Mudcard

- **How do we actually do the gradient descent function, I know you said not to use the code that you did? Is it a simple sklearn code like it is for linear regression?**
  - Yes, when you call `.fit` of linear or logistic regression, it does gradient descent for you
- **can the learning rate be anything?**
  - If it's too small, it will take a long time to converge.
  - If it's too large, it will introduce numerical instabilities.
  - Learning rates in the range if 1e-4 and 1e-2 usually work well.
- **Could you explain the gradient descent a little more?**
  - We need to move on but there are countless online resources to learn about it
- **how are iterations determined while running gradient descent?**
  - do you mean the number of iterations?
  - You can do a couple of things:
    - have a preset number of iterations like in my code
    - have a preset number of operations but stop sooner if a convergence criteria is fulfilled
    - use a while loop and iterate until a convergence criteria is fulfilled (not usually done because the while loop can go forever)

Regularization

By the end of this lecture, you will be able to

- Describe why regularization is important and what are the two types of regularization
- Describe how regularized linear regression works
- Describe how regularized logistic regression works

Regularization

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- **Describe why regularization is important and what are the two types of regularization**
- Describe how regularized linear regression works
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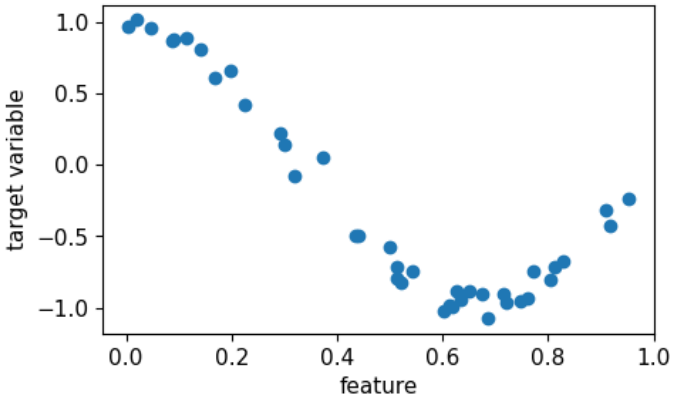
Polynomial regression

Let's work with a new example dataset

In [1]:
```
# load packages
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
from sklearn.preprocessing import PolynomialFeatures
import matplotlib
matplotlib.rcParams.update({'font.size': 11})

df = pd.read_csv('data/regularization_example.csv')
X_ori = df['x0'].values.reshape(-1, 1)
y = df['y'].values
print(np.shape(X_ori))
print(np.shape(y))

# visualize the data
plt.figure(figsize=(5,3))
plt.scatter(X_ori,y)
plt.xlabel('feature')
plt.ylabel('target variable')
plt.show()
```

```
(40, 1)
(40,)
```



In [2]:
```
# lets generate more features because a linear model will obviously be insufficient to fit the data well
pf = PolynomialFeatures(degree = 20,include_bias=False)
X = pf.fit_transform(X_ori)
print(np.shape(X))
print(pf.get_feature_names_out())
```

```
(40, 20)
['x0' 'x0^2' 'x0^3' 'x0^4' 'x0^5' 'x0^6' 'x0^7' 'x0^8' 'x0^9' 'x0^10'
 'x0^11' 'x0^12' 'x0^13' 'x0^14' 'x0^15' 'x0^16' 'x0^17' 'x0^18' 'x0^19'
 'x0^20']
```

We split data into train and validation!

In [3]:
```
from sklearn.model_selection import train_test_split
X_train, X_val, y_train, y_val = train_test_split(X, y, test_size=0.2, random_state=1)
print(np.shape(X_train),np.shape(y_train))
print(np.shape(X_val),np.shape(y_val))
```

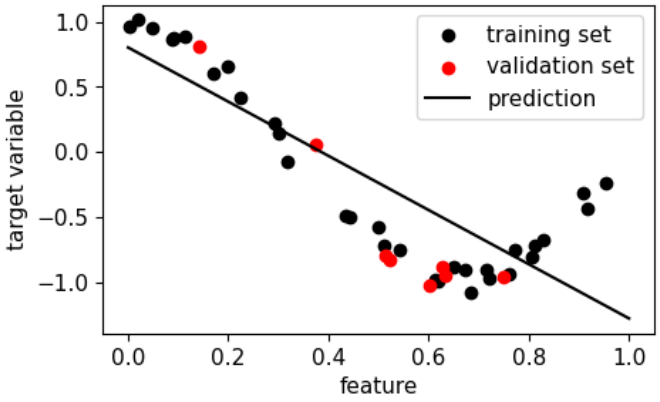
```
(32, 20) (32,)
(8, 20) (8,)
```

Let's train and validate some linear regression models

Use the first feature only

In [4]:
```
from sklearn.linear_model import LinearRegression
from sklearn.metrics import mean_squared_error

# let's use only the first feature
linreg = LinearRegression(fit_intercept=True)
linreg.fit(X_train[:,:1], y_train)
print('intercept:',linreg.intercept_)
print('w:',linreg.coef_)

train_MSE = mean_squared_error(y_train,linreg.predict(X_train[:,:1]))
val_MSE = mean_squared_error(y_val,linreg.predict(X_val[:,:1]))
print('train MSE:',train_MSE)
print('val MSE:',val_MSE)


# let's visualuze the model
x_model = np.linspace(0,1,100)
plt.figure(figsize=(5,3))
plt.scatter(X_train[:,0],y_train,color='k',label='training set')
plt.scatter(X_val[:,0],y_val,color='r',label='validation set')
plt.plot(x_model,linreg.predict(x_model.reshape(-1,1)),color='k',label='prediction')
plt.xlabel('feature')
plt.ylabel('target variable')
plt.legend()
plt.show()
```

