Statistics – Answers to Tutorial Questions

1. Consider the following data set: 1, 2, 3, 3, 4, 4, 4, 6, 8, 8, 10, 11, 11

    Calculate: a) The mean   b) The mode   c) The median   d) The range   e) The interquartile range

    Solution

    a) $\text{mean} = \dfrac{1+2+3+3+4+4+4+6+8+8+10+11+11}{13} = \dfrac{75}{13} = 5.77$

    b) The mode is the most common value, 4.

    c) The median is the middle value, 4.

    d) Range = 11 – 1 = 10.

    e) $\dfrac{13}{4} = 3.25$ Round down to 3

       The 3rd item is 3 – this is the lower quartile.

       $\dfrac{3 \times 13}{4} = 9.75$ Round up to 10

       The 10th item is 8 – this is the upper quartile.

       The interquartile range = 8 – 3 = 5.

2. A survey is conducted on a number of similar households to find how much they spend per week on food. The results are summarised in a box plot, as shown.

100   110   120   130   140   150   160   170   180   190   200   210

Weekly spend (pounds sterling)
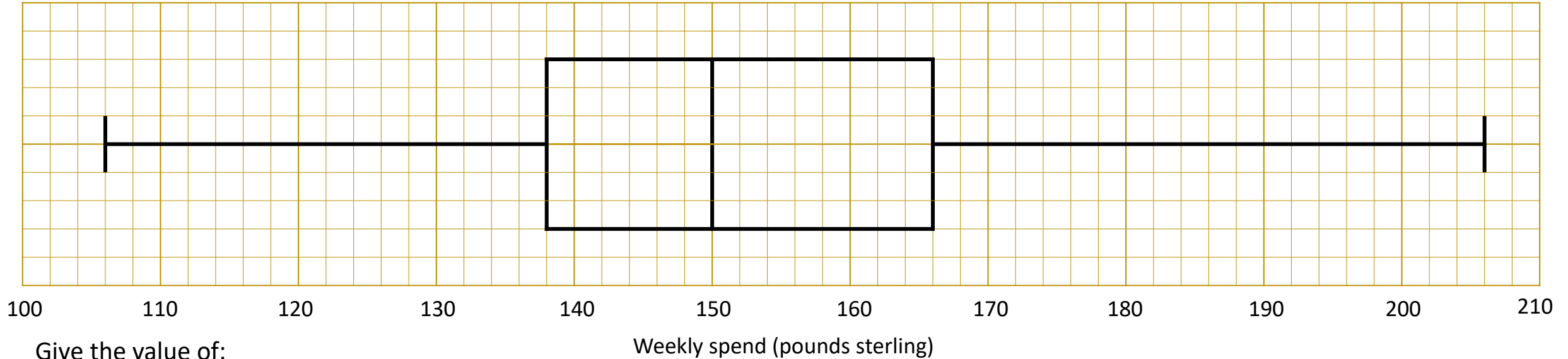
Give the value of:

a) The lowest value recorded.   b) The highest value recorded.   c) The range.
d) The median.   e) The lower quartile.   f) The upper quartile
g) The interquartile range.

Solution

a) 106   b) 206   c) 206 – 106 = 100

d) 150   e) 138   f) 166

g) 166 – 138 = 28

3. A group of 75 UK students were surveyed to find out how many (if any) GCSE exams they had passed. The results are shown in the table on the right.

Number of GCSE passes, n	Frequency f
0	1
1	2
2	2
3	5
4	7
5	9
6	10
7	14
8	12
9	8
10	5
Total	75

From the table find the following: a) The mean
b) The mode
c) The median
d) The interquartile range

Solution

a) Add a column for the product fn.

Number of GCSE passes, n	Frequency f	Frequency fn
0	1	0
1	2	2
2	2	4
3	5	15
4	7	28
5	9	45
6	10	60
7	14	98
8	12	96
9	8	72
10	5	50
Total	75	470

$$\text{mean} = \frac{\Sigma fn}{\Sigma f} = \frac{470}{75} = 6.3$$

b) The most commonly occurring total is 7, which occurs 14 times. The mode is 7.

For c) and d) we need a column for the cumulative frequency.

Number of GCSE passes, n	Frequency f	Cumulative frequency
0	1	1
1	2	3
2	2	5
3	5	10
4	7	17
5	9	26
6	10	36
7	14	50
8	12	62
9	8	70
10	5	75
Total	75	

c) There are 75 items. The median is therefore the 38th item.

