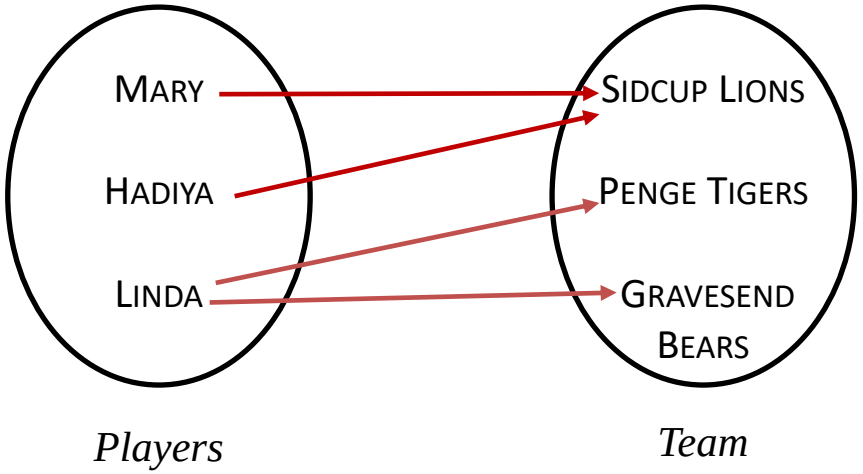
Relations and Functions – Answers to Tutorial Questions

1. The diagram below represents the relation "*plays for*" between the set *Players* and the set *Teams*:



a) Represent the relation in terms of a set of ordered pairs.

b) Write in words: MARY $\mathscr{R}$ SIDCUP LIONS

Solution

a) R = {(MARY, SIDCUP LIONS), (HADIYA, SIDCUP LIONS), (LINDA, PENGE TIGERS), (LINDA, GRAVESEND BEARS)}

b) Mary plays for Sidcup Lions.

2. A and B are two sets and R is a relation from set A to set B .

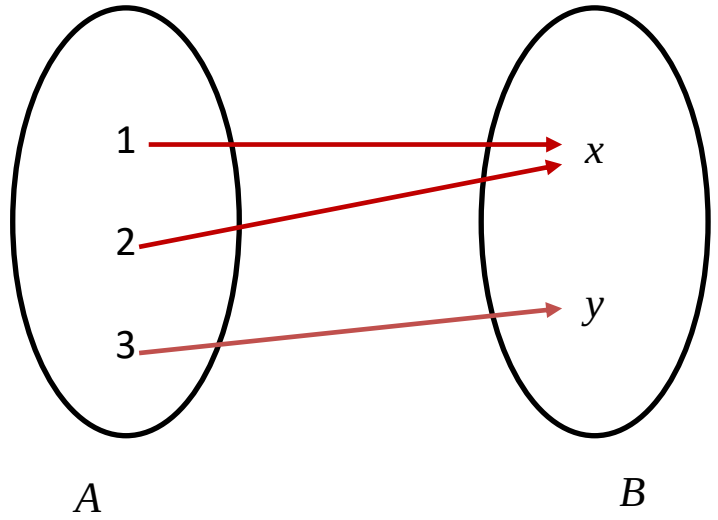
$A = \{ 1, 2, 3\}$

$B = \{ x, y\}$

$R = \{(1, x), (2, x), (3, y)\}$

Represent the relation R pictorially.

Solution



3. A relation R is specified as follows:

$$R = \{ (a, 2), (d, 9), (b, 4), (c, 7), (a, 1) \}$$

Give the value of R^{-1}, the inverse of this relation.

Solution

$$R^{-1} = \{ (2, a), (9, d), (4, b), (7, c), (1, a) \}$$

4. Consider the relation "*is less than*" on the set of integers.

   Is this relation: a) symmetric
   b) reflexive
   c) transitive?

   In each case give a reason for your answer.