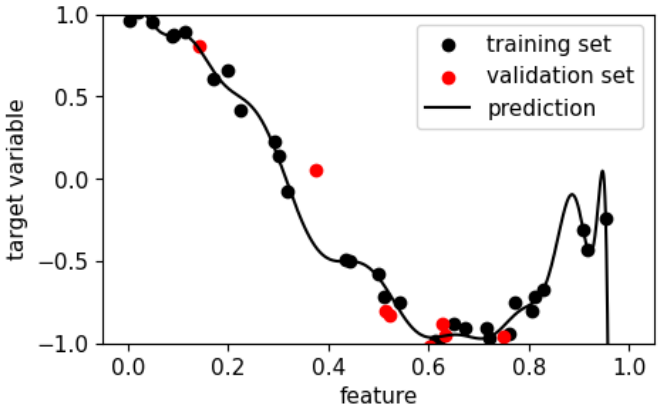
```
intercept: 0.8018842867499771
w: [-2.08151827]
train MSE: 0.13964692457239292
val MSE: 0.17142516062337293
```



Use all features

In [5]:
```
from sklearn.linear_model import LinearRegression
from sklearn.metrics import mean_squared_error

# use all features
linreg = LinearRegression(fit_intercept=True)
linreg.fit(X_train, y_train)
print('intercept:',linreg.intercept_)
print('ws:',linreg.coef_)

train_MSE = mean_squared_error(y_train,linreg.predict(X_train))
val_MSE = mean_squared_error(y_val,linreg.predict(X_val))
print('train MSE:',train_MSE)
print('val MSE:',val_MSE)


# let's visualuze the model
x_model = np.linspace(0,1,1000)
plt.figure(figsize=(5,3))
plt.scatter(X_train[:,0],y_train,color='k',label='training set')
plt.scatter(X_val[:,0],y_val,color='r',label='validation set')
plt.plot(x_model,linreg.predict(pf.transform(x_model.reshape(-1,1))),color='k',label='prediction')
plt.ylim([-1,1])
plt.xlabel('feature')
plt.ylabel('target variable')
plt.legend()
plt.show()
```

```
intercept: 0.9521398936541472
ws: [ 2.94556467e+00  1.78575034e+02 -1.07852992e+04  7.71588159e+04
  3.57083482e+06 -9.66895422e+07  1.20126203e+09 -9.39908662e+09
  5.13303355e+10 -2.05802411e+11  6.23129909e+11 -1.44706005e+12
  2.59415710e+12 -3.58595231e+12  3.78788495e+12 -3.00091345e+12
  1.72536419e+12 -6.79474340e+11  1.63872612e+11 -1.82456642e+10]
train MSE: 0.0022236975806119337
val MSE: 0.032870916690386186
```



What to do?

- the model is visibly performs poorly when only the original feature is used

- the model performs very good on the training set but poorly on the validation set when all features are used
  - the ws are huge!

Regulazation solves this problem!

Regularization

By the end of this lecture, you will be able to

- Describe why regularization is important and what are the two types of regularization
- **Describe how regularized linear regression works**
- Describe how regularized logistic regression works

Regularization to the rescue!

- let's change the cost function and add a penalty term for large ws
- **Lasso regression**: regularize using the l1 norm of w:

$$L(w) = \frac{1}{n}\sum_{i=1}^{n}[(w_0 + \sum_{j=1}^{m} w_j x_{ij} - y_i)^2] + \alpha \sum_{j=0}^{m} |w_j|$$

- **Ridge regression**: regularize using the l2 norm of w:

$$L(w) = \frac{1}{n}\sum_{i=1}^{n}[(w_0 + \sum_{j=1}^{m} w_j x_{ij} - y_i)^2] + \alpha \sqrt{\sum_{j=0}^{m} w_j^2}$$

- α is the regularization parameter (positive number), it describes how much we penalize large ws
- With the cost function changed, the derivatives in gradient descent need to be updated too!

Feature selection with Lasso regularization

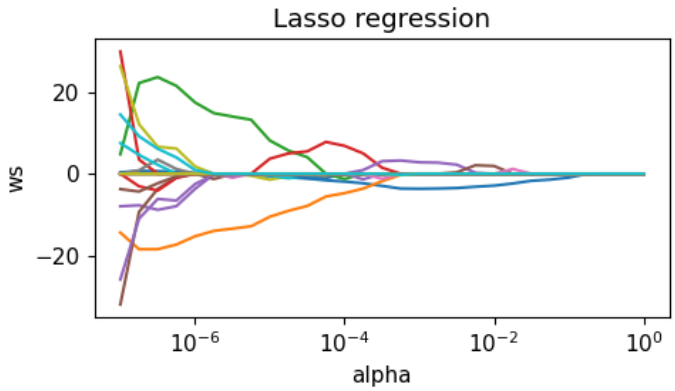
- Least Absolute Shrinkage and Selection Operator
- cost = MSE + α * l1 norm of w

$$L(w) = \frac{1}{n}\sum_{i=1}^{n}[(w_0 + \sum_{j=1}^{m} w_j x_{ij} - y_i)^2] + \alpha \sum_{j=0}^{m} |w_j|$$

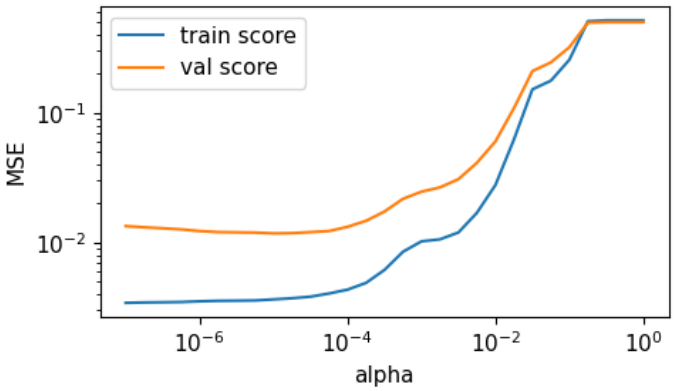
- ideal for feature selection
- as α increases, more and more feature weights are reduced to 0.

