Mathematical Logic, Part 1 – Answers to Tutorial Questions

1. If P, Q and R represent the following statements: P: 2 x 25 = 50

Q: Germany is in Asia.

R: 3.65 is an integer

What is the value of a) $P \wedge R$ b) $P \vee Q$ c) $Q \vee R$?

Solution

P is true
Q is false
R is false

a) $P \wedge R \equiv \mathsf{T} \wedge \mathsf{F} \equiv \mathsf{F}$

b) $P \vee Q \equiv \mathsf{T} \vee \mathsf{F} \equiv \mathsf{T}$

c) $Q \vee R \equiv \mathsf{F} \vee \mathsf{F} \equiv \mathsf{F}$

2. Let P be "It is summer" and Q be "Leon is playing tennis". Give simple sentences which represent the following statements:

a) $\neg P$ b) $P \wedge \neg Q$