Advertising on the Web

CS246: Mining Massive Datasets
Jure Leskovec, Stanford University
http://cs246.stanford.edu

Infinite data: Online Algorithms

High dim. data
- Locality sensitive hashing
- Clustering
- Dimensionality reduction

Graph data
- **PageRank,** SimRank
- Community Detection
- Spam Detection

Infinite data
- Filtering data streams
- Web advertising
- Queries on streams

Machine learning
- SVM
- Decision Trees
- Perceptron, kNN

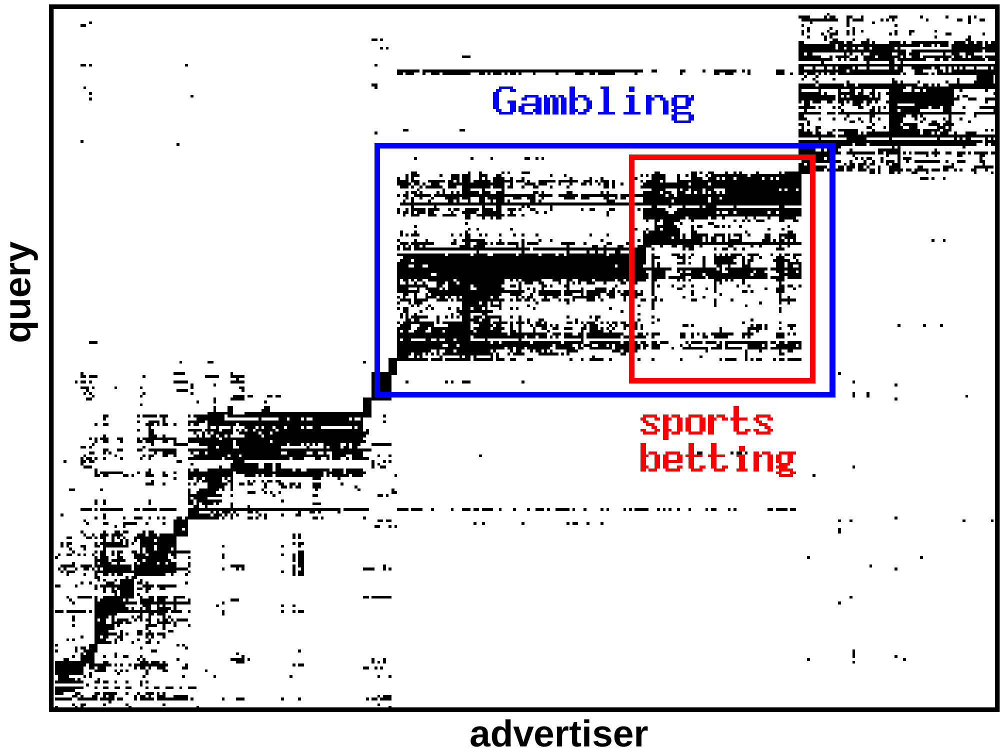
Apps
- Recommender systems
- Association Rules
- Duplicate document detection

Online Algorithms

- **Classic model of algorithms**
  - You get to see the entire input, then compute some function of it
  - In this context, "**offline** algorithm"
- **Online Algorithms**
  - You get to see the input one piece at a time, and need to make irrevocable decisions along the way
  - **Similar to the data stream model**

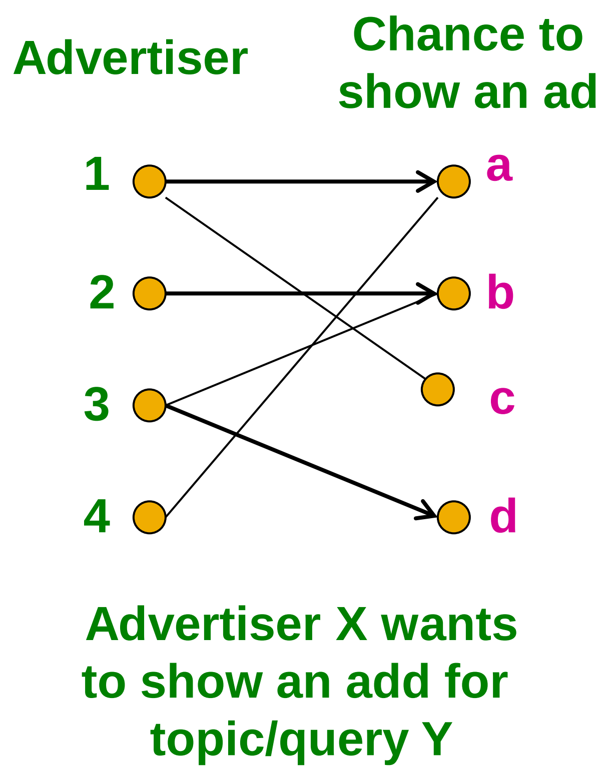
Sponsored Search: Ads

- **Query-to-advertiser graph:**



[Andersen, Lang: Communities from seed sets, 2006]