The median is 7.

d) $\dfrac{75}{4} = 18.75$ Round up to 19

The 19th item is 5. This is the lower quartile.

$$\dfrac{3 \times 75}{4} = 56.25$$

Round down to 56

The 56th item is 8. This is the upper quartile.

The interquartile range is 8 – 5 = 3

IQ	Frequency, f
$85 \le n < 90$	3
$90 \le n < 95$	12
$95 \le n < 100$	26
$100 \le n < 105$	22
$105 \le n < 110$	19
$110 \le n < 115$	8
$115 \le n < 120$	5
$120 \le n < 125$	3
$125 \le n < 130$	2
Total	100

4. 100 students were given an IQ test. The results are summarised in the table on right.

a) Find the mean IQ of this group.
b) What is the modal group?
c) What is the median group?

Solution

a) We need a column for the mid-point, x, and a column for the product, fx.

IQ	Mid-point, x	Frequency, f	Product, fx
$85 \le n < 90$	87	3	261
$90 \le n < 95$	92	12	1104
$95 \le n < 100$	97	26	2522
$100 \le n < 105$	102	22	2244
$105 \le n < 110$	107	19	2033
$110 \le n < 115$	112	8	896
$115 \le n < 120$	117	5	585
$120 \le n < 125$	122	3	366
$125 \le n < 130$	127	2	254
Total		100	10265

$$\text{mean} = \frac{\Sigma fx}{\Sigma f} = \frac{10265}{100} = 102.65$$

b) The modal group is $95 \le n < 100$, which contains 26 items.

c) To find the median group, we need to add a column for the cumulative frequency

IQ	Frequency, f
$85 \le n < 90$	3
$90 \le n < 95$	12
$95 \le n < 100$	26
$100 \le n < 105$	22
$105 \le n < 110$	19
$110 \le n < 115$	8
$115 \le n < 120$	5
$120 \le n < 125$	3
$125 \le n < 130$	2
Total	100

IQ	Frequency, f	Cumulative frequency
$85 \le n < 90$	3	3
$90 \le n < 95$	12	15
$95 \le n < 100$	26	41
$100 \le n < 105$	22	63
$105 \le n < 110$	19	82
$110 \le n < 115$	8	90
$115 \le n < 120$	5	95
$120 \le n < 125$	3	98
$125 \le n < 130$	2	100
Total	100	

There are 100 items. The median is therefore the mean of the 50th and the 51st items.