In [6]:
```
from  sklearn.linear_model import Lasso
from sklearn.metrics import mean_squared_error

alpha = np.logspace(-7,0,29)
ws = []
models = []
train_MSE = np.zeros(len(alpha))
val_MSE = np.zeros(len(alpha))

# do the fit
for i in range(len(alpha)):
    # load the linear regression model
    lin_reg = Lasso(alpha=alpha[i],max_iter=100000000)
    lin_reg.fit(X_train, y_train)
    ws.append(lin_reg.coef_)
    models.append(lin_reg)
    train_MSE[i] = mean_squared_error(y_train,lin_reg.predict(X_train))
    val_MSE[i] = mean_squared_error(y_val,lin_reg.predict(X_val))
```

In [7]:
```
plt.figure(figsize=(5,3))
plt.plot(alpha, ws)
plt.semilogx()
plt.xlabel('alpha')
plt.ylabel('ws')
plt.title('Lasso regression')
plt.tight_layout()
plt.savefig('figures/lasso_coefs.png',dpi=300)
plt.show()
```



In [8]:
```
plt.figure(figsize=(5,3))
plt.plot(alpha,train_MSE,label='train score')
plt.plot(alpha,val_MSE,label='val score')
plt.semilogy()
plt.semilogx()
plt.xlabel('alpha')
plt.ylabel('MSE')
plt.legend()
plt.tight_layout()
plt.savefig('figures/train_val_MSE_lasso.png',dpi=300)
plt.show()
```



Bias vs variance

- Bias: the model performs poorly on both the train and validation sets
  - high alpha in our example
- the model performs very well on the training set but it performs poorly on the validation set
  - low alpha in our example
  - lowering the alpha further would improve the train score but the validation score would increase
  - we don't do it because of convergence issues

The bias-variance trade off

- the curve of the validation score as a function of a hyper-parameter usually has a U shape if evaluation metric needs to be minimized, or an inverted U if the metric needs to be maximized
- choose the hyper-parameter value that gives you the best validation score

Quiz

Which alpha value gives the best validation score? Visualize that model!

In [ ]:
```
```

The bias-variance tradeoff with Ridge regularization

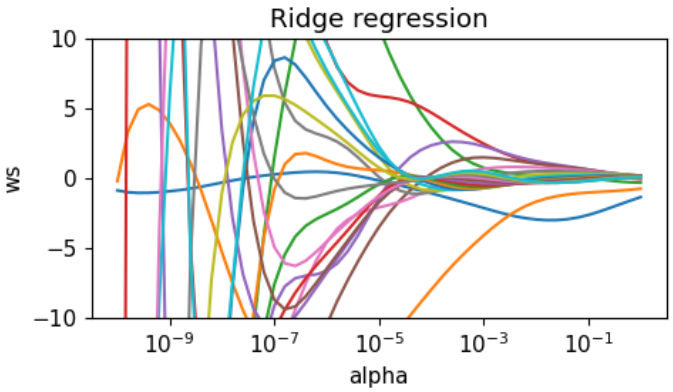
- cost = MSE + α * l2 norm of w

$$L(w) = \frac{1}{n}\sum_{i=1}^{n}[(w_0 + \sum_{j=1}^{m} w_j x_{ij} - y_i)^2] + \alpha\sqrt{\sum_{j=0}^{m} w_j^2}$$

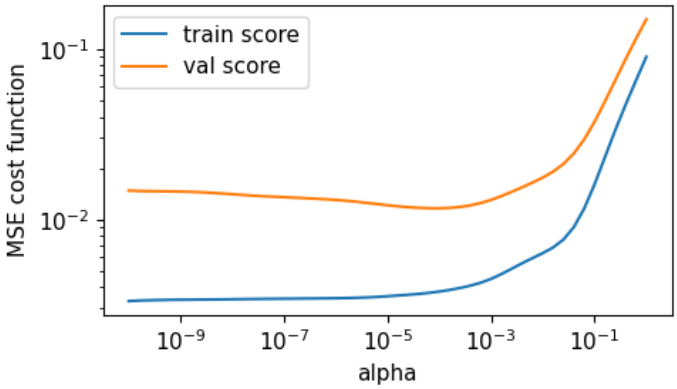
- as α approaches 0, we reproduce the linear regression weights
- small α creates high variance
- large α creates high bias

In [9]:
```
from  sklearn.linear_model import Ridge
from sklearn.metrics import mean_squared_error

alpha = np.logspace(-10,0,51)

# arrays to save train and test MSE scores
train_MSE = np.zeros(len(alpha))
val_MSE = np.zeros(len(alpha))

ws = []

# do the fit
for i in range(len(alpha)):
    # load the linear regression model
    lin_reg = Ridge(alpha=alpha[i])
    lin_reg.fit(X_train, y_train)
    ws.append(lin_reg.coef_)
    # train and test scores
    train_MSE[i] = mean_squared_error(y_train,lin_reg.predict(X_train))
    val_MSE[i] = mean_squared_error(y_val,lin_reg.predict(X_val))
```

In [10]:
```
plt.figure(figsize=(5,3))
plt.plot(alpha, ws)
plt.semilogx()
plt.ylim([-1e1,1e1])
plt.xlabel('alpha')
plt.ylabel('ws')
plt.title('Ridge regression')
plt.tight_layout()
plt.savefig('figures/ridge_coefs.png',dpi=300)
plt.show()
```



In [11]:
```
plt.figure(figsize=(5,3))
plt.plot(alpha,train_MSE,label='train score')
plt.plot(alpha,val_MSE,label='val score')
plt.semilogy()
plt.semilogx()
plt.xlabel('alpha')
plt.ylabel('MSE cost function')
plt.legend()
plt.tight_layout()
plt.savefig('figures/train_val_MSE_ridge.png',dpi=300)
plt.show()
```



Quiz

Which α gives us the best tradeoff between bias and variance?