Graph Matching for Advertising



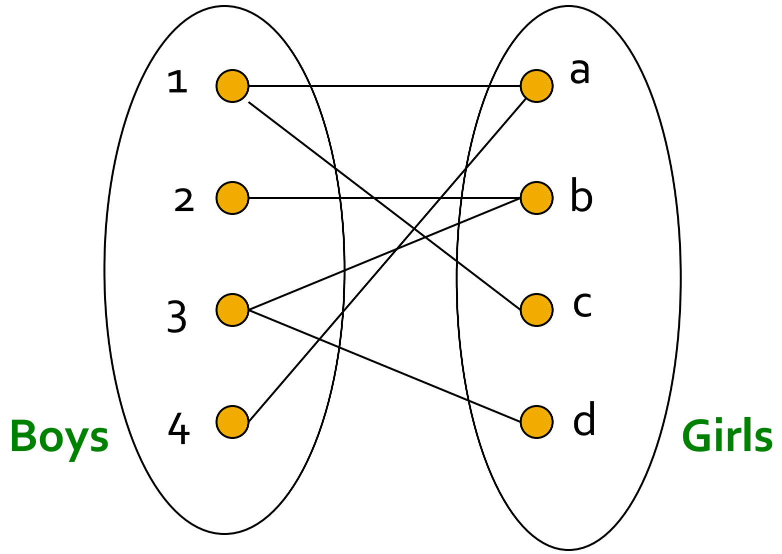
Which advertiser gets picked

(1,a)
(2,b)
(3,d)

This is an online problem: We have to make decisions as queries/topics show up. We do not know what topics will show up in the future.

