*. This important remark is due to BACK [1994].

3 Nonlinear Equilibria: Risk Neutral Case

A judicious guess of the pricing is useful to solve the problem. For example, the linear class considered by KYLE is very fruitful. This methodology, however, is not convenient to solve the problem in a general way.

We can consider our problem as a classical Stackelberg game: either the insider or the market-maker moves first, and the other adjusts optimally. In our case, it would be natural to consider the insider as the leader of the game. She computes first the α-rational pricing rule, denoted by H_α, for any given α, in order to obtain the family of couples (H_α,α) verifying the market efficiency condition. Next, she determines α^*, by restricting herself to the family of couples (H_α,α) obtained before, so that (H_{α^*},α^*) satisfies the optimality condition as well; consequently (H_{α^*},α^*) is an equilibrium.

For simplicity, we assume in this section that the insider is risk neutral: the utility function is $u(w) = w$ for all w. Notice that, in this case, the initial position X_0 as well as the noise Ξ is irrelevant: without loss of generality, we assume that $X_0 = 0$ and $\mathrm{Var}(\Xi) \doteq \eta^2 = 0$. In particular, $X = \alpha(V)$ and the insider's information $\mathcal{I}$ is just the realization v of V.

To obtain the α-rational pricing H_α for a given strategy α, let us use Bayes' formula, which yields (see Appendix A):

$$\text{(1)} \qquad H_\alpha(y) = E[\, V \mid \alpha(V) + Z = y] = \frac{E[\, V f_Z(y - \alpha(V))\,]}{E[\, f_Z(y - \alpha(V))\,]},$$

where f_Z is the probability density function of Z. Given that Z is Normally $(0,\sigma^2)$ distributed, the α-rational pricing rule is

$$H_\alpha(y) = \frac{E\,[\,V\exp\{\frac{1}{\sigma^2}(\alpha(V)y - \alpha^2(V)/2)\}\,]}{E\,[\exp\{\frac{1}{\sigma^2}(\alpha(V)y - \alpha^2(V)/2)\}\,]}, \tag{2}$$

which classifies the family of couples (H_α,α) verifying E1.

The next step is to determine α^* so that the couple (H_{α^*},α^*) satisfies E2 as well. That is, we want to find α^* such that

α^*=Argmax$_\alpha$ $E[W_{1+}(H_{\alpha^*},\alpha)]$, where $W_{1+}(H_{\alpha^*},\alpha) = \alpha(v)\{v - H_{\alpha^*}(\alpha(v)+Z)\}$.

Hereafter, H replaces H_α for notational convenience. Notice that H given in (2) is sufficiently smooth so that we can exchange the order of the differential operator and the expectation. The first order condition for a strategy α $(x := \alpha(v))$ to be H-optimal is that it solves the equation

$$\frac{d}{dx}E[W_{1+}(H,x)] = E[v - H(x+Z) - xH'(x+Z)] = 0.$$

Similarly, we can obtain the second order condition, and hence the following optimality criterion whose elementary proof is omitted.

LEMMA 2: If α is H-optimal,then $x = \alpha(v)$ verifies the following conditions:
C1 $E[\,H(x+Z) + xH'(x+Z)\,] = v, \quad \forall v \in \mathrm{Supp}\{V\}$.
C2 $E[\,2H'(x+Z) + xH''(x+Z)\,] > 0$.
Conversely, if $x = \alpha(v)$ is a unique solution to C1 and verifies C2 as well, then α is a unique H-optimal strategy.

The following is a simpler version of the above classical optimal criterion (the proof can be found in Appendix A).

LEMMA 3: Let α be a smooth function of v. Then α is a unique H-optimal strategy if and only if $x = \alpha(v)$ verifies C1 and α is a strictly increasing function on the support of V.

For computational simplicity it is very useful to rewrite C1 without the derivative function H': by changing the probability (or variable),

$$E[H(x+Z)] = E[H(Z)\varepsilon^{xZ-x^2/2}], \text{ where } \varepsilon \doteq e^{1/\sigma^2},$$

by deriving with respect to x and by changing once again the probability,

$$E[H'(x+Z)] = \frac{1}{\sigma^2}E[(Z-x)H(Z)\,\varepsilon^{xZ-x^2/2}] = \frac{1}{\sigma^2}E[ZH(x+Z)],$$

therefore, we have the following equivalent expression for C1

$$\mathbf{C1'}\;\; E[(1 + xZ/\sigma^2)H(x+Z)] = v, \quad \forall v \in \mathrm{Supp}\{V\}.$$

We summerize in the following:

PROPOSITION 4: A pair (H^*, α^*) is an equilibrium if, and only if,
1. α^* solves the equation C1$'$.
2. α^* is strictly increasing on the support of V.
3. H^* is as given in (2) with $\alpha = \alpha^*$.

Despite of this simplification, it is yet too complex to determine H-optimal strategy for a pricing rule H of the form (2): we need to go further the computation of H, and it is possible in particular when (i) V is a discrete variable, or (ii) V is an "*appropriate*" function of a Gaussian variable and α is a suitable function of V. We consider, first, the simple case where V is a discrete variable with two values, and then, consider a class of continuous distributions with convex support.

