Sets and Groups Part 1 – Answers to Tutorial Questions

1. Express the following specification of a set M in words:

$$M = \{x \in \mathbb{N} | x \geq 50\}$$

Solution

M is the set of natural numbers greater than or equal to 50.

2. Using set comprehension, specify a set A that contains all the integers greater than -5 and less than 5.

Solution

$$A = \{x \in \mathbb{Z} | -5 < x < 5\}$$

3. State whether each of the following sets is finite or infinite:

   a) The set of natural numbers between 50 and 100.
   b) The set of real numbers less than 10.
   c) The set of chemical elements discovered so far.

   Solution

   a) finite b) infinite c) finite

4. Consider the following sets:

$A = \{ a, b, d, e, g, h, x\}$ $B = \{ a, b , c, d \}$ $C = \{ g, h, x, a\}$ $D = \{ h, x, a, g\}$

For each of the following, state whether the expression is true or false:

a) $C \subset A$ b) $A \subset C$ c) $C = D$ d) $B \not\subset A$ e) $D \subset C$ f) $C \subseteq D$

Solution

a) True

b) False

c) True

d) True

e) False

f) True

5. Consider the following sets:

A = { APPLE, ORANGE, PEAR, BANANA, PLUM, LEMON}
B = { APPLE, MANGO, ORANGE }
C = { ORANGE, GRAPE, CHERRY}
D = { BANANA }

a) Evaluate the following:

i) $A \cap B$ ii) $B \cup C$ iii) $A \setminus B$ iv) $B \cap D$ v) B x D vi) $n(C)$

b) If the universal set is {APPLE, ORANGE, PEAR, BANANA, PLUM, LEMON, MANGO, GRAPE, CHERRY, PINEAPPLE }, what is the value of $\overline{A}$?

Solution

a) i) $A \cap B$ = { APPLE, ORANGE}

ii) $B \cup C$ = { APPLE, MANGO, ORANGE, GRAPE, CHERRY}

iii) $A \setminus B$ = { PEAR, BANANA, PLUM, LEMON }

iv) $B \cap D = \varnothing$

v) B x D = { (APPLE, BANANA), (MANGO, BANANA), (ORANGE, BANANA) }

vi) $n(C) = 3$

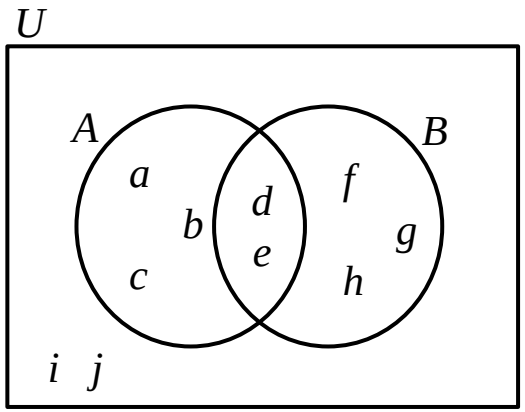
b) $\overline{A}$ = { MANGO, GRAPE, CHERRY, PINEAPPLE}

6. $A = \{a, b, c, d, e\}$ $B = \{f, d, e, g, h\}$

The universal set $U = \{a, b, c, d, e, f, g, h, i, j\}$

Represent this information on a Venn diagram.

Solution



7. Consider the following sets:

A = { APPLE, ORANGE, PEAR, BANANA, PLUM, LEMON}
B = { APPLE, MANGO, ORANGE }
C = { ORANGE, GRAPE, CHERRY}
D = { BANANA }

Evaluate the following: $A \Delta B$

Solution

$$A \Delta B = A \setminus B \cup B \setminus A$$

$$= \{ \text{PEAR, BANANA, PLUM, LEMON} \} \cup \{ \text{MANGO} \} = \{ \text{PEAR, BANANA, PLUM, LEMON , MANGO} \}$$

8. This question refers to 30 people who were surveyed about the type of vehicles they own.

B is the set of people who own bicycles, and C is the set of people who own cars.

15 people own bicycles, and 12 own cars. 4 people own both.

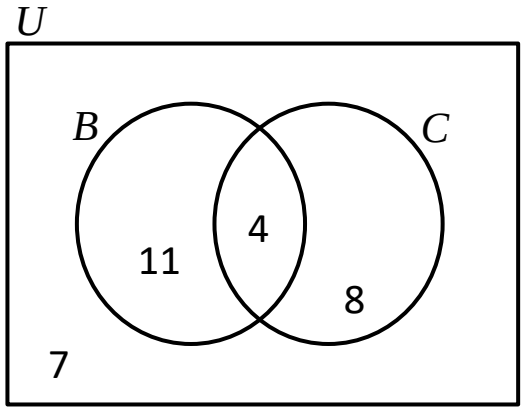
a) Represent this information on a Venn diagram.

b) Give values for the following:

i) $n(B \cap C)$ ii) $n(B \cup C)$ iii) $n(B \setminus C)$ iv) $n(\overline{B \cup C})$

Solution

a)



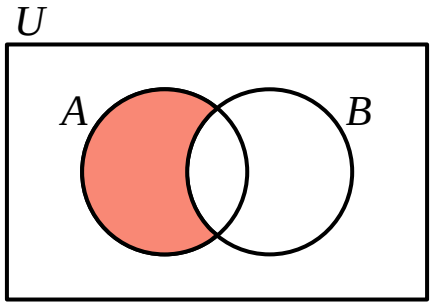
b) i) $n(B \cap C) = 4$

ii) $n(B \cup C) = 23$

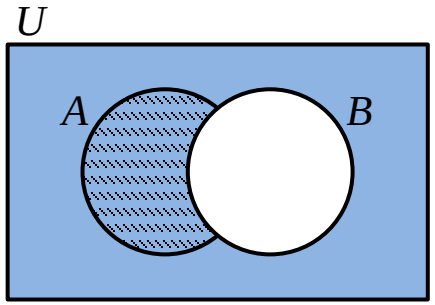
iii) $n(B \setminus C) = 11$

iv) $n(\overline{B \cup C}) = 7$

9. By drawing a Venn diagram show that: $A \backslash B = A \cap \overline{B}$



$A \backslash B$



$\overline{B}$

$A \cap \overline{B}$

10. If A is the set $\{x, y, z\}$, what is the power set, $P(A)$?

Solution

$P(A) = \{\varnothing, \{x\}, \{y\}, \{z\}, \{x, y\}, \{x, z\}, \{y, z\}, \{x, y, z\}\}$

11. a) If a set has a cardinality of 4, then how many elements will be in the power set?

b) How many **proper** subsets does the above set have?

Solution

a) Number of elements in the power set = $2^4 = 16$

b) Number of proper subsets contained in the power set = 15.