The 50th and 51st item lie in the group $100 \le n < 105$. This is the median group.
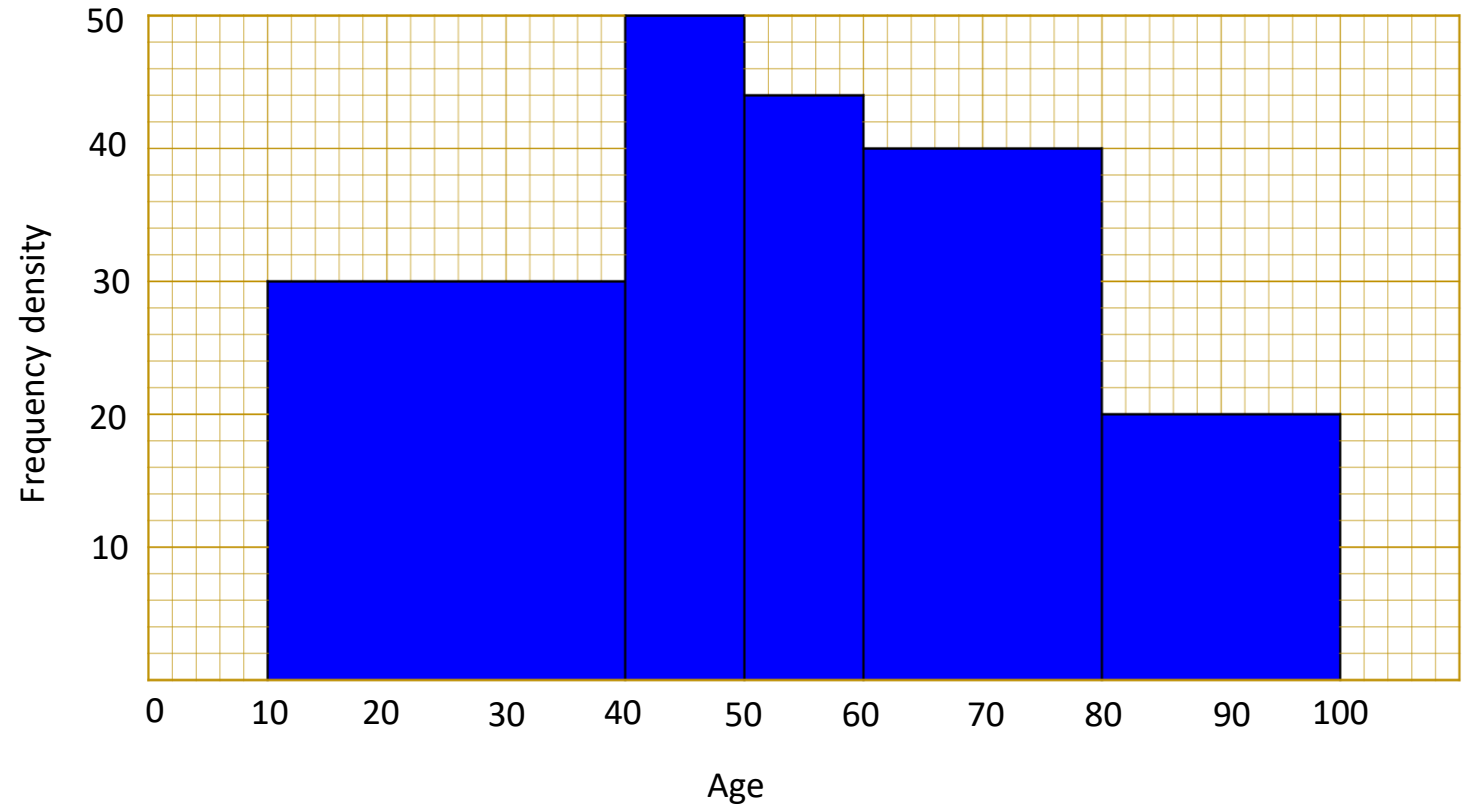
5. A survey was undertaken to find out the ages of people (from 10 onwards) visiting a particular museum over the bank holiday period. A histogram was produced, and is shown below.

a) How many visitors were there in the age range of $60 \leq n < 80$?

b) In which range was there a total of 900 visitors? Explain your answer.

Solution

a) *frequency = frequency density* x *width*

= 20 x 40 = 800

b) There were 900 visitors in the range $10 \leq n < 40$

frequency density x *width* = 30 x 30 = 900

6. A class of 190 students took an examination. The results were entered into a table and a histogram was produced.

Incomplete versions of the table and histogram are show here.

a) Work out the scale of the vertical axis and show this on the table.

b) Complete the table and histogram.

Marks, x	Frequency, f
$0 \leq x < 30$	
$30 \leq x < 40$	34
$40 \leq x < 50$	38
$50 \leq x < 60$	
$60 \leq x < 80$	32
$80 \leq x \leq 100$	