3.1 Case of a Two-Point Distribution

Suppose that there are two numbers v_1 and v_0 (good news and bad news, respectively) with $v_0 < v_1$ such that

$$(3) \qquad P[V = v_1] = \pi_1 \ \ (0 < \pi_1 < 1) \quad \text{and} \quad P[V = v_0] = 1 - \pi_1 \doteq \pi_0.$$

Set $\alpha(v_i) \doteq \alpha_i$, $i = 0, 1$, so that the informed trading quantity can be written as

$$(4) \qquad \alpha(V) = \alpha_1 \mathbf{1}_{\{V=v_1\}} + \alpha_0 \mathbf{1}_{\{V=v_0\}}.$$

Then by (2), the α-rational pricing is

$$(5) \qquad H(y) = \frac{\pi_1 v_1 \varepsilon^{\alpha_1 y - \alpha_1^2/2} + \pi_0 v_0 \varepsilon^{\alpha_0 y - \alpha_0^2/2}}{\pi_1 \varepsilon^{\alpha_1 y - \alpha_1^2/2} + \pi_0 \varepsilon^{\alpha_0 y - \alpha_0^2/2}}$$
$$= \frac{\pi_1 v_1 \varepsilon^{(\alpha_1 - \alpha_0) y - (\alpha_1^2 - \alpha_0^2)/2} + \pi_0 v_0}{\pi_1 \varepsilon^{(\alpha_1 - \alpha_0) y - (\alpha_1^2 - \alpha_0^2)/2} + \pi_0}.$$

In order to determine an H-optimal strategy, we apply the first order condition C1$'$ for $v \in \{v_1, v_0\}$:

$$(6) \qquad \begin{aligned} v_1 &= E[(1 + \alpha_1 Z/\sigma^2) H(\alpha_1 + Z)] \quad \text{(case of good news)},\\ v_0 &= E[(1 + \alpha_0 Z/\sigma^2) H(\alpha_0 + Z)] \quad \text{(case of bad news)}. \end{aligned}$$

Set

$$L^1 \doteq \varepsilon^{(\alpha_1 - \alpha_0) Z + (\alpha_1 - \alpha_0)^2/2} \text{ and } L^0 \doteq \varepsilon^{-(\alpha_1 - \alpha_0) Z + (\alpha_1 - \alpha_0)^2/2}.$$

Then according to (5), we have

$$(7) \quad H(\alpha_1 + Z) = \frac{\pi_1 v_1 \varepsilon^{(\alpha_1 - \alpha_0) Z + (\alpha_1 - \alpha_0)^2/2} + \pi_0 v_0}{\pi_1 \varepsilon^{(\alpha_1 - \alpha_0) Z + (\alpha_1 - \alpha_0)^2/2} + \pi_0} = \frac{\pi_1 v_1 L^1 + \pi_0 v_0}{\pi_1 L^1 + \pi_0}$$

and

$$H(\alpha_0 + Z) = \frac{\pi_1 v_1 \varepsilon^{(\alpha_1-\alpha_0)Z-(\alpha_1-\alpha_0)^2/2} + \pi_0 v_0}{\pi_1 \varepsilon^{(\alpha_1-\alpha_0)Z-(\alpha_1-\alpha_0)^2/2} + \pi_0} = \frac{\pi_1 v_1 + \pi_0 v_0 L^0}{\pi_1 + \pi_0 L^0}.$$

Notice that by (7),

$$H(\alpha_1 + Z) = v_1 - \pi_0(v_1 - v_0)\frac{1}{\pi_1 L^1 + \pi_0}.$$

Since $\pi_0 \neq 0$ and $v_1 - v_0 \neq 0$, condition (6) becomes[5]

$$E\left[\frac{1+\alpha_1 Z/\sigma^2}{\pi_1 L^1 + \pi_0}\right] = 0 \quad \text{and} \quad E\left[\frac{1+\alpha_0 Z/\sigma^2}{\pi_1 + \pi_0 L^0}\right] = 0. \tag{8}$$

By solving now the system of two equations (with two unknowns α_0 and α_1) we obtain the following result.

PROPOSITION 5: Assume (3). In order that the strategy α in (4) be H_α-optimal, it is necessary that

$$\alpha_1 \cdot \alpha_0 = -\sigma^2. \tag{9}$$

A unique equilibrium is the pair (H^*, α^*) of functions given by[6]

$$H^*(y) = \frac{\pi_1 v_1 \varepsilon^{(\alpha_1^*+\sigma^2/\alpha_1^*)y-(\alpha_1^{*2}-\sigma^2/\alpha_1^{*2})/2} + \pi_0 v_0}{\pi_1 \varepsilon^{(\alpha_1^*+\sigma^2/\alpha_1^*)y-(\alpha_1^{*2}-\sigma^2/\alpha_1^{*2})/2} + \pi_0},$$

with $\varepsilon \doteq \exp\{\sigma^{-2}\}$, and

$$\alpha^*(v) = \alpha_1^* \mathbf{1}_{\{v=v_1\}} + \alpha_0^* \mathbf{1}_{\{v=v_0\}},$$

where α_1^* is a unique positive solution of the equation (in x)

$$E\left[\frac{1+xZ/\sigma^2}{\pi_1 \varepsilon^{(x+\sigma^2/x)Z+(x+\sigma^2/x)^2/2} + \pi_0}\right] = 0, \tag{10}$$

and then α_0^* can be determined by (9).

In the symmetric case of $\pi_1 = \pi_0 = 0.5$, one would intuitively expect $\alpha_1^* = -\alpha_0^* = \sqrt{\sigma^2}$. Indeed, we can easily check that this is actually the case.[7]

5. Here we have justified the first equality. To prove the second equality, observe in (7) and (8) that $H(\alpha_0 + Z)$ is obtained from $H(\alpha_1 + Z)$ by the exchanges $\alpha_1 \to \alpha_0$, $\pi_1 \to \pi_0$ and $v_1 \to v_0$.