Online Bipartite Matching

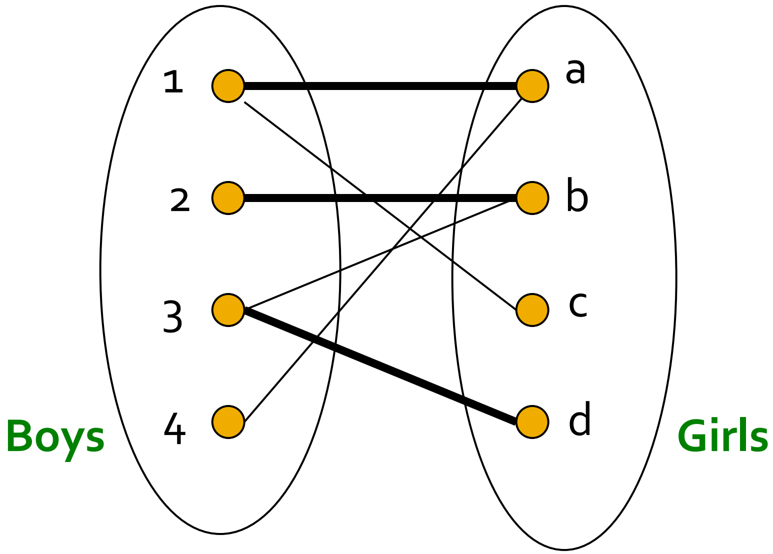
Example: Bipartite Matching



Nodes: Boys and Girls; Links: Preferences

Goal: Match boys to girls so that maximum number of preferences is satisfied

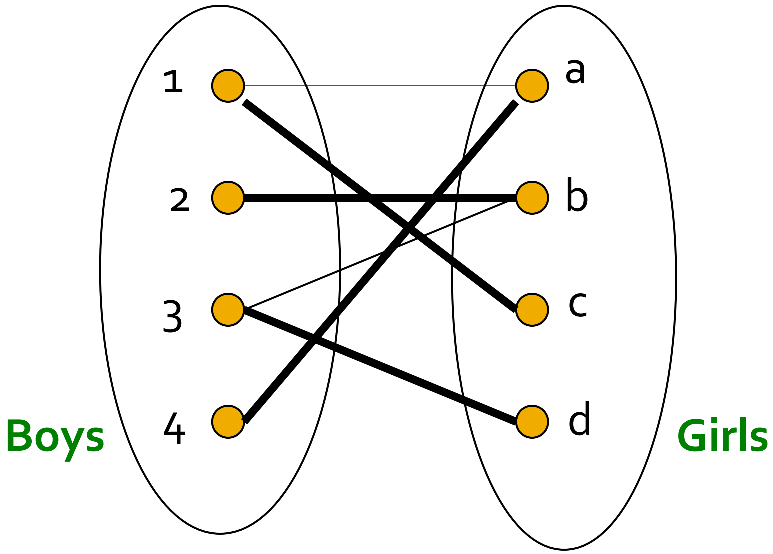
Example: Bipartite Matching



M = {(1,a),(2,b),(3,d)} is a **matching**

Cardinality of matching = |M| = 3

Example: Bipartite Matching



M = {(1,c),(2,b),(3,d),(4,a)} is a
perfect matching

Perfect matching … all vertices of the graph are matched
Maximum matching … a matching that contains the largest possible number of matches

Matching Algorithm

- **Problem:** **Find a maximum matching for a given bipartite graph**
  - A perfect one if it exists

- There is a polynomial-time offline algorithm based on augmenting paths (Hopcroft & Karp 1973, see http://en.wikipedia.org/wiki/Hopcroft-Karp_algorithm)

- **But what if we do not know the entire graph upfront?**

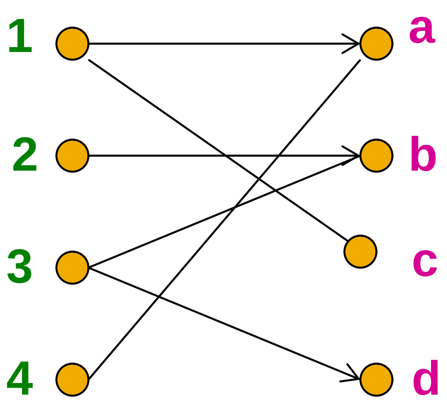
Online Graph Matching Problem

- Initially, we are given the set **boys**
- In each **round**, **one girl's choices are revealed**
  - That is, girl's **edges** are revealed
- **At that time, we have to decide to either:**
  - Pair the **girl** with a **boy**
  - Do not pair the **girl** with any **boy**

- **Example of application:**

  Assigning tasks to servers

Online Graph Matching: Example



(1,a)
(2,b)
(3,d)

Greedy Algorithm

- **Greedy algorithm for the online graph matching problem:**
  - Pair the new girl with **any** eligible boy
    - If there is none, do not pair girl
- **How good is the algorithm?**

Competitive Ratio

- For input $\boldsymbol{I}$, suppose greedy produces matching $\boldsymbol{M_{greedy}}$ while an optimal matching is $\boldsymbol{M_{opt}}$

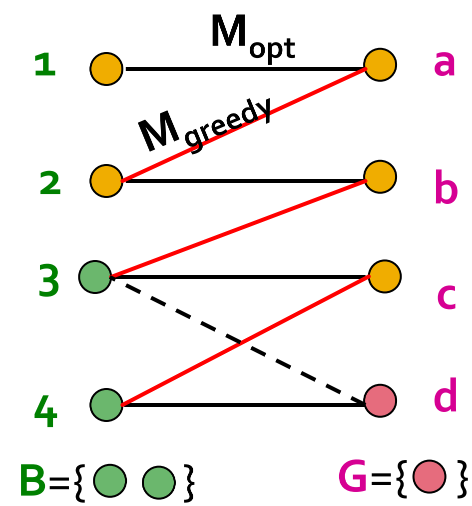
Competitive ratio =

$$\min_{\text{all possible inputs } I} (|M_{greedy}|/|M_{opt}|)$$

(what is greedy's <u>worst</u> performance <u>over all possible</u> inputs *I*)

Analyzing the Greedy Algorithm

- Consider a case: $M_{greedy} \neq M_{opt}$
- Consider the set G of girls matched in M_{opt} but not in M_{greedy}