Frequency density

Marks (out of 100)
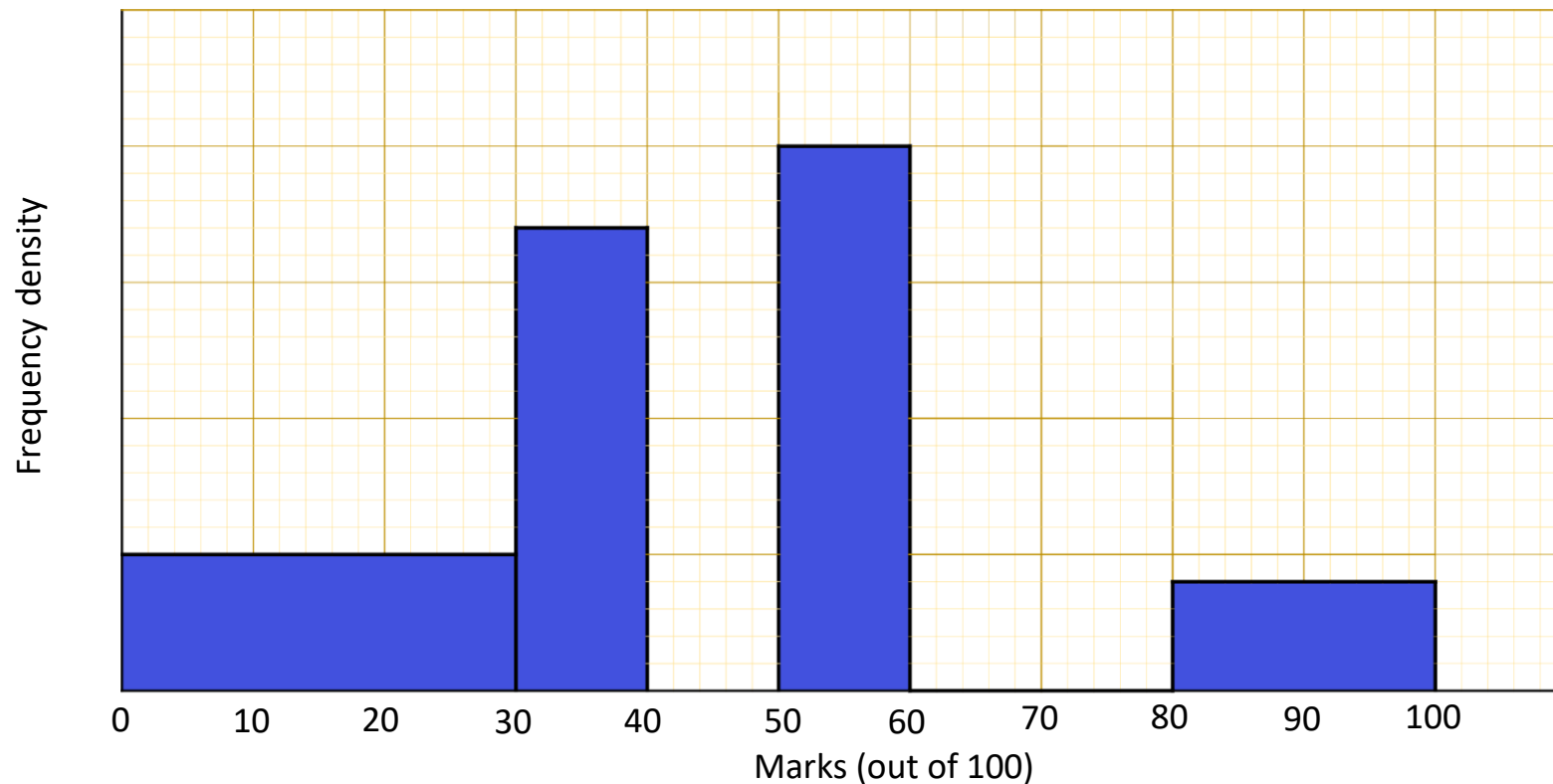
a) We need a group that is filled in both in the table and on the chart. We can use the second group.

$$frequency\ density = \frac{frequency}{width} = {}^{34}/_{10} = 3.4$$

The height of this bar is 3.4, enabling us to calibrate the axis – the height of each square represents a frequency density of 1.

b) We can now use the formula $frequency = frequency\ density \times width$ to complete the table and the histogram.

Marks, x	Frequency, f
$0 \leq x < 30$	30
$30 \leq x < 40$	34
$40 \leq x < 50$	38
$50 \leq x < 60$	40
$60 \leq x < 80$	32
$80 \leq x \leq 100$	16