6. See Appendix B (Figures 3 and 4) for the graphical presentation.

7. It is clear that $\alpha_1^* = -\alpha_0^* = \sqrt{\sigma^2}$ verifies (9). Furthermore, by plugging $\pi_1 = \pi_0 = 0.5$ and $x = \alpha_1^* = \sqrt{\sigma^2}$ into (10), we see that the left-hand side of equation (10) is the integral on the whole line of an (integrable) odd function.

Naturally, the higher the variance of the noise trading is, the deeper the market is, and thus the larger the optimal informed trading quantity is (see Figure 4). Note also that α_1^* (nor α_0^*) does not depend on (v_1,v_0). In particular, the optimal trading strategy is not a function of the difference between the signal and the expectation of the *ex-ante* asset value, which is usually assumed in the literature. Of course, the post-announcement wealth W_{1+} depends on the difference $v_1 - v_0$.

3.2 Case of Continuous Distribution

We assume now that the asset value V is an absolutely continuous variable with convex support, so that the inverse of its distribution function F^{-1} is well-defined (and continuous) on the interval $(0,1)$. Then, the function $F^{-1} \circ N$, where N is the distribution function of a Normal random variable, is well-defined on $\mathbf{R}$ and strictly increasing. Therefore, our hypothesis can be equivalently stated as follows.

HYPOTHESIS 6: There is a continuous and strictly increasing function h for which the asset value writes $V = h(\Theta)$ for a standard Normal variable Θ.

As noted before, it is not easy to compute H-optimal strategy for H given in (2). In order to obtain sufficiently simple expression for the pricing rule given in (2), let us begin by restricting the informed trading strategies.That is, we assume that there is an affine function $l(\theta) = a\theta + b$ $(a > 0)$ such that $\alpha \circ h = l$, or equivalently,

$$\alpha(V) = ah^{-1}(V) + b, \quad \text{for} \quad (a,b) \in \mathbf{R}_+ \times \mathbf{R}. \tag{11}$$

What we study hereafter is the existence of equilibria in which the order strategy α is of the form (11): such equilibrium will be termed a *quasilinear* equilibrium.

Recall that $Y = \alpha(V) + Z$ and so for α of the form (11), $Y = a\Theta + b + Z$, thus (Θ,Y) is a Gaussian vector. Hence the conditional distribution of Θ given $Y = y$ is $\mathcal{N}(\hat{m},\hat{\sigma}^2)$ where

$$\hat{m} = \zeta(y - b), \quad \text{with} \quad \zeta = \frac{a}{a^2 + \sigma^2} \quad \text{and} \quad \hat{\sigma}^2 = \frac{\sigma^2}{a^2 + \sigma^2}. \tag{12}$$

The α-rational pricing rule H is then

$$H(y) \doteq E[h(\Theta)|Y = y] = E[h(\zeta y + \Sigma)], \quad \text{where } \Sigma \sim \mathcal{N}(-\zeta b,\hat{\sigma}^2). \tag{13}$$

It remains now to check that the above pricing admits the corresponding optimal strategy of the form (11). In other words, by Lemma 3, a quasilinear equilibrium occurs if and only if a strategy α of the form (11) verifies condition C1 for a given pricing rule of the form (13).

PROPOSITION 7: Under Hypothesis 6, there is a unique quasilinear equilibrium if and only if there is a linear function $l(\theta) = a\theta + b$ $(a > 0)$ for which the function h verifies

$$(14) \quad E[h(\zeta l(\theta) + \tilde{\Sigma}) + \zeta l(\theta) \cdot h'(\zeta l(\theta) + \tilde{\Sigma})] = h(\theta), \text{ for any } \theta \in \mathbf{R},$$

where $\tilde{\Sigma}$ is a $\mathcal{N}(-\zeta b, \zeta^2\sigma^2 + \hat{\sigma}^2)$ random variable, for ζ and $\hat{\sigma}^2$ given in (12). If this is the case, the couple $(H^*, \alpha^*) = (H, \alpha)$ with H in (13) and α in (11) forms a unique quasilinear equilibrium.

Suppose that the *ex-ante* asset value is Normally distributed, or equivalently, h is a linear function.[8] Then, we can easily show that KYLE's equilibrium is the unique quasilinear equilibrium:

COROLLARY 8: If v is a Gaussian random variable, then there exists a unique quasilinear equilibrium, and this is exactly KYLE's linear equilibrium (see Proposition 9 with $\eta^2 = 0$).

Let us consider now some nonNormal distributions in order to obtain some nonlinear equilibrium. A plausible candidate would be a Lognormal distribution, in which case, h is the exponential function. Unfortunately, the exponential function does not satisfy condition (14). Indeed,

$$E[h(\zeta l(\theta) + \tilde{\Sigma}) + \zeta l(\theta) \cdot h'(\zeta l(\theta) + \tilde{\Sigma})] = \{1 + \zeta l(\theta)\} e^{\zeta\{l(\theta) - b\} + (\zeta^2\sigma^2 + \hat{\sigma}^2)/2},$$

and it can not be of the form e^{θ}, for any $\theta \in \mathbf{R}$. This would imply that there is no explicitely-expressed equilibrium when V is Lognormally distributed.

4 Risk Aversion Versus Risk Neutral

To see how risk aversion affects the equilibrium, for tractability reason, assume that the asset value V is Normally (m, Γ) distributed. Assume further that the price is a linear function of Y. We allow, in this section, the possibility that the insider receives a noisy signal $S = V + \Xi$, where Ξ is $\mathcal{N}(0, \eta^2)$ with $0 \leqslant \eta^2 \leqslant \Gamma$.