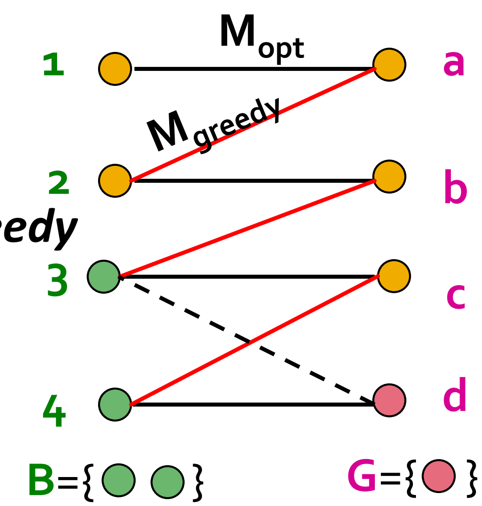


- **(1)** By definition of G:
  $|M_{opt}| \leq |M_{greedy}| + |G|$
- **(2)** Every boy B linked to girls in G is already matched in M_{greedy}:
  - If there would exist such non-matched (by M_{greedy}) boy adjacent to a non-matched girl then greedy would have matched them

  So: $|M_{greedy}| \geq |B|$

Analyzing the Greedy Algorithm

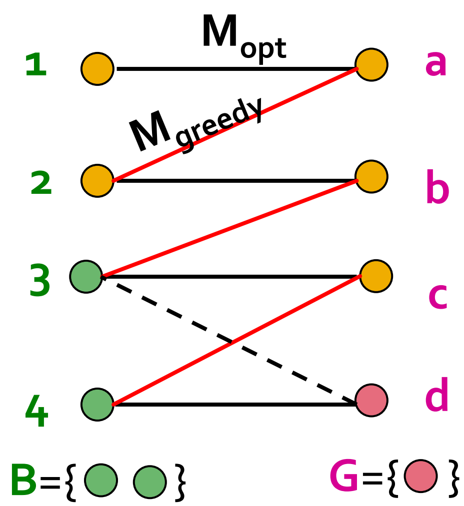
- **Summary so far:**
  - Girls G matched in M_{opt} but not in M_{greedy}
  - Boys B adjacent to girls G
  - **(1)** $|M_{opt}| \leq |M_{greedy}| + |G|$
  - **(2)** $|M_{greedy}| \geq |B|$



- Optimal matches all girls in G to (some) boys in B
  - **(3)** $|G| \leq |B|$
- Combining **(2)** and **(3)**:
  - $|G| \leq |B| \leq |M_{greedy}|$

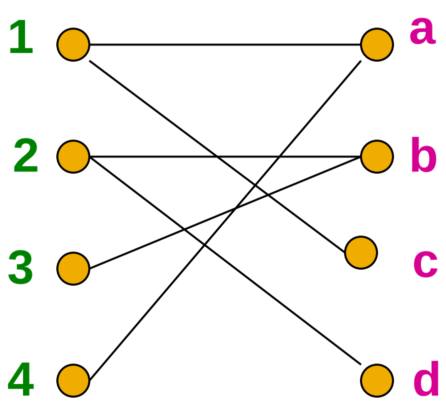
Analyzing the Greedy Algorithm

- **So we have:**
  - **(1)** $|M_{opt}| \leq |M_{greedy}| + |G|$
  - **(4)** $|G| \leq |B| \leq |M_{greedy}|$



- Combining **(1)** and **(4)**:
  - Worst case is when $|G| = |B| = |M_{greedy}|$
  - $|M_{opt}| \leq |M_{greedy}| + |M_{greedy}|$
  - **Then $|M_{greedy}|/|M_{opt}| \geq 1/2$**

Worst-case Scenario



(1,a)
(2,b)

Web Advertising

History of Web Advertising

- **Banner ads (1995-2001)**
  - Initial form of web advertising
  - Popular websites charged *X*$ for every 1,000 "impressions" of the ad
    - Called "**CPM**" rate (Cost per thousand impressions)
    - Modeled similar to TV, magazine ads
  - From **untargeted** to **demographically targeted**
  - **Low click-through rates**
    - Low ROI for advertisers

CPM…cost per *mille*
Mille…thousand in Latin

Performance-based Advertising

- **Introduced by Overture around 2000**
  - Advertisers **bid** on **search keywords**
  - When someone searches for that keyword, the **highest bidder's ad is shown**
  - Advertiser is charged only if the ad is clicked on
- Similar model adopted by Google with some changes around 2002
  - Called **Adwords**