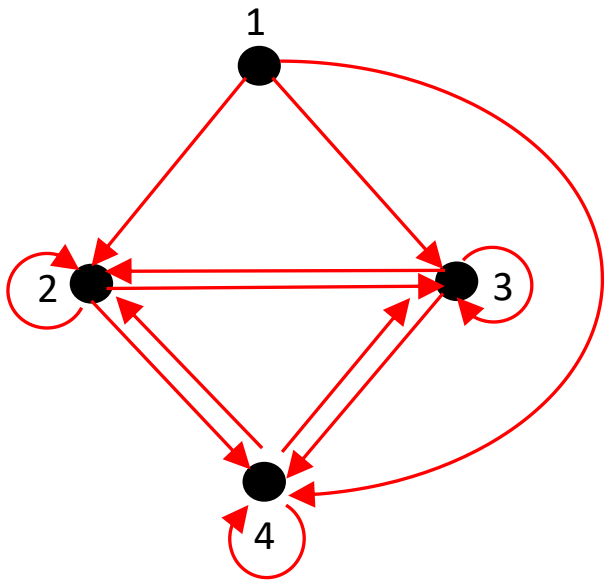
   <u>Solution</u>

   a) It is not symmetric because if $a < b$, then $b \not< a$. For example 4 is less than 5, but 5 is not less than 4.

   b) It is not reflexive because in no case is $a < a$.

   c) It is transitive, because in every case if $a < b$ and $b < c$, then $a < c$. For example 3 is less then 5 and 5 is less than 6 – of course 3 is also less than 6, and this is true in every case.

5. Consider the digraph below which shows a relation on the set {1, 2, 3, 4} :



Is this relation:

a) Reflexive? b) Symmetric c) Transitive?

Solution

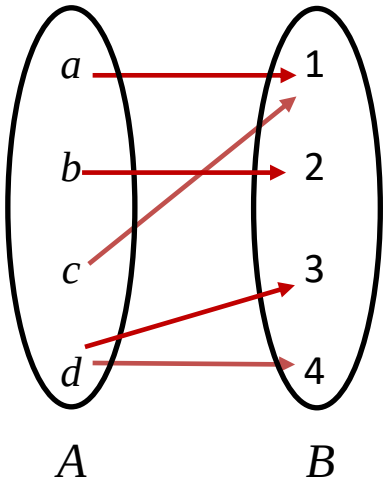
a) It is not reflexive because 1 $\not\mathcal{R}$ 1.

b) It is not symmetric because, for example, 1 $\mathcal{R}$ 3 but 3 $\not\mathcal{R}$ 1.

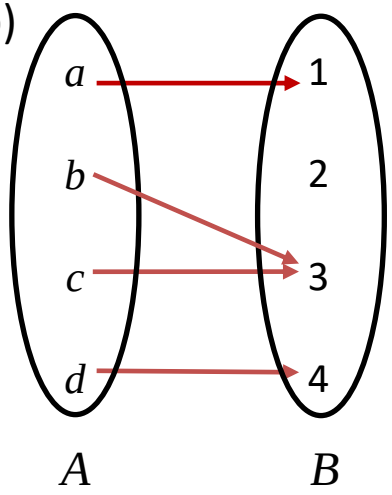
c) it is transitive because in every case if $a \mathcal{R} b$ and $b \mathcal{R} c$ then $a \mathcal{R} c$.

6. Which of the diagrams below represents a function?

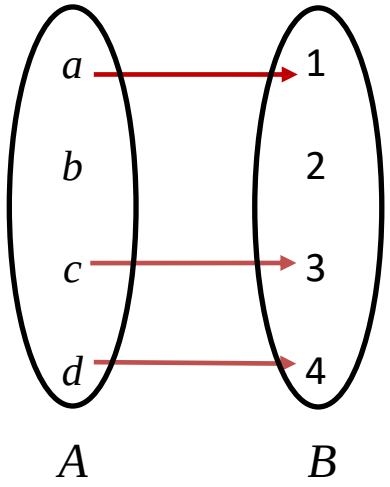
a)



b)



c)



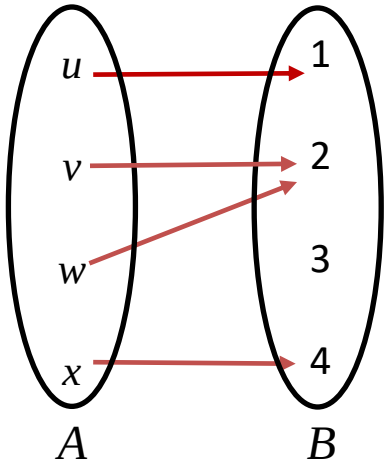
Solution

a) This is not a function because d maps on to two elements in the codomain.

b) This is a function.

c) This is not a function because b has no image in the codomain.