Consider first risk neutral case. A risk neutral informed agent maximizes the expected value of the wealth. For arbitrarily given linear pricing rule $H(y) = \lambda y + \mu$, with $\lambda > 0$, the value function of the insider is

$$J(s) \doteq \sup_{x \in \mathbf{R}} E[W_{1^+} \mid V = s - \Xi] = \sup_{x \in \mathbf{R}} x(s - \lambda x - \mu).$$

8. Recall that h is assumed continuous and strictly increasing. Without this assumption, we can find a counter example: $h(x) = \mathbf{1}_{[-1,1]}(x) - \mathbf{1}_{\mathbf{R}\setminus[-1,1]}(x)$.

The unique H-optimal strategy, denoted by α_H, is thus

$$\alpha_H(s) = \phi(s - \mu), \quad \text{where } \phi = 1/2\lambda, \tag{15}$$

and this classifies the family of couples (H,α_H) verifying E2.

The next step is to determine explicitly H, so that the couple (H,α_H) verifies E1 as well. Notice that (V,Y_H) with $Y_H = \alpha_H(V) + Z$ is a Gaussian vector, thus

$$E[V|Y_H = y] = \frac{\Gamma\phi}{\phi^2(\Gamma + \eta^2) + \sigma^2}\{y - \phi(m - \mu)\} + m.$$

On the other hand, as $E[V|Y_H = y] = H(y)$ for any $y \in \mathbf{R}$, we have

$$\lambda = \frac{\Gamma\phi}{\phi^2(\Gamma + \eta^2) + \sigma^2} \quad \text{and} \quad \mu = m - \lambda\phi(m - \mu). \tag{16}$$

Now, by solving the system of equations (15) and (16), we obtain $\mu = m$, while λ and ϕ are given in the following proposition (some additional properties are given in Appendix C).

PROPOSITION 9: Assume that V is a Gaussian variable with mean m and variance Γ. Then the couple (H^*,α^*) defined by

$$H^*(Y) = \lambda^* Y + m \quad \text{and} \quad \alpha^*(s) = \phi^*(s - m),$$

with

$$2\lambda^* = \sqrt{(\Gamma - \eta^2)/\sigma^2} \quad \text{and} \quad 2\lambda^*\phi^* = 1$$

is a unique linear equilibrium.

As η^2 increases to Γ, the insider observes nothing more than the market-maker: the price pressure λ approaches to zero as if there were no informed agent, hence the insider trading quantity tends to infinity. This strange equilibrium property is due to the unrealistic assumption that the insider is risk neutral. Let us now study the case where the informed agent is risk averse: specifically, consider a negative exponential utility function:

$$u(W) = \gamma\exp\{\gamma W\}, \quad \text{with } \gamma < 0, \tag{17}$$

and the absolute value $|\gamma|$ is termed the risk aversion rate. The informed agent maximizes

$$E\left[\gamma\exp\{\gamma W_{1^+}\}\,|S{=}s, X_0{=}x_0\right], \text{where } W_{1^+} = VX_0 + \{V - H(X+Z)\}X.$$

For a given linear pricing $H(y) = \lambda y + \mu$, the value function is

$$J(s) \doteq \sup_{x \in \mathbf{R}} E\left[\gamma \exp\{\gamma(Vx_0 + (V - P_1)x)\} \mid V = s - \Xi\right]$$

$$= \gamma \inf_{x \in \mathbf{R}} E\left[\exp\{\gamma(x(s - \lambda x - \mu) + x_0 s) - \gamma((x_0 + x)\Xi + \lambda x Z)\}\right]$$

$$= \gamma \inf_{x \in \mathbf{R}} \exp\left[\gamma\{x(s - \lambda x - \mu) + x_0 s\} + \frac{\gamma^2}{2}\{(x_0 + x)^2\eta^2 + \sigma^2\lambda^2 x^2\}\right].$$

Thus, the unique H-optimal strategy is

$$(18) \quad \alpha_H(s) = \operatorname{argmax}_x \left[x(s - \lambda x - \mu) + x_0 s + \frac{\gamma}{2}\{(x_0 + x)^2\eta^2 + \sigma^2\lambda^2 x^2\}\right]$$

$$= \phi(s - \mu + \gamma\eta^2 x_0), \text{ with } \phi^{-1} = 2\lambda - \gamma(\eta^2 + \sigma^2\lambda^2),$$

because the second order condition $\gamma(\eta^2 + \sigma^2\lambda^2) - 2\lambda < 0$ is verified for $\lambda > 0$.

However, a crucial difficulty remains for the second step: in general, the market-maker cannot observe the insider's initial position X_0. Despite unrealistic aspect, let us assume that $X_0 = x_0$ is a common knowledge.

PROPOSITION 10: Assume that V is a Gaussian (m, Γ) random variable, and that the informed agent has a utility function given in (17). Define

$$H_\gamma^*(Y) = \lambda_\gamma^* Y + m - \frac{\gamma\eta^2 x_0 \lambda_\gamma^* \phi_\gamma^*}{1 - \lambda_\gamma^* \phi_\gamma^*} \quad \text{and}$$

$$\alpha_\gamma^*(s) = \phi_\gamma^* \left\{ s - \left(m - \frac{\gamma\eta^2 x_0}{1 - \lambda_\gamma^* \phi_\gamma^*} \right) \right\},$$

where ϕ_γ^* is a unique strictly positive solution of the following equation:

$$\gamma\eta^2(\Gamma + \eta^2)^2\phi^5 - (\Gamma + \eta^2)(\Gamma - \eta^2)\phi^4$$

$$+ \gamma\sigma^2\left\{\Gamma^2 + 2\eta^2(\Gamma + \eta^2)\right\}\phi^3 + 2\sigma^2\eta^2\phi^2 + \gamma\sigma^4\eta^2\phi + \sigma^4 = 0,$$

and

$$\lambda_\gamma^* = \Gamma\phi_\gamma^* \Big/ \left\{\phi_\gamma^{*2}(\Gamma + \eta^2) + \sigma^2\right\}.$$

Then, the couple $(H_\gamma^*, \alpha_\gamma^*)$ is a unique linear equilibrium. Moreover, this equilibrium converges to the risk neutral equilibrium (given in Proposition 9) as γ tends to zero.[9]

9. As γ approaches to zero, the utility function $u(W) = \gamma\exp\{\gamma W\}$ converges to an affine function. That is, the insider becomes less and less risk averse.