Ads vs. Search Results

Web Results **1** - **10** of about **2,230,000** for **geico**. (**0.04** seco

GEICO Car Insurance. Get an auto insurance quote and save today ...
GEICO auto insurance, online car insurance quote, motorcycle insurance quote, online insurance sales and service from a leading insurance company.
www.**geico**.com/ - 21k - Sep 22, 2005 - Cached - Similar pages
Auto Insurance - Buy Auto Insurance
Contact Us - Make a Payment
More results from www.geico.com »

Geico, Google Settle Trademark Dispute
The case was resolved out of court, so advertisers are still left without legal guidance on use of trademarks within ads or as keywords.
www.clickz.com/news/article.php/3547356 - 44k - Cached - Similar pages

Google and **GEICO** settle AdWords dispute | The Register
Google and car insurance firm **GEICO** have settled a trade mark dispute over **...** Car insurance firm **GEICO** sued both Google and Yahoo! subsidiary Overture in **...**
www.theregister.co.uk/2005/09/09/google_**geico**_settlement/ - 21k - Cached - Similar pages

GEICO v. Google
... involving a lawsuit filed by Government Employees Insurance Company (**GEICO**). **GEICO** has filed suit against two major Internet search engine operators, **...**
www.consumeraffairs.com/news04/**geico**_google.html - 19k - Cached - Similar pages

Sponsored Links

Great Car Insurance Rates
Simplify Buying Insurance at Safeco
See Your Rate with an Instant Quote
www.Safeco.com

Free Insurance Quotes
Fill out one simple form to get
multiple quotes from local agents.
www.HometownQuotes.com

5 Free Quotes. 1 Form.
Get 5 Free Quotes In Minutes!
You Have Nothing To Lose. It's Free
sayyessoftware.com/Insurance
Missouri

Web 2.0

- **Performance-based advertising works!**
  - Multi-billion-dollar industry
- **Interesting problem:**
  What ads to show for a given query?
  - (Today’s lecture)
- **If I am an advertiser, which search terms should I bid on and how much should I bid?**
  - (Not focus of today’s lecture)

Adwords Problem

- A stream of queries arrives at the search engine: $q_1, q_2, \ldots$
- Several advertisers bid on each query
- When query q_i arrives, search engine must pick a subset of advertisers whose ads are shown

- **Goal: Maximize search engine's revenues**
  - **Simple solution:** Instead of raw bids, use the "**expected revenue per click**" (i.e., **Bid*CTR**)
- **Clearly we need an online algorithm!**

The Adwords Innovation

Advertiser	Bid	CTR	Bid * CTR
A	$1.00	1%	1 cent
B	$0.75	2%	1.5 cents
C	$0.50	2.5%	1.125 cents

Click through rate

Expected revenue

The Adwords Innovation

Advertiser	Bid	CTR	Bid * CTR
B	$0.75	2%	1.5 cents
C	$0.50	2.5%	1.125 cents
A	$1.00	1%	1 cent

Instead of sorting advertisers by bid, sort by expected revenue

Limitations of Simple Algorithm

Instead of sorting advertisers by bid, sort by expected revenue

Advertiser	Bid	CTR	Bid * CTR
B	$0.75	2%	1.5 cents
C	$0.50	2.5%	1.125 cents
A	$1.00	1%	1 cent

Challenges:

- CTR of an ad is unknown
- Advertisers have limited budges and bid on multiple queries

Complications: Budget

- **Two further complications:**
  - **Budget**
  - **CTR of an ad is unknown**

1) Budget: Each advertiser has a limited budget

  - **Search engine guarantees that the advertiser will not be charged more than their daily budget**

Complications: CTR

- **2) CTR (Click-Through Rate): Each ad-query pair has a different likelihood of being clicked**
  - **Advertiser 1** bids $2, click probability = 0.1
  - **Advertiser 2** bids $1, click probability = 0.5
- **CTR** is measured **historically**
  - Averaged over a time period

- **Some complications we will <u>not</u> cover:**
  - **1) CTR is position dependent:**
    - Ad #1 is clicked more than Ad #2

Complications: CTR

- **Some complications we will cover (next lecture):**
  - **2) Exploration vs. exploitation**
    Exploit: Should we keep showing an ad for which we have good estimates of click-through rate?
    or
    Explore: Shall we show a brand new ad to get a better sense of its click-through rate?