30 x 1.0 = 30

20 x 0.8 = 16

10 x 4.0 = 40

38 ÷ 10 = 3.8
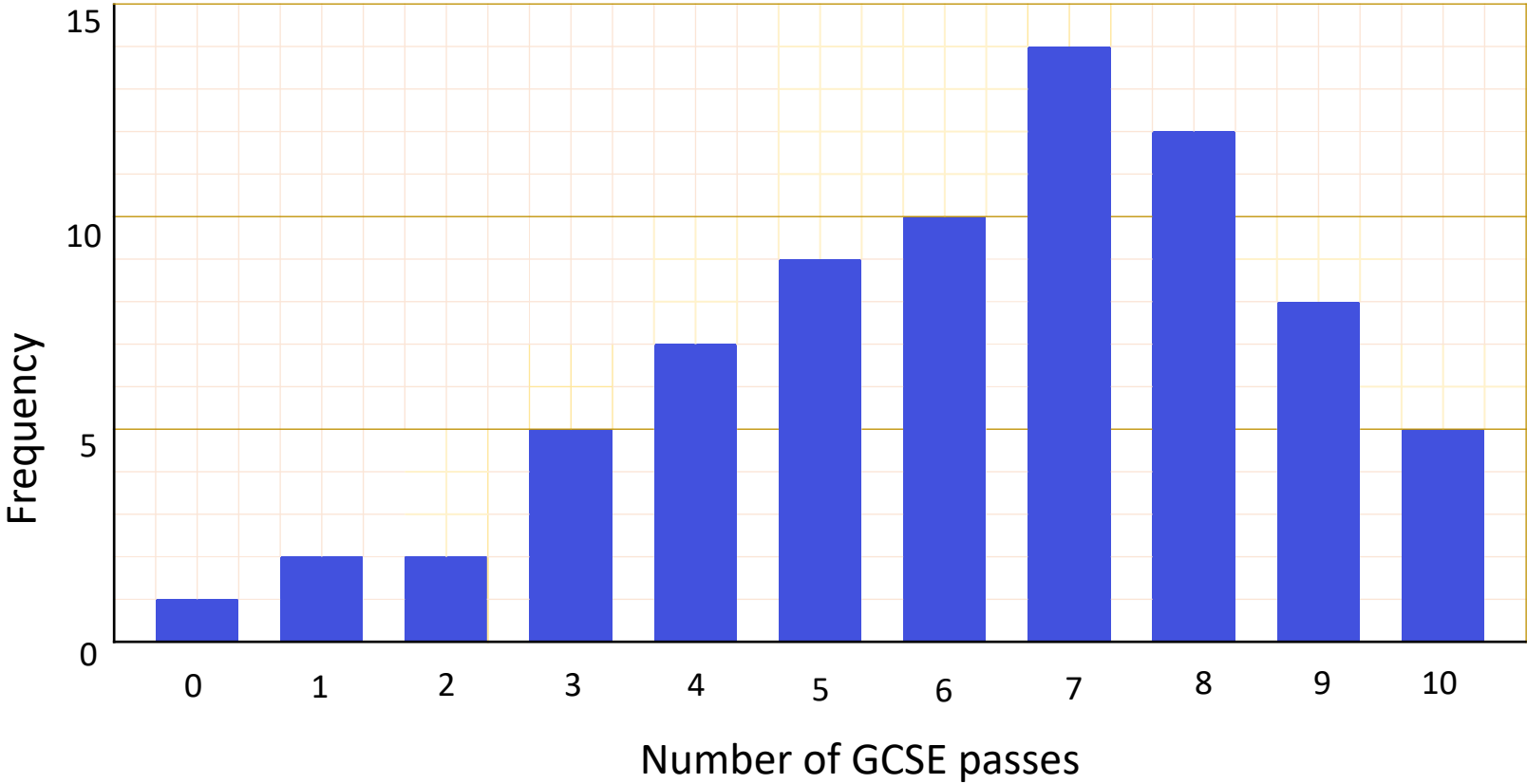
32 ÷ 20 = 1.6