Note that $y = 0$ does not imply $P_1 = E[V]$ and that α^* is not of the form $\alpha^*(s) = \phi(s - m)$ unless $x_0 = 0$ (see Appendix C). It is easy to check that ϕ_γ^* is increasing from zero to ϕ^* (that of risk neutral case, given in Proposition 9) as γ increases from $-\infty$ to zero. In particular, the trading quantity diminishes (towards zero) as the risk aversion rate increases (towards infinity), hence the price pressure λ_γ^* approaches to zero (and to λ^* as aversion rate decreases towards zero, that is, $\gamma \to 0$). By contrast to the risk neutral case, the trading quantity does not explode even though the price pressure is very small.

Note also that if the insider observes the asset value without noise, that is, if $\eta^2 = 0$, then the initial position X_0 becomes irrelevant. However, if this is not the case, the problem could not be solved in a real sense: we made an unrealistic assumption that the market-maker observes X_0. Although, we could solve in a quite straightforward way by assuming that X_0 is Normally distributed and independent of all other variables, the question remains an open issue.

5 Concluding Remarks

This paper analyzed some questions arising from the paper of KYLE [1985], which has been a standard model in the study of microstructure of financial markets in the presence of asymmetric information. Relying on the KYLE's original setting, we investigated the existence of nonlinear equilibria.

No restriction is made neither on the form of equilibrium nor on the probability distribution of the *ex-ante* asset value. Equilibrium is then understood as a fixed point of an operator, and a necessary and sufficient condition for the existence of an equilibrium is specified. However, we were not able to prove neither existence nor unicity of the equilibrium in this general setting.

A unique nonlinear equilibrium is explicitly constructed in the simple case where the asset value takes only two values (good news and bad news). In particular, the pricing rule is strongly nonlinear, and matches quite well with the recent empirical research of KEMPF and KORN [1998], who show that the price pressure is not constant but is decreasing with respect to the bias of bid and ask orders. Moreover, we observe that the optimal trading strategy is not a (linear) function of the difference between the signal and the expectation of the *ex-ante* asset value, which is usually assumed in the literature.

In the case where the *ex-ante* asset value is a variable with convex support, we specify a condition for a probability distribution of the asset value to admit a unique quasilinear equilibrium. The Normal distribution is the only example that we have found. This is quite different from the continuous-time model studied by BACK [1992] and CHO [1997]: there exists a unique nonlinear (quasilinear) equilibrium for any distribution of convex support, such as Lognormal or Uniform distribution. As noted in BACK [1992], a key aspect of the continuous-time model is that the informed agent can move continuously up or down the residual supply curve. This is possible because the insider can observe the noise trading by inverting the current price, and it makes the insider's information strictly superior to the market-maker's. Another interesting comparison would be possible with ROCHET and VILA [1994] who prove the existence of a unique equilibrium in a "*one shot*" trading model, assuming that the informed agent can observe the noise trading quantity before her order submission. The similarity of the continuous-time model and ROCHET and VILA's setting is that the insider observes the noise trading. Presumably the discretized version of the continuous-time model would imply ROCHET and VILA's equilibrium rather than KYLE's original one.

Finally, we treated the risk aversion problem in which the insider observes asset value with some noise and proved the existence of a unique linear equilibrium. As expected, it converges to KYLE's linear equilibrium as the insider becomes less and less risk averse. ■

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APPENDIX A

Proof of equation (1)

The Bayes rule says that

$$E[V|Y=y] \doteq \int_{\mathbf{R}} v\, f_{V|Y}(v|y)dv = \frac{\displaystyle\int_{\mathbf{R}} v\, f_{Y|V}(y|v) f_V(v)dv}{\displaystyle\int_{\mathbf{R}} f_{Y|V}(y|v) f_V(v)dv},$$

where $f_{Y|V}$ is the conditional density function of Y given V and f_V the marginal density function of V. Note that

$$F_{Y|V}(y|v) \doteq [Y \leqslant y | V = v] = [Z \leqslant y - \alpha(v)] \doteq F_Z(y - \alpha(v)),$$

because $Y = \alpha(V) + Z$ and V and Z are independent. Therefore, $f_{Y|V}(y|v) = f_Z(y - \alpha(v))$, which implies the desired result.

Proof of Lemma 3

Suppose that $x = \alpha(v)$ verifies C1:

$$E\left[H(\alpha(v) + Z) + \alpha(v)\, H'(\alpha(v) + Z)\right] = v, \quad \text{for} \quad v \in \text{Supp}\{V\}.$$

Then by differentiating this with respect to v in the interior of Supp$\{V\}$ (in the case where V has finite support, the function α is a smooth extension of the original function on the smallest interval that contains the support of V), we have

$$\text{(19)} \qquad \alpha'(v) \cdot E\left[2H'(\alpha(v) + Z) + \alpha(v)\, H''(\alpha(v) + Z)\right] = 1.$$

Therefore, $x = \alpha(v)$ verifies C2 if and only if $\alpha'(v) > 0$, *i.e.* α is a strictly increasing function of v.