Online Algorithms
The BALANCE Algorithm

Adwords Problem

- **Given:**
  - **1.** A set of bids by advertisers for search queries
  - **2.** A click-through rate for each advertiser-query pair
  - **3.** A budget for each advertiser (say for 1 month)
  - **4.** A limit on the number of ads to be displayed with each search query
- **Respond to each search query with a set of advertisers such that:**
  - **1.** The size of the set is no larger than the limit on the number of ads per query
  - **2.** Each advertiser has bid on the search query
  - **3.** Each advertiser has enough budget left to pay for the ad if it is clicked upon

Greedy Algorithm

- **Our setting: Simplified environment**
  - There is **1** ad shown for each query
  - All advertisers have the same budget ***B***
  - All ads are equally likely to be clicked
  - Value of each ad is the same (=**1**)

- **Simplest algorithm is greedy:**
  - For a query pick any advertiser who has bid **1** for that query
  - **Competitive ratio of greedy is 1/2**

Bad Scenario for Greedy

- **Two advertisers A and B**
  - ***A*** bids on query ***x***, ***B*** bids on ***x*** and ***y***
  - Both have budgets of **$4**
- **Query stream:** ***x x x x y y y y***
  - Worst case greedy choice: ***B B B B*** **_ _ _ _**
  - Optimal: **A A A A B B B B**
  - **Competitive ratio = ½**
- **This is the worst case!**
  - **Note:** Greedy algorithm is deterministic – it always resolves draws in the same way

BALANCE Algorithm [MSVV]

- **BALANCE** Algorithm by Mehta, Saberi, Vazirani, and Vazirani
  - **For each query, pick the advertiser with the <u>largest unspent budget</u>**
    - Break ties arbitrarily (**but in a deterministic way**)

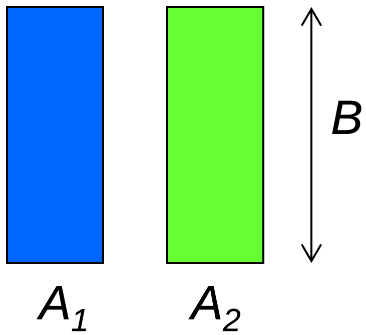
Example: BALANCE

- Two advertisers A and B
  - **A** bids on query ***x***, **B** bids on ***x*** and ***y***
  - Both have budgets of **$4**
- Query stream: ***x x x x y y y y***
- BALANCE choice: **A B A B B B _ _**
  - Optimal: **A A A A B B B B**
- **In general:** For **BALANCE** on **2** advertisers
  Competitive ratio = ¾

Analyzing BALANCE

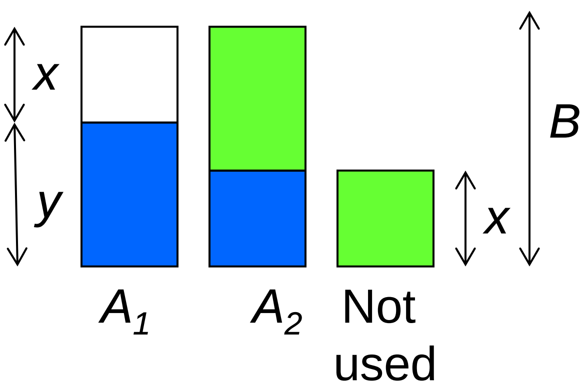
- **Consider simple case (w.l.o.g.):**
  - **2** advertisers, $\mathbf{A_1}$ and $\mathbf{A_2}$, each with budget **B** (≥ 1)
  - Optimal solution exhausts both advertisers' budgets
- **BALANCE must exhaust at least one budget:**
  - **If not, we can allocate more queries**
    - Whenever BALANCE makes a mistake (both advertisers bid on the query), advertiser's unspent budget only decreases
    - Since optimal exhausts both budgets, one will for sure get exhausted
  - Assume BALANCE exhausts $\boldsymbol{A_2}$'s budget, but allocates ***x*** queries fewer than the optimal
    - **So revenue of *BALANCE = 2B – x* (where OPT is 2B)**
  - **Let's work out what x is!**

Analyzing BALANCE: What's x?