7. A function f, which maps from a set A to a set B, is represented pictorially below:



What is the value of the following?

a) $f(u)$ b) $f(v)$ c) $f(w)$ d) $f(x)$

Solution

a) $f(u) = 1$ b) $f(v) = 2$ c) $f(w) = 2$ d) $f(x) = 4$

8. A function f is specified as follows:

$$f : \mathbb{Z} \rightarrow \mathbb{Z}$$
$$f(x) = 4x^2 - 5$$

What is the value of the following?

a) $f(3)$ b) $f(-1)$ c) $f(0)$

Solution

a) $f(3) = 4 \times 3^2 - 5 = 4 \times 9 - 5 = 31$

b) $f(-1) = 4 \times (-1)^2 - 5 = -1$

c) $f(0) = -5$

9. Consider the following function:

$$f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$$
$$f(x, y) = 2x^2 + 3y$$

State the value of:

a) $f(2, 0)$ b) $f(1, -1)$

Solution

a) $f(2, 0) = 2 \times 2^2 + 3 \times 0 = 8$

b) $f(1, -1) = 2 \times 1^2 + 3 \times -1 = -1$

10. $g(x) = 3x + 1$ $f(x) = x^3$

Calculate: $f(g(3))$

Solution

$g(3) = 3 \times 3 + 1 = 10$

$f(10) = 10^3 = 1000$

11. Write a complete specification (signature and behaviour) of a function that accepts two integers and outputs a number which is twice the sum of these two integers.

    Solution

$$f: \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$$

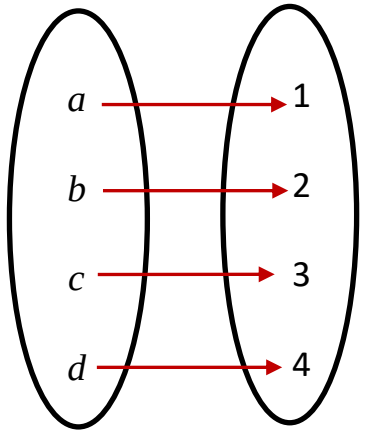
$$f(x, y) = 2(x + y)$$

12. Write a Java method that implements the example in question 11.

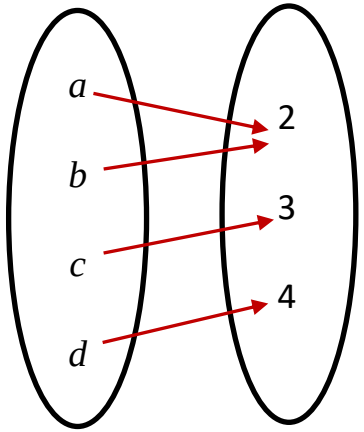
```
int myFunction(int firstIn, int secondIn)
{
    return 2 * (firstIn + secondIn);
}
```

13. Consider the functions below. For each one, say whether it is an onto function, a one-to-one function, neither or both.

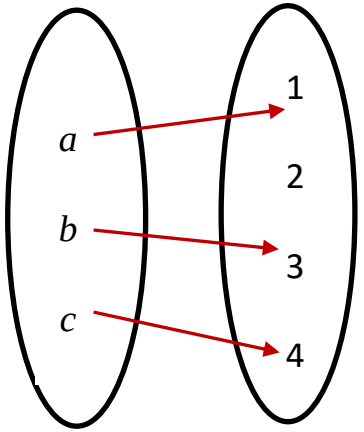
a)



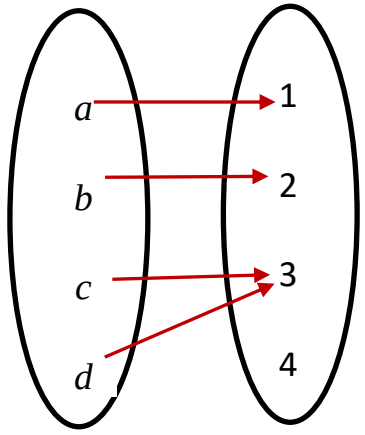
b)



c)



d)



Solution

a) This is both one-to-one and onto (it is a bijective function).

b) This is onto, but not one-to-one.

c) This is one-to-one, but not onto.

d) This is neither one-to-one nor onto.

14. Consider the following function:

$$f: \mathbb{R} \to \mathbb{R}$$
$$f(x) = x^2$$

Is this function:

a) onto?
b) one-to-one?

Solution

a) It is not an onto function, because the negative numbers in the codomain are not images of any number in the domain.

b) It is not a one-to-one function, because more than one element from the domain maps to the same element in the codomain. For example, both 2 and -2 map onto 4.