Analysis I (Tao)

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Exercises 2.1

Apparently lemma 2.2.1 and induction is necessary to prove that $b++=a$, does that mean my proof is wrong?

Exercises 3.3

Ex. 3.3.2. (Direct Proof) If f and g are both injective (if $f,\ g\ :\ \hookrightarrow ?$) then $x \neq x' \implies f(x) \neq f(x')$ and $f(x) \neq f(x') \implies (g \circ f)(x) \neq (g \circ f)(x')$, from which $x \neq x' \implies (g \circ f)(x)$.

Suppose f and g are both surjective. Then $\forall y \in Y$, $\exists x\ f(x) = y$, and also $\forall z \in Z$, $\exists y\ g(y) = z$, and $|X| \geq |Y| \geq |Z|$. It follows that $\forall z \in Z$, $\exists x$ $(g \circ f)(x) = z$, i.e. $g \circ f$ is also surjective.

Ex. 3.3.4. (Direct Proof) This one was trickier than the others, and I kicked myself when I found a simple solution (What made the solution tricky?).

If $g \circ f = g \circ h$ and $g \colon Y \hookrightarrow Z \implies f = h$, then $f \neq h \implies \neg(g \colon X \hookrightarrow Y)$ or $g \circ f \neq g \circ h$, and this contrapositive is easy to show.

Ex. 3.3.5. (Proof by Contradiction) This exercise is the cousin of 3.3.2., investigating its claims from the opposite direction.

if $(g \circ f)(x)$ and $f : X \twoheadrightarrow Y$