In [ ]:

Regularization

By the end of this lecture, you will be able to

- Describe why regularization is important and what are the two types of regularization
- Describe how regularized linear regression works
- **Describe how regularized logistic regression works**

Logistic regression

- Recap: the logloss metric is the cost function

$$L(w) = -\frac{1}{N}\sum_{i=1}^{n}[y_i \ln(y_i') + (1-y_i)\ln(1-y_i')]$$
$$L(w) = -\frac{1}{N}\sum_{i=1}^{n}[y_i \ln(\frac{1}{1+e^{-w_0+\sum_{j=1}^{m} w_j x_{ij}}}) + (1-y_i)\ln(1-\frac{1}{1+e^{-w_0+\sum_{j=1}^{m} w_j x_{ij}}})]$$

- the logloss metric with l1 regularization

$$L(w) = -\frac{1}{N}\sum_{i=1}^{n}[y_i \ln(\frac{1}{1+e^{-w_0+\sum_{j=1}^{m} w_j x_{ij}}}) + (1-y_i)\ln(1-\frac{1}{1+e^{-w_0+\sum_{j=1}^{m} w_j x_{ij}}}))] + \alpha\sum_{j=0}^{m}|w_j|$$

- the logloss metric with l2 regularization

$$L(w) = -\frac{1}{N}\sum_{i=1}^{n}[y_i \ln(\frac{1}{1+e^{-w_0+\sum_{j=1}^{m} w_j x_{ij}}}) + (1-y_i)\ln(1-\frac{1}{1+e^{-w_0+\sum_{j=1}^{m} w_j x_{ij}}}))] + \alpha\sqrt{\sum_{j=0}^{m} w_j^2}$$

Logistic regression in sklearn

In [12]:
```
from sklearn.linear_model import LogisticRegression

log_reg_l1 = LogisticRegression(penalty='l1', C = 1/alpha) # C is the inverse of the regularization strength
log_reg_l2 = LogisticRegression(penalty='l2', C = 1/alpha)
# fit, predict, predict_proba are available
# log_reg.coef_ returns the w values
```

