CN4004 Maths for Computing

Sets and Groups 2 : Tutorial

1. Use the laws of set algebra to simplify the following expression:

$$A \cup (\overline{A} \cap B)$$

2. Use set algebra to show that: $B \cap (\overline{A \cap B}) = B \cap \bar{A}$

3. In the last tutorial you drew Venn diagrams to prove the following:

$$A \backslash B = A \cap \overline{B}$$

   a) Bearing this in mind, use set algebra to show that:

$$B \cup (A \backslash B) = B \cup A$$

   b) Verify that this is true by drawing a Venn diagram.

4. Consider the following numbers: 2.35 -335 2 -3.75 0

   Which of these numbers are:

   a) Real numbers b) Integers c) Natural numbers?

5. Which of the following is a rational number? Give a reason for your answer.

   7.5 e (Euler's constant) $\sqrt{7}$ $\sqrt{25}$

6. Simplify the following expressions containing complex numbers:

   a) $i(3 + i)$ b) $(2 - 3i)(4 + i)$ c) $(3 - i)^2$ d) $(1 + i)^3$

   e) $\frac{3+4i}{1+i}$

7. Consider the following sets:

A = the set of integers less than 10

B = the set of natural numbers less than 10

S = the set of people living in London

T = the set of real numbers greater than 1000

In each case, state whether the set is: (a) countable or non-countable
(b) finite or infinite.

8. Consider the following combinations of a set and an operation, and state, giving reasons, which of them constitute a formal group:

a) Natural numbers under subtraction

b) The set {0} under addition

c) The set {1} under multiplication

9. Show that the set of natural numbers starting from 0 ($\mathbb{N}_0$) under addition is a monoid but not a group.

10. Show that the set of natural numbers starting from 1 ($\mathbb{N}_1$) under addition is a semigroup but not a group.

11. (This is a very hard question, so don't worry if you have trouble solving it!)

Is the set of rational numbers countable? Can you provide a proof for your answer?