Suppose further that α is an increasing function of v on the support of V. It suffices then to show, according to Lemma 2, that for each v, $x = \alpha(v)$ is the unique solution of C1. To do this, define

$$f_H(x) \doteq E\left[H(x + Z) + x\, H'(x + Z)\right].$$

Then, $x = \alpha(v)$ is a solution of $f_H(x) = v$, for any $v \in \text{Supp}\{V\}$. This implies that the range of the function f_H contains the support of V. Moreover, by (19),

$$f_H'(x) = E\left[2H'(x + Z) + x\, H''(x + Z)\right] > 0, \quad \text{for} \quad x \in f_H^{-1}[\text{Supp}\{V\}],$$

which implies that f_H is strictly increasing. Therefore, the uniqueness is established.

Proof of Proposition 5

To solve system (8), write

$$E\left[\frac{1+\alpha_0 Z/\sigma^2}{\pi_1+\pi_0 L^0}\right]=E\left[\frac{1+\alpha_0 Z/\sigma^2}{\pi_1 \varepsilon^{(\alpha_1-\alpha_0)Z-(\alpha_1-\alpha_0)^2/2}+\pi_0}\varepsilon^{(\alpha_1-\alpha_0)Z-(\alpha_1-\alpha_0)^2/2}\right]$$

The effect of the last factor in the expectation is to replace Z by $Z+\alpha_1-\alpha_0$ in the numerator and denominator of the fraction, so this is equal to

$$E\left[\frac{1+\alpha_0\{Z+(\alpha_1-\alpha_0)\}/\sigma^2}{\pi_1 \varepsilon^{(\alpha_1-\alpha_0)Z+(\alpha_1-\alpha_0)^2/2}+\pi_0}\right]=E\left[\frac{1+\alpha_0(\alpha_1-\alpha_0)/\sigma^2+\alpha_0 Z/\sigma^2}{\pi_1 L^1+\pi_0}\right].$$

We now conclude that

$$E\left[\frac{1+\alpha_0(\alpha_1-\alpha_0)/\sigma^2+\alpha_0 Z/\sigma^2}{\pi_1 L^1+\pi_0}\right]=0=E\left[\frac{1+\alpha_1 Z/\sigma^2}{\pi_1 L^1+\pi_0}\right],$$

or more conveniently,

$$E\left[\frac{\alpha_1+\alpha_0\alpha_1(\alpha_1-\alpha_0)/\sigma^2+\alpha_0\alpha_1 Z/\sigma^2}{\pi_1 L^1+\pi_0}\right]=E\left[\frac{\alpha_0+\alpha_0\alpha_1 Z/\sigma^2}{\pi_1 L^1+\pi_0}\right].$$

This equality holds if, and only if,

$$\alpha_1+\alpha_0\alpha_1(\alpha_1-\alpha_0)/\sigma^2=\alpha_0,$$

because $E[1/(\pi_1 L^1+\pi_0)]$ never vanishes. By doing some elementary algebra, as $\alpha_0 \neq \alpha_1$ (indeed, $\alpha_0 < \alpha_1$ by Lemma 3), we see that relation (9) is a necessary condition for optimality. Hence, α_1^* is, if it exists, a solution of (10). Equation (10) cannot be solved analytically, so we use a numerical approach, which indicates the existence of a unique solution. In addition, we can remark that the solution is indeed the maximum of $E[W_{1+}(H,\alpha)]$ (second order condition). Numerical results and some discussions are reported in Appendix B.

Proof of Proposition 7

As it is already shown that the α-rational pricing rule is the function H given in (13), for α of the form (11), it remains to prove that the couple (H,α) satisfies the optimality condition E2 as well. Applying Lemma 3, as the function α is strictly increasing in v, there is a unique quasilinear equilibrium if and only if (H,α) verifies the condition C1. As

$$H(x+z)=E\left[h\left(\zeta x+\zeta z+\Sigma\right)\right]=\int_{\mathbf{R}} h(\zeta x+\zeta z+u)f_\Sigma(u)du,$$

where f_Σ is the probability density function of Σ, we have

$$E[H(x+Z)]=\int_{\mathbf{R}^2} h(\zeta x+\zeta z+u)f_\Sigma(u)f_Z(z)dudz=E\left[h(\zeta x+\tilde{\Sigma})\right],$$

and thus $E[H'(x+Z)] = E[\zeta\, h'(\zeta x + \tilde{\Sigma})]$. Plugging these into the equation in C1, we obtain(14).

Proof of Corollary 8

Suppose that V is Normally distributed, or equivalently, h is an affine function. It is easy to check then that h satisfies condition (14), and hence, by Proposition 7, we have the existence of a unique quasilinear equilibrium. Indeed, for any linear function $h(\theta) = c\,\theta + d$ $(c > 0)$, condition (14) is verified by the linear function $l(\theta) = a\theta + b$ $(a > 0)$ with $a = \sqrt{\sigma^2}$ and $b = 0$. This implies that $\zeta = 1/(2\sqrt{\sigma^2})$ and thus the equilibrium pricing rule is, by (13),

$$H(y) = \lambda y + d, \quad \text{with} \quad \lambda = c/(2\sqrt{\sigma^2}),$$

and the equilibrium strategy is, by (11),

$$\alpha(v) = ah^{-1}(v) + b = \phi(v - d), \quad \text{with} \quad \phi = a/c = 1/2\lambda,$$

Finally, suppose that V is Normally (m,Γ) distributed. Then, we can write $V = h(\Theta)$, where $h(\theta) = \sqrt{\Gamma}\,\theta + m$, and in this case, we have KYLE's linear equilibrium.