In [13]:
```
help(LogisticRegression)
```

```
Help on class LogisticRegression in module sklearn.linear_model._logistic:

class LogisticRegression(sklearn.linear_model._base.LinearClassifierMixin, sklearn.linear_model._base.SparseCoefMi
xin, sklearn.base.BaseEstimator)
 |  LogisticRegression(penalty='l2', *, dual=False, tol=0.0001, C=1.0, fit_intercept=True, intercept_scaling=1, cl
ass_weight=None, random_state=None, solver='lbfgs', max_iter=100, multi_class='auto', verbose=0, warm_start=False,
n_jobs=None, l1_ratio=None)
 |
 |  Logistic Regression (aka logit, MaxEnt) classifier.
 |
 |  In the multiclass case, the training algorithm uses the one-vs-rest (OvR)
 |  scheme if the 'multi_class' option is set to 'ovr', and uses the
 |  cross-entropy loss if the 'multi_class' option is set to 'multinomial'.
 |  (Currently the 'multinomial' option is supported only by the 'lbfgs',
 |  'sag', 'saga' and 'newton-cg' solvers.)
 |
 |  This class implements regularized logistic regression using the
 |  'liblinear' library, 'newton-cg', 'sag', 'saga' and 'lbfgs' solvers. **Note
 |  that regularization is applied by default**. It can handle both dense
 |  and sparse input. Use C-ordered arrays or CSR matrices containing 64-bit
 |  floats for optimal performance; any other input format will be converted
 |  (and copied).
 |
 |  The 'newton-cg', 'sag', and 'lbfgs' solvers support only L2 regularization
 |  with primal formulation, or no regularization. The 'liblinear' solver
 |  supports both L1 and L2 regularization, with a dual formulation only for
 |  the L2 penalty. The Elastic-Net regularization is only supported by the
 |  'saga' solver.
 |
 |  Read more in the :ref:`User Guide <logistic_regression>`.
 |
 |  Parameters
 |  ----------
 |  penalty : {'l1', 'l2', 'elasticnet', None}, default='l2'
 |      Specify the norm of the penalty:
 |
 |      - `None`: no penalty is added;
 |      - `'l2'`: add a L2 penalty term and it is the default choice;
 |      - `'l1'`: add a L1 penalty term;
 |      - `'elasticnet'`: both L1 and L2 penalty terms are added.
 |
 |      .. warning::
 |         Some penalties may not work with some solvers. See the parameter
 |         `solver` below, to know the compatibility between the penalty and
 |         solver.
 |
 |      .. versionadded:: 0.19
 |         l1 penalty with SAGA solver (allowing 'multinomial' + L1)
 |
 |      .. deprecated:: 1.2
 |         The 'none' option was deprecated in version 1.2, and will be removed
 |         in 1.4. Use `None` instead.
 |
 |  dual : bool, default=False
 |      Dual or primal formulation. Dual formulation is only implemented for
 |      l2 penalty with liblinear solver. Prefer dual=False when
 |      n_samples > n_features.
 |
 |  tol : float, default=1e-4
 |      Tolerance for stopping criteria.
 |
 |  C : float, default=1.0
 |      Inverse of regularization strength; must be a positive float.
 |      Like in support vector machines, smaller values specify stronger
 |      regularization.
 |
 |  fit_intercept : bool, default=True
 |      Specifies if a constant (a.k.a. bias or intercept) should be
 |      added to the decision function.
 |
 |  intercept_scaling : float, default=1
 |      Useful only when the solver 'liblinear' is used
 |      and self.fit_intercept is set to True. In this case, x becomes
 |      [x, self.intercept_scaling],
 |      i.e. a "synthetic" feature with constant value equal to
 |      intercept_scaling is appended to the instance vector.
 |      The intercept becomes ``intercept_scaling * synthetic_feature_weight``.
 |
 |      Note! the synthetic feature weight is subject to l1/l2 regularization
```

```
|      as all other features.
|      To lessen the effect of regularization on synthetic feature weight
|      (and therefore on the intercept) intercept_scaling has to be increased.
|
|  class_weight : dict or 'balanced', default=None
|      Weights associated with classes in the form ``{class_label: weight}``.
|      If not given, all classes are supposed to have weight one.
|
|      The "balanced" mode uses the values of y to automatically adjust
|      weights inversely proportional to class frequencies in the input data
|      as ``n_samples / (n_classes * np.bincount(y))``.
|
|      Note that these weights will be multiplied with sample_weight (passed
|      through the fit method) if sample_weight is specified.
|
|      .. versionadded:: 0.17
|         *class_weight='balanced'*
|
|  random_state : int, RandomState instance, default=None
|      Used when ``solver`` == 'sag', 'saga' or 'liblinear' to shuffle the
|      data. See :term:`Glossary <random_state>` for details.
|
|  solver : {'lbfgs', 'liblinear', 'newton-cg', 'newton-cholesky', 'sag', 'saga'},             default='lbfgs'
|
|      Algorithm to use in the optimization problem. Default is 'lbfgs'.
|      To choose a solver, you might want to consider the following aspects:
|
|          - For small datasets, 'liblinear' is a good choice, whereas 'sag'
|            and 'saga' are faster for large ones;
|          - For multiclass problems, only 'newton-cg', 'sag', 'saga' and
|            'lbfgs' handle multinomial loss;
|          - 'liblinear' is limited to one-versus-rest schemes.
|          - 'newton-cholesky' is a good choice for `n_samples` >> `n_features`,
|            especially with one-hot encoded categorical features with rare
|            categories. Note that it is limited to binary classification and the
|            one-versus-rest reduction for multiclass classification. Be aware that
|            the memory usage of this solver has a quadratic dependency on
|            `n_features` because it explicitly computes the Hessian matrix.
|
|      .. warning::
|         The choice of the algorithm depends on the penalty chosen.
|         Supported penalties by solver:
|
|         - 'lbfgs'           -   ['l2', None]
|         - 'liblinear'       -   ['l1', 'l2']
|         - 'newton-cg'       -   ['l2', None]
|         - 'newton-cholesky' -   ['l2', None]
|         - 'sag'             -   ['l2', None]
|         - 'saga'            -   ['elasticnet', 'l1', 'l2', None]
|
|      .. note::
|         'sag' and 'saga' fast convergence is only guaranteed on features
|         with approximately the same scale. You can preprocess the data with
|         a scaler from :mod:`sklearn.preprocessing`.
|
|      .. seealso::
|         Refer to the User Guide for more information regarding
|         :class:`LogisticRegression` and more specifically the
|         :ref:`Table <Logistic_regression>`
|         summarizing solver/penalty supports.