Queries allocated to $\boldsymbol{A_1}$ in the optimal solution
Queries allocated to $\boldsymbol{A_2}$ in the optimal solution

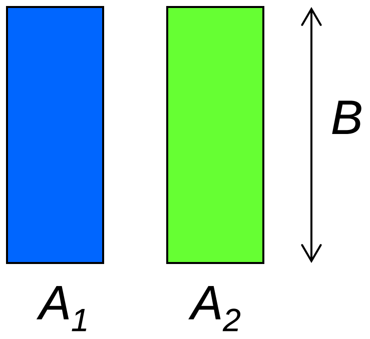
Optimal revenue = **2B**
Assume Balance gives revenue **= 2B-x = B+y**
Assume we exhausted A_2's budget



Unassigned queries should be assigned to A_2
(if we could assign to $\mathbf{A_1}$ we would since we still have the budget)
Goal: Show we have **y ≥ B/2**

Case 1) BALANCE assigns at least B/2 blue queries to A_1.
So, $\boldsymbol{y} \geq \boldsymbol{B}/\mathbf{2}$

Analyzing BALANCE: What's x?



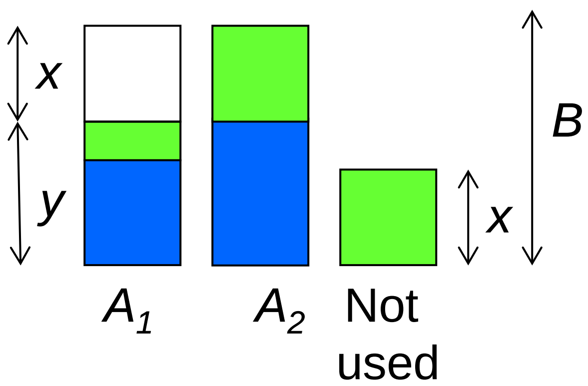
Queries allocated to A_1 in the optimal solution

Queries allocated to A_2 in the optimal solution

Optimal revenue = **2B**

Assume Balance gives revenue **= 2B-x = B+y**

Assume we exhausted A_2's budget



Unassigned queries should be assigned to A_2

(if we could assign to $\mathbf{A_1}$ we would since we still have the budget)

Goal: Show we have **y ≥ B/2**

Balance revenue is minimum for $x = y = B/2$

Minimum Balance revenue = $3B/2$

Competitive Ratio: BAL/OPT = 3/4

Case 2) BALANCE assigns more than B/2 blue queries to A_2.

Consider the last blue query assigned to A_2.

At that time, A_2's unspent budget must have been at least as big as A_1's.

That means at least as many queries have been assigned to A_1 as to A_2.

At this point, we have already assigned at least B/2 queries to A_2.

So, $x \leq B/2$ **and** $x + y = B$ **then** $y > B/2$

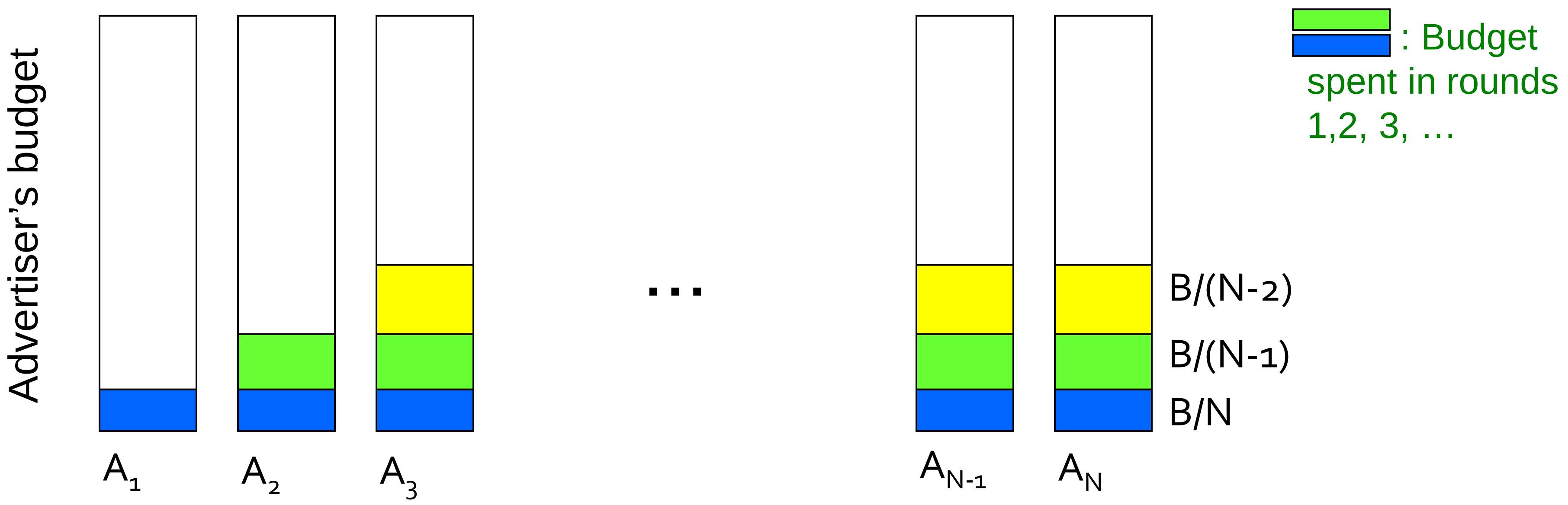
BALANCE: General Result

- **In the general case, worst competitive ratio of BALANCE is 1–1/e = approx. 0.63**
  - ***e*** **= 2.7182**
  - Interestingly, no online algorithm has a better competitive ratio!
- **Let's see the worst case example that gives this ratio**

Worst case for BALANCE

- *N* **advertisers:** $A_1, A_2, \ldots A_N$
  - Each with budget $B > N$
- **Queries:**
  - $N \cdot B$ queries appear in N rounds of B queries each
- **Bidding:**
  - Round **1** queries: bidders $A_1, A_2, \ldots, A_N$
  - Round **2** queries: bidders $A_2, A_3, \ldots, A_N$
  - Round *i* queries: bidders $A_i, \ldots, A_N$
- **Optimum allocation:**
  Allocate round *i* queries to A_i
  - Optimum revenue $N \cdot B$

BALANCE Allocation



BALANCE assigns each of the queries in round 1 to **N** advertisers.
After ***k*** rounds, sum of allocations to each of advertisers $\boldsymbol{A_k,\ldots,A_N}$ is

$$\boldsymbol{S_k = S_{k+1} = \cdots = S_N = \sum_{i=1}^{k-1} \frac{B}{N-(i-1)}}$$

If we find the smallest *k* such that $S_k \geq B$, then after *k* rounds we cannot allocate any queries to any advertiser

BALANCE: Analysis

B/1 B/2 B/3 … B/(N-(k-1)) … B/(N-1) B/N

S_1

S_2

S_k **= B**

1/1 1/2 1/3 … 1/(N-(k-1)) … 1/(N-1) 1/N

S_1

S_2

S_k **= 1**

BALANCE: Analysis

- **Fact:** $H_n = \sum_{i=1}^{n} 1/i \approx \ln(n)$ for large n
  - Result due to Euler

1/1 1/2 1/3 … 1/(N-(k-1)) … 1/(N-1) 1/N

ln(N)

ln(N)-1 $S_k = 1$

- $S_k = 1$ implies: $H_{N-k} = ln(N) - 1 = ln(\frac{N}{e})$
- We also know: $H_{N-k} = ln(N-k)$
- So: $N - k = \frac{N}{e}$
- Then: $k = N(1 - \frac{1}{e})$

N terms sum to ln(*N*).
Last *k* terms sum to 1.
First *N*-*k* terms sum to ln(*N*-*k*) but also to ln(*N*)-1

BALANCE: Analysis

- So after the first **k=N(1-1/e)** rounds, we cannot allocate a query to any advertiser
- **Revenue = B·N (1-1/e)**
- **Competitive ratio = 1-1/e**

General Version of the Problem

- **Arbitrary bids and arbitrary budgets!**
- Consider we have 1 query q, advertiser i
  - Bid = x_i
  - Budget = b_i
- **In a general setting BALANCE can be terrible**
  - Consider two advertisers A_1 and A_2
  - A_1: $x_1 = 1$, $b_1 = 110$
  - A_2: $x_2 = 10$, $b_2 = 100$
  - Consider we see **10** instances of **q**
  - BALANCE always selects A_1 and earns **10**
  - Optimal earns **100**

Generalized BALANCE

- **Arbitrary bids:** consider query $\boldsymbol{q}$, bidder $\boldsymbol{i}$
  - Bid = $\boldsymbol{x_i}$
  - Budget = $\boldsymbol{b_i}$
  - Amount spent so far = $\boldsymbol{m_i}$
  - Fraction of budget left over $\boldsymbol{f_i = 1 - m_i/b_i}$
  - Define $\boldsymbol{\psi_i(q) = x_i(1-e^{-f_i})}$
- Allocate query $\boldsymbol{q}$ to bidder $\boldsymbol{i}$ with largest value of $\boldsymbol{\psi_i(q)}$
- **Same competitive ratio (1-1/*e*) = 0.63**