Proof of Proposition 10

Notice that (V, Y_H), with $Y_H = \phi^*(V + \Xi - \mu + \gamma\eta^2 x_0) + Z$, is a Gaussian vector. Thus, the conditional distribution of V given $Y_H = y$ is Gaussian with conditional mean

$$E[V|Y_H = y] = \frac{\Gamma\phi}{\phi^2(\Gamma + \eta^2) + \sigma^2}\left\{y - \phi(m - \mu + \gamma\eta^2 x_0)\right\} + m.$$

Equating this with the pricing $H(y) = \lambda y + \mu$, we have

$$\lambda = \frac{\Gamma\phi}{\phi^2(\Gamma + \eta^2) + \sigma^2} \quad \text{and} \quad \mu = m - \lambda\phi(m - \mu + \gamma\eta^2 x_0).$$

Combining it with (18), we obtain $\mu = m - \lambda\phi \cdot \gamma\eta^2 x_0/(1 - \lambda\phi)$, λ and ϕ. It suffices now to show that the equation

$$f(\phi) \doteq -\gamma\eta^2(\Gamma + \eta^2)^2\phi^5 + (\Gamma + \eta^2)(\Gamma - \eta^2)\phi^4$$
$$-\gamma\sigma^2\left\{\Gamma^2 + 2\eta^2(\Gamma + \eta^2)\right\}\phi^3 - 2\sigma^2\eta^2\phi^2 - \gamma\sigma^4\eta^2\phi - \sigma^4 = 0$$

has a unique strictly positive solution. This is true because the function $f(\phi)$ is continuous and strictly increasing from $-\sigma^4$ to $+\infty$ on the interval $[0, +\infty[$. The convergence is obvious.

APPENDIX B

Numerical Results

We consider the case of the good news. The case of the bad news follows by duality. The noise trading variance is taken to be 1 ($\sigma^2 = 1$), except in Figure 4. In this good news case, the informed trading quantity is strictly positive, that is, insider buys (see Lemma 3 and Proposition 5). Computed by the Monte-Carlo simulation method.

FIGURE 1

The variation in the expected wealth of the insider with respect to the trading quantity $x = \alpha_1$ (from 0 to 8) for π_1 ranging from 0.1 to 0.9. We can observe that, for every $\pi_1 \in]0,1[$, there exists a unique optimal trading quantity: the equation $d_x E[W_{1^+}(H_x, x; v_1)] = 0$ has a unique solution.

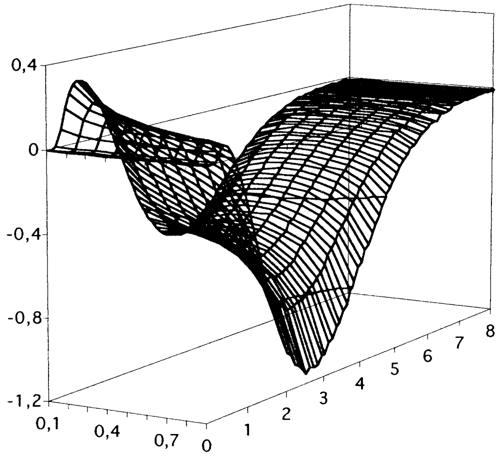


FIGURE 2

The section of the surface with the $x\pi_1$-plane in Figure 1: the insider's optimal trading quantity for each π_1 ranging from 0.1 to 0.9. The optimal trading quantity decreases as π_1 (the a prior probability for the good news) gets closer to 1.

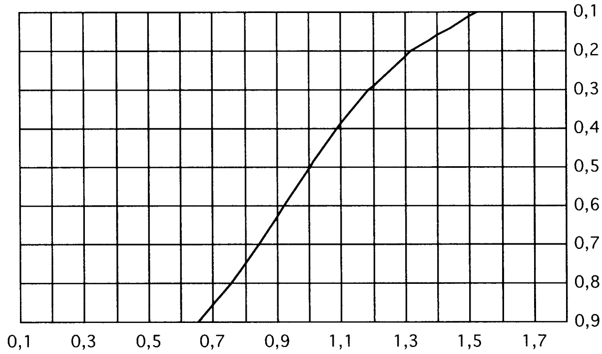


FIGURE 3

A posterior probability $\hat{\pi}_1(y) \doteq P[V = v_1 | Y = y]$ of the good news after having observed the demand y (from -3 to 3) for π_1: 0.25, 0.5 and 0.75. The price is just an appropriate scaling and translation:

$$P_1(y) = v_0(1 - \hat{\pi}_1(y)) + v_1\hat{\pi}_1(y) = v_0 + (v_1 - v_0)\hat{\pi}_1(y).$$

It is interesting to observe that the unbiased order submission $y = 0$ does not necessarily imply the price change $\Delta P = 0$ (That is the case only for $\pi_1 = 0.5$).