|
|      .. versionadded:: 0.17
|         Stochastic Average Gradient descent solver.
|      .. versionadded:: 0.19
|         SAGA solver.
|      .. versionchanged:: 0.22
|          The default solver changed from 'liblinear' to 'lbfgs' in 0.22.
|      .. versionadded:: 1.2
|         newton-cholesky solver.
|
|  max_iter : int, default=100
|      Maximum number of iterations taken for the solvers to converge.
|
|  multi_class : {'auto', 'ovr', 'multinomial'}, default='auto'
|      If the option chosen is 'ovr', then a binary problem is fit for each
|      label. For 'multinomial' the loss minimised is the multinomial loss fit
|      across the entire probability distribution, *even when the data is
|      binary*. 'multinomial' is unavailable when solver='liblinear'.
|      'auto' selects 'ovr' if the data is binary, or if solver='liblinear',
```

```
|      and otherwise selects 'multinomial'.
|
|      .. versionadded:: 0.18
|         Stochastic Average Gradient descent solver for 'multinomial' case.
|      .. versionchanged:: 0.22
|          Default changed from 'ovr' to 'auto' in 0.22.
|
|  verbose : int, default=0
|      For the liblinear and lbfgs solvers set verbose to any positive
|      number for verbosity.
|
|  warm_start : bool, default=False
|      When set to True, reuse the solution of the previous call to fit as
|      initialization, otherwise, just erase the previous solution.
|      Useless for liblinear solver. See :term:`the Glossary <warm_start>`.
|
|      .. versionadded:: 0.17
|         *warm_start* to support *lbfgs*, *newton-cg*, *sag*, *saga* solvers.
|
|  n_jobs : int, default=None
|      Number of CPU cores used when parallelizing over classes if
|      multi_class='ovr'". This parameter is ignored when the ``solver`` is
|      set to 'liblinear' regardless of whether 'multi_class' is specified or
|      not. ``None`` means 1 unless in a :obj:`joblib.parallel_backend`
|      context. ``-1`` means using all processors.
|      See :term:`Glossary <n_jobs>` for more details.
|
|  l1_ratio : float, default=None
|      The Elastic-Net mixing parameter, with ``0 <= l1_ratio <= 1``. Only
|      used if ``penalty='elasticnet'``. Setting ``l1_ratio=0`` is equivalent
|      to using ``penalty='l2'``, while setting ``l1_ratio=1`` is equivalent
|      to using ``penalty='l1'``. For ``0 < l1_ratio <1``, the penalty is a
|      combination of L1 and L2.
|
|  Attributes
|  ----------
|
|  classes_ : ndarray of shape (n_classes, )
|      A list of class labels known to the classifier.
|
|  coef_ : ndarray of shape (1, n_features) or (n_classes, n_features)
|      Coefficient of the features in the decision function.
|
|      `coef_` is of shape (1, n_features) when the given problem is binary.
|      In particular, when `multi_class='multinomial'`, `coef_` corresponds
|      to outcome 1 (True) and `-coef_` corresponds to outcome 0 (False).
|
|  intercept_ : ndarray of shape (1,) or (n_classes,)
|      Intercept (a.k.a. bias) added to the decision function.
|
|      If `fit_intercept` is set to False, the intercept is set to zero.
|      `intercept_` is of shape (1,) when the given problem is binary.
|      In particular, when `multi_class='multinomial'`, `intercept_`
|      corresponds to outcome 1 (True) and `-intercept_` corresponds to
|      outcome 0 (False).
|
|  n_features_in_ : int
|      Number of features seen during :term:`fit`.
|
|      .. versionadded:: 0.24
|
|  feature_names_in_ : ndarray of shape (`n_features_in_`,)
|      Names of features seen during :term:`fit`. Defined only when `X`
|      has feature names that are all strings.
|
|      .. versionadded:: 1.0
|
|  n_iter_ : ndarray of shape (n_classes,) or (1, )
|      Actual number of iterations for all classes. If binary or multinomial,
|      it returns only 1 element. For liblinear solver, only the maximum
|      number of iteration across all classes is given.
|
|      .. versionchanged:: 0.20
|
|          In SciPy <= 1.0.0 the number of lbfgs iterations may exceed
|          ``max_iter``. ``n_iter_`` will now report at most ``max_iter``.
|
|  See Also
|  --------
```

```
 |  SGDClassifier : Incrementally trained logistic regression (when given
 |      the parameter ``loss="log_loss"``).
 |  LogisticRegressionCV : Logistic regression with built-in cross validation.
 |
 |  Notes
 |  -----
 |  The underlying C implementation uses a random number generator to
 |  select features when fitting the model. It is thus not uncommon,
 |  to have slightly different results for the same input data. If
 |  that happens, try with a smaller tol parameter.
 |
 |  Predict output may not match that of standalone liblinear in certain
 |  cases. See :ref:`differences from liblinear <liblinear_differences>`
 |  in the narrative documentation.
 |
 |  References
 |  ----------
 |
 |  L-BFGS-B -- Software for Large-scale Bound-constrained Optimization
 |      Ciyou Zhu, Richard Byrd, Jorge Nocedal and Jose Luis Morales.
 |      http://users.iems.northwestern.edu/~nocedal/lbfgsb.html
 |
 |  LIBLINEAR -- A Library for Large Linear Classification
 |      https://www.csie.ntu.edu.tw/~cjlin/liblinear/
 |
 |  SAG -- Mark Schmidt, Nicolas Le Roux, and Francis Bach
 |      Minimizing Finite Sums with the Stochastic Average Gradient
 |      https://hal.inria.fr/hal-00860051/document
 |
 |  SAGA -- Defazio, A., Bach F. & Lacoste-Julien S. (2014).
 |          :arxiv:`"SAGA: A Fast Incremental Gradient Method With Support
 |          for Non-Strongly Convex Composite Objectives" <1407.0202>`
 |
 |  Hsiang-Fu Yu, Fang-Lan Huang, Chih-Jen Lin (2011). Dual coordinate descent
 |      methods for logistic regression and maximum entropy models.
 |      Machine Learning 85(1-2):41-75.
 |      https://www.csie.ntu.edu.tw/~cjlin/papers/maxent_dual.pdf
 |
 |  Examples
 |  --------
 |  >>> from sklearn.datasets import load_iris
 |  >>> from sklearn.linear_model import LogisticRegression
 |  >>> X, y = load_iris(return_X_y=True)
 |  >>> clf = LogisticRegression(random_state=0).fit(X, y)
 |  >>> clf.predict(X[:2, :])
 |  array([0, 0])
 |  >>> clf.predict_proba(X[:2, :])
 |  array([[9.8...e-01, 1.8...e-02, 1.4...e-08],
 |         [9.7...e-01, 2.8...e-02, ...e-08]])
 |  >>> clf.score(X, y)
 |  0.97...
 |
 |  Method resolution order:
 |      LogisticRegression
 |      sklearn.linear_model._base.LinearClassifierMixin
 |      sklearn.base.ClassifierMixin
 |      sklearn.linear_model._base.SparseCoefMixin
 |      sklearn.base.BaseEstimator
 |      sklearn.utils._metadata_requests._MetadataRequester
 |      builtins.object
 |
 |  Methods defined here:
 |
 |  __init__(self, penalty='l2', *, dual=False, tol=0.0001, C=1.0, fit_intercept=True, intercept_scaling=1, class_
weight=None, random_state=None, solver='lbfgs', max_iter=100, multi_class='auto', verbose=0, warm_start=False, n_j
obs=None, l1_ratio=None)
 |      Initialize self.  See help(type(self)) for accurate signature.
 |
 |  fit(self, X, y, sample_weight=None)
 |      Fit the model according to the given training data.
 |
 |      Parameters
 |      ----------
 |      X : {array-like, sparse matrix} of shape (n_samples, n_features)
 |          Training vector, where `n_samples` is the number of samples and
 |          `n_features` is the number of features.
 |
 |      y : array-like of shape (n_samples,)
 |          Target vector relative to X.
```

```
 |
 |      sample_weight : array-like of shape (n_samples,) default=None
 |          Array of weights that are assigned to individual samples.
 |          If not provided, then each sample is given unit weight.
 |
 |          .. versionadded:: 0.17
 |             *sample_weight* support to LogisticRegression.
 |
 |      Returns
 |      -------
 |      self
 |          Fitted estimator.
 |
 |      Notes
 |      -----
 |      The SAGA solver supports both float64 and float32 bit arrays.
 |
 |  predict_log_proba(self, X)
 |      Predict logarithm of probability estimates.
 |
 |      The returned estimates for all classes are ordered by the
 |      label of classes.
 |
 |      Parameters
 |      ----------
 |      X : array-like of shape (n_samples, n_features)
 |          Vector to be scored, where `n_samples` is the number of samples and
 |          `n_features` is the number of features.
 |
 |      Returns
 |      -------
 |      T : array-like of shape (n_samples, n_classes)
 |          Returns the log-probability of the sample for each class in the
 |          model, where classes are ordered as they are in ``self.classes_``.
 |
 |  predict_proba(self, X)
 |      Probability estimates.
 |
 |      The returned estimates for all classes are ordered by the
 |      label of classes.
 |
 |      For a multi_class problem, if multi_class is set to be "multinomial"
 |      the softmax function is used to find the predicted probability of
 |      each class.
 |      Else use a one-vs-rest approach, i.e calculate the probability
 |      of each class assuming it to be positive using the logistic function.
 |      and normalize these values across all the classes.
 |
 |      Parameters
 |      ----------
 |      X : array-like of shape (n_samples, n_features)
 |          Vector to be scored, where `n_samples` is the number of samples and
 |          `n_features` is the number of features.
 |
 |      Returns
 |      -------
 |      T : array-like of shape (n_samples, n_classes)
 |          Returns the probability of the sample for each class in the model,
 |          where classes are ordered as they are in ``self.classes_``.
 |
 |  set_fit_request(self: sklearn.linear_model._logistic.LogisticRegression, *, sample_weight: Union[bool, NoneTyp
e, str] = '$UNCHANGED$') -> sklearn.linear_model._logistic.LogisticRegression
 |      Request metadata passed to the ``fit`` method.
 |
 |      Note that this method is only relevant if
 |      ``enable_metadata_routing=True`` (see :func:`sklearn.set_config`).
 |      Please see :ref:`User Guide <metadata_routing>` on how the routing
 |      mechanism works.
 |
 |      The options for each parameter are:
 |
 |      - ``True``: metadata is requested, and passed to ``fit`` if provided. The request is ignored if metadata i
s not provided.
 |
 |      - ``False``: metadata is not requested and the meta-estimator will not pass it to ``fit``.
 |
 |      - ``None``: metadata is not requested, and the meta-estimator will raise an error if the user provides it.
 |
 |      - ``str``: metadata should be passed to the meta-estimator with this given alias instead of the original n
```

```
ame.
 |
 |      The default (``sklearn.utils.metadata_routing.UNCHANGED``) retains the
 |      existing request. This allows you to change the request for some
 |      parameters and not others.
 |
 |      .. versionadded:: 1.3
 |
 |      .. note::
 |          This method is only relevant if this estimator is used as a
 |          sub-estimator of a meta-estimator, e.g. used inside a
 |          :class:`pipeline.Pipeline`. Otherwise it has no effect.
 |
 |      Parameters
 |      ----------
 |      sample_weight : str, True, False, or None,                     default=sklearn.utils.metadata_routing.UNCH
ANGED
 |          Metadata routing for ``sample_weight`` parameter in ``fit``.
 |
 |      Returns
 |      -------
 |      self : object
 |          The updated object.
 |
 |  set_score_request(self: sklearn.linear_model._logistic.LogisticRegression, *, sample_weight: Union[bool, NoneT
ype, str] = '$UNCHANGED$') -> sklearn.linear_model._logistic.LogisticRegression
 |      Request metadata passed to the ``score`` method.
 |
 |      Note that this method is only relevant if
 |      ``enable_metadata_routing=True`` (see :func:`sklearn.set_config`).
 |      Please see :ref:`User Guide <metadata_routing>` on how the routing
 |      mechanism works.
 |
 |      The options for each parameter are:
 |
 |      - ``True``: metadata is requested, and passed to ``score`` if provided. The request is ignored if metadata
is not provided.
 |
 |      - ``False``: metadata is not requested and the meta-estimator will not pass it to ``score``.
 |
 |      - ``None``: metadata is not requested, and the meta-estimator will raise an error if the user provides it.
 |
 |      - ``str``: metadata should be passed to the meta-estimator with this given alias instead of the original n
ame.
 |
 |      The default (``sklearn.utils.metadata_routing.UNCHANGED``) retains the
 |      existing request. This allows you to change the request for some
 |      parameters and not others.
 |
 |      .. versionadded:: 1.3
 |
 |      .. note::
 |          This method is only relevant if this estimator is used as a
 |          sub-estimator of a meta-estimator, e.g. used inside a
 |          :class:`pipeline.Pipeline`. Otherwise it has no effect.
 |
 |      Parameters
 |      ----------
 |      sample_weight : str, True, False, or None,                     default=sklearn.utils.metadata_routing.UNCH
ANGED
 |          Metadata routing for ``sample_weight`` parameter in ``score``.
 |
 |      Returns
 |      -------
 |      self : object
 |          The updated object.
 |
 |  ----------------------------------------------------------------------
 |  Data and other attributes defined here:
 |
 |  __annotations__ = {'_parameter_constraints': <class 'dict'>}
 |
 |  ----------------------------------------------------------------------
 |  Methods inherited from sklearn.linear_model._base.LinearClassifierMixin:
 |
 |  decision_function(self, X)
 |      Predict confidence scores for samples.
 |
 |      The confidence score for a sample is proportional to the signed
```

```
|      distance of that sample to the hyperplane.
|
|      Parameters
|      ----------
|      X : {array-like, sparse matrix} of shape (n_samples, n_features)
|          The data matrix for which we want to get the confidence scores.
|
|      Returns
|      -------
|      scores : ndarray of shape (n_samples,) or (n_samples, n_classes)
|          Confidence scores per `(n_samples, n_classes)` combination. In the
|          binary case, confidence score for `self.classes_[1]` where >0 means
|          this class would be predicted.
|
|  predict(self, X)
|      Predict class labels for samples in X.
|
|      Parameters
|      ----------
|      X : {array-like, sparse matrix} of shape (n_samples, n_features)
|          The data matrix for which we want to get the predictions.
|
|      Returns
|      -------
|      y_pred : ndarray of shape (n_samples,)
|          Vector containing the class labels for each sample.
|
|  ----------------------------------------------------------------------
|  Methods inherited from sklearn.base.ClassifierMixin:
|
|  score(self, X, y, sample_weight=None)
|      Return the mean accuracy on the given test data and labels.
|
|      In multi-label classification, this is the subset accuracy
|      which is a harsh metric since you require for each sample that
|      each label set be correctly predicted.
|
|      Parameters
|      ----------
|      X : array-like of shape (n_samples, n_features)
|          Test samples.
|
|      y : array-like of shape (n_samples,) or (n_samples, n_outputs)
|          True labels for `X`.
|
|      sample_weight : array-like of shape (n_samples,), default=None
|          Sample weights.
|
|      Returns
|      -------
|      score : float
|          Mean accuracy of ``self.predict(X)`` w.r.t. `y`.
|
|  ----------------------------------------------------------------------
|  Data descriptors inherited from sklearn.base.ClassifierMixin:
|
|  __dict__
|      dictionary for instance variables (if defined)
|
|  __weakref__
|      list of weak references to the object (if defined)
|
|  ----------------------------------------------------------------------
|  Methods inherited from sklearn.linear_model._base.SparseCoefMixin:
|
|  densify(self)
|      Convert coefficient matrix to dense array format.
|
|      Converts the ``coef_`` member (back) to a numpy.ndarray. This is the
|      default format of ``coef_`` and is required for fitting, so calling
|      this method is only required on models that have previously been
|      sparsified; otherwise, it is a no-op.
|
|      Returns
|      -------
|      self
|          Fitted estimator.
|
|  sparsify(self)
```

```
|      Convert coefficient matrix to sparse format.
|
|      Converts the ``coef_`` member to a scipy.sparse matrix, which for
|      L1-regularized models can be much more memory- and storage-efficient
|      than the usual numpy.ndarray representation.
|
|      The ``intercept_`` member is not converted.
|
|      Returns
|      -------
|      self
|          Fitted estimator.
|
|      Notes
|      -----
|      For non-sparse models, i.e. when there are not many zeros in ``coef_``,
|      this may actually *increase* memory usage, so use this method with
|      care. A rule of thumb is that the number of zero elements, which can
|      be computed with ``(coef_ == 0).sum()``, must be more than 50% for this
|      to provide significant benefits.
|
|      After calling this method, further fitting with the partial_fit
|      method (if any) will not work until you call densify.
|
|  ----------------------------------------------------------------------
|  Methods inherited from sklearn.base.BaseEstimator:
|
|  __getstate__(self)
|      Helper for pickle.
|
|  __repr__(self, N_CHAR_MAX=700)
|      Return repr(self).
|
|  __setstate__(self, state)
|
|  __sklearn_clone__(self)
|
|  get_params(self, deep=True)
|      Get parameters for this estimator.
|
|      Parameters
|      ----------
|      deep : bool, default=True
|          If True, will return the parameters for this estimator and
|          contained subobjects that are estimators.
|
|      Returns
|      -------
|      params : dict
|          Parameter names mapped to their values.
|
|  set_params(self, **params)
|      Set the parameters of this estimator.
|
|      The method works on simple estimators as well as on nested objects
|      (such as :class:`~sklearn.pipeline.Pipeline`). The latter have
|      parameters of the form ``<component>__<parameter>`` so that it's
|      possible to update each component of a nested object.
|
|      Parameters
|      ----------
|      **params : dict
|          Estimator parameters.
|
|      Returns
|      -------
|      self : estimator instance
|          Estimator instance.
|
|  ----------------------------------------------------------------------
|  Methods inherited from sklearn.utils._metadata_requests._MetadataRequester:
|
|  get_metadata_routing(self)
|      Get metadata routing of this object.
|
|      Please check :ref:`User Guide <metadata_routing>` on how the routing
|      mechanism works.
|
|      Returns
```

```
|      -------
|      routing : MetadataRequest
|          A :class:`~utils.metadata_routing.MetadataRequest` encapsulating
|          routing information.
|
|  ----------------------------------------------------------------------
|  Class methods inherited from sklearn.utils._metadata_requests._MetadataRequester:
|
|  __init_subclass__(**kwargs) from builtins.type
|      Set the ``set_{method}_request`` methods.
|
|      This uses PEP-487 [1]_ to set the ``set_{method}_request`` methods. It
|      looks for the information available in the set default values which are
|      set using ``__metadata_request__*`` class attributes, or inferred
|      from method signatures.
|
|      The ``__metadata_request__*`` class attributes are used when a method
|      does not explicitly accept a metadata through its arguments or if the
|      developer would like to specify a request value for those metadata
|      which are different from the default ``None``.
|
|      References
|      ----------
|      .. [1] https://www.python.org/dev/peps/pep-0487
```

Mudcard

In [ ]: