[PS 4]

$\boxed{\#1. \text{ see ps 4-1.ipynb}}$

#2. At convergence: $M = \sum_i \sum_{z^i} Q_i^*(z^i) \log \frac{P(x^i, z^i; \theta)}{Q_i^*(z^i)}$

$\left(Q_i^*(z^i) = \frac{P(x^i, z^i; \theta^*)}{P(x^i; \theta^*)} \right)$

$$0 = \nabla_\theta M = \sum_i \sum_{z_i} \frac{P(x^i, z^i; \theta^*)}{P(x^i; \theta^*)} \frac{\nabla_\theta P(x^i, z^i; \theta)|_{\theta=\theta^*}}{P(x^i, z^i; \theta^*)}$$

$$= \sum_i \frac{\nabla_\theta P(x^i; \theta)|_{\theta=\theta^*}}{P(x^i; \theta^*)} = \nabla_\theta \sum \log P(x^i; \theta)|_{\theta=\theta^*}$$

$$= \nabla_\theta \ell(\theta)|_{\theta=\theta^*}$$

#3. $\frac{1}{m} \sum_i \|x^i - f_u(x^i)\|_2^2$

$$= \frac{1}{m} \sum_i (x^i - (u^T x^i) u)^T (x^i - (u^T x^i) u)$$

$$= \frac{1}{m} \sum_i ((x^i)^T x^i - (u^T x^i)^2) \quad (\because u^T u = 1,\ u^T x^i \text{, constant})$$

$$= \frac{1}{m} \sum_i (x^i)^T x^i - \left(\frac{1}{m} \sum_i u^T x^i\right)^2 - Var(u^T x^i)$$

This means maximizing variance (which gives first principle component u) is equivalent to minimizing MSE.

$\boxed{\#4. \text{ see ps 4-4.ipynb}}$

#5.

(a) Let $\arg\max_{s \in S} \|B(V_1)(s) - B(V_2)(s)\| = s^+$

$\|B(V_1) - B(V_2)\|_\infty$

$$= \gamma \| \max_a \sum_{s'} P_{s^+a}(s') V_1(s') - \max_a \sum_{s'} P_{s^+a}(s') V_2(s') |$$

$$= \gamma \| \sum_{s'} P_{s^+a_1}(s') V_1(s') - \sum_{s'} P_{s^+a_2}(s') V_2(s')$$

$$\le \gamma \sum_{s'} P_{s^+a_1}(s') \| V_1(s') - V_2(s') \| \quad \cdots ① \quad (\because a_2 \text{ is argmax of subtracted value})$$

$$\le \gamma \| V_1 - V_2 \|_\infty \cdots ② \quad (\because ① \text{ is average of } |V_1(s) - V_2(s)|$$

while ② is maximum of $|V_1(s) - V_2(s)|$)

(b) Assume two fixed point V_1, V_2, then

$\|V_1 - V_2\|_\infty = \|B(V_1) - B(V_2)\|_\infty \le \gamma \|V_1 - V_2\|_\infty$. Since $\gamma < 1$, $\|V_1 - V_2\|_\infty$ must be 0. $\therefore V_1 = V_2$

$\boxed{\#6. \text{ see ps4-6.ipynb}}$