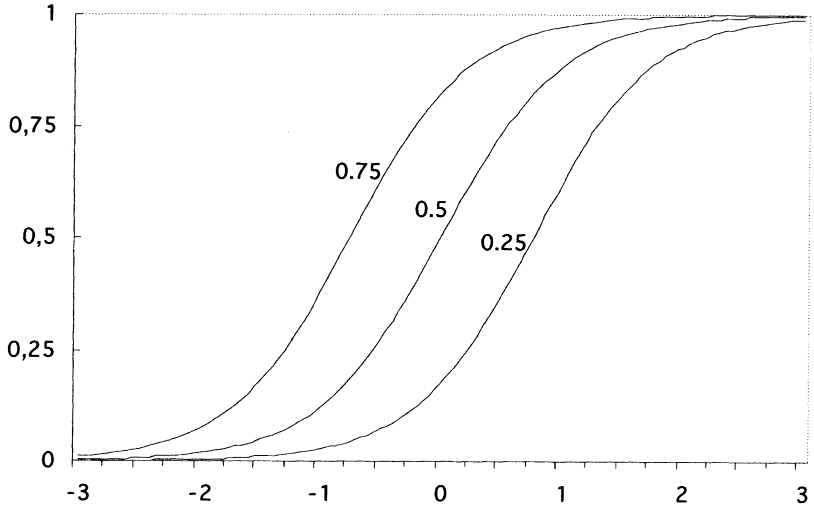
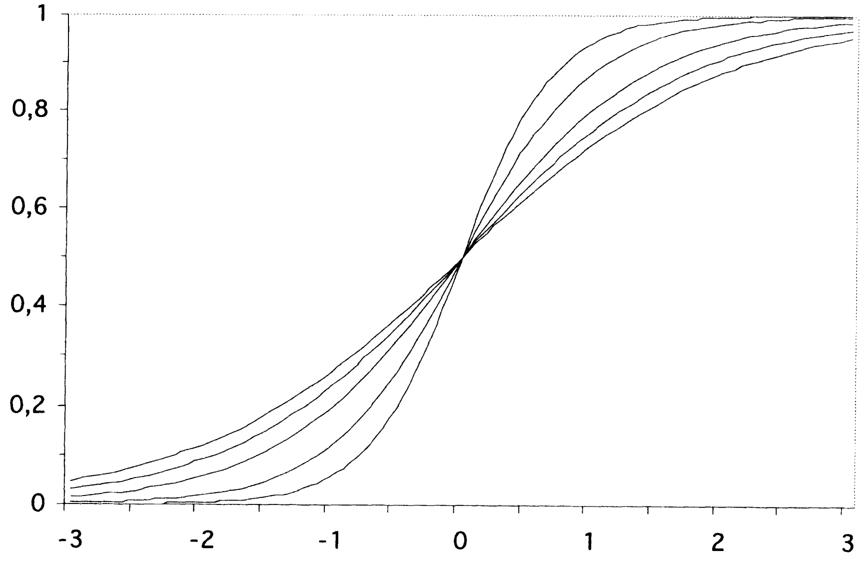


FIGURE 4

The same function $\hat{\pi}_1(y)$ as in Figure 3, but now by varying the value of σ^2: 0.5, 1, 2, 3 and 4, and fixing $\pi_1 = 0.5$. Naturally, the increasing speed of the curve slows down (or equivalently, the price pressure becomes smaller) as the market becomes deeper, that is, as σ^2 increases.



APPENDIX C

Linear Equilibrium Properties

Consider first the case of risk neutral insider. As shown in (15), the H-optimal trading strategy is $\alpha_H(s) = \frac{1}{2} H^{-1}(s)$ for any given linear pricing rule H: a risk neutral insider trades a half of the quantity that yields the price $P_1 = s$ (see Figure 5 below). The market-maker believes that the half of the order bias is from the informed agent: $E[X|Y] = \frac{1}{2}Y$. It is different from ROCHET and VILA [1994] in which $E[X|Y] = 0$, which can be understood as a no expected trade property. This difference is due to the additional assumption in ROCHET and VILA [1994] that the informed agent observes the noise trading before her own order submission.

The insider expects to earn $(s-m)^2/4\lambda^*$, whereas the market-maker estimates the insider's profit at $E\left[(V-m)^2|Y\right]/2\lambda - \lambda Y^2/2 = \hat{\sigma}^2/2\lambda^*$. As the market-maker believes zero payoffs on average for himself, the latter quantity is the market-maker's estimation for the noise trader's loss, while the insider's estimation for the market-maker's loss is $(s-m)^2/4\lambda^* - \hat{\sigma}^2/2\lambda^*$. It is quite natural that the price pressure becomes lower when the insider's observation is more perturbed. On the contrary, the quantity traded by the informed agent becomes larger when the quality of her observation is lower. This is, in part, because of lower price pressure. This strange phenomenon can be better explained when the insider is risk averse.

Consider now the case where the informed agent has CARA utility (thus is risk averse). To fix the idea, consider an insider who has a long position, for example a corporate insider. It is then natural that the insider has greater incentive to bid than to ask (it is inacceptable for a risk averse agent to lose money, while it does not matter to miss the chance to earn more money), hence the price becomes bigger than the risk neutral price, unless the aggregated market order is bigger than some sufficiently big number. Indeed, as $1 - \lambda_\gamma^* \phi_\gamma^* > 0$, the insider would like to sell when the signal is the same or even greater (to some extent) than $E[V]$, and the price for $y = 0$ is clearly $m_\gamma := m - \dfrac{\gamma \eta^2 x_0 \lambda_\gamma^* \phi_\gamma^*}{1 - \lambda_\gamma^* \phi_\gamma^*} > m$. That is, if the aggregate order is unbiased or even negative (to some extent), the rational price is strictly bigger than $E[V]$ (see Figure 5).

FIGURE 5

Linear pricing rules H_γ (risk aversion) and H (risk neutral) and optimal trading quantities $H^{-1}(s_1)$ and $H^{-1}(s_2)$ corresponding respectively to the signals s_1 and s_2. The pricing H_γ corresponds to the case of $x_0 > 0$.