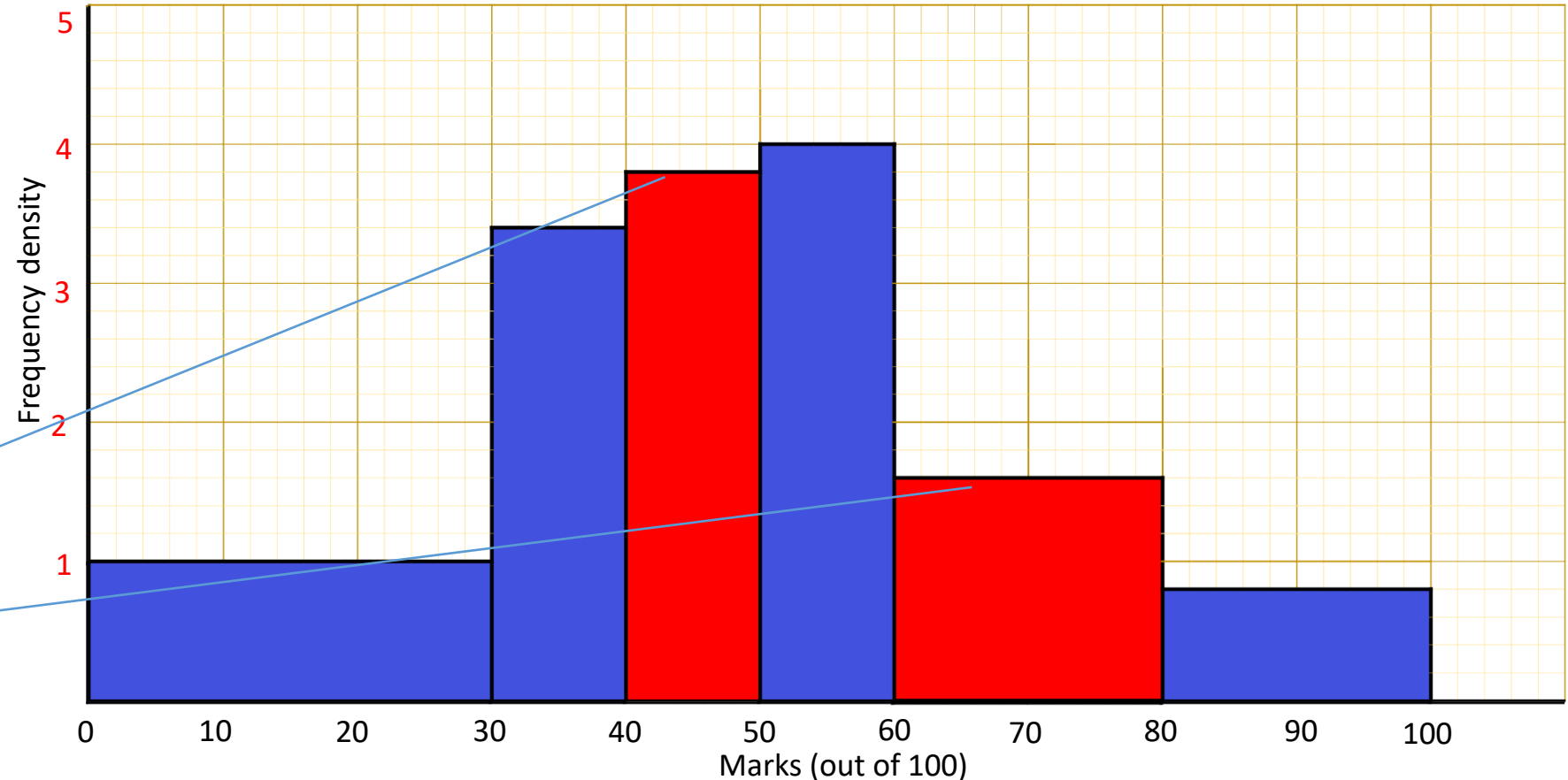
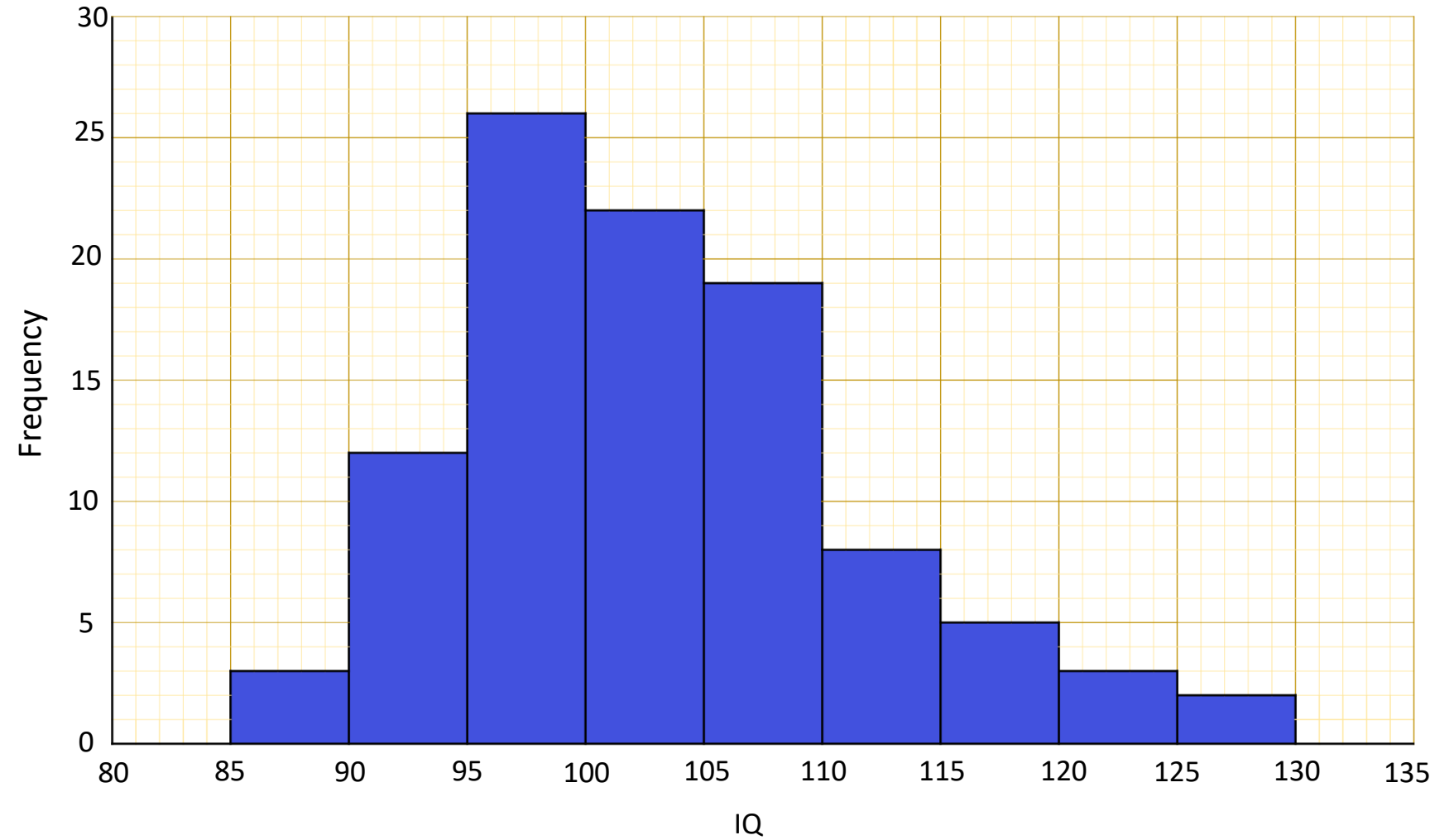7. Draw a bar chart to represent the information in question 3.

Solution

Number of GCSE passes, n	Frequency f
0	1
1	2
2	2
3	5
4	7
5	9
6	10
7	14
8	12
9	8
10	5
Total	75

8. Draw a frequency polygon to represent the data in question 4.

Solution

IQ	Frequency, f
$85 \leq n < 90$	3
$90 \leq n < 95$	12
$95 \leq n < 100$	26
$100 \leq n < 105$	22
$105 \leq n < 110$	19
$110 \leq n < 115$	8
$115 \leq n < 120$	5
$120 \leq n < 125$	3
$125 \leq n < 130$	2
Total	100

Advanced Question

9. Using formulae, estimate the median and the mode of the data shown in question 4.

IQ	Frequency, f	Cumulative frequency
$85 \le n < 90$	3	3
$90 \le n < 95$	12	15
$95 \le n < 100$	26	41
$100 \le n < 105$	22	63
$105 \le n < 110$	19	82
$110 \le n < 115$	8	90
$115 \le n < 120$	5	95
$120 \le n < 125$	3	98
$125 \le n < 130$	2	100
Total	**100**	

Solution

$$Estimated\ Median = L + \frac{n/2 - cf_b}{f_m} \times w$$

Where: L is the lower class boundary of the median group
n is the total number of items
cf_b is the cumulative frequency of the groups before the median group.
f_m is frequency of the median group
w is the group width

We saw that the median group is $100 \le n < 105$

$$Estimated\ Median = 100 + \frac{100/2 - 41}{22} \times 5 \approx 102$$

$$Estimated\ Mode = L + \frac{f_m - f_{m-1}}{(f_m - f_{m-1}) + (f_m - f_{m+1})} \times w$$

Where: L is the lower class boundary of the modal group
f_{m-1} is the frequency of the group before the modal group
f_m is the frequency of the modal group
f_{m+1} is the frequency of the group after the modal group
w is the group width

The modal group is $95 \le n < 100$

$$Estimated\ Mode = 95 + \frac{26 - 12}{(26 - 12) + (26 - 22)} \times 5 \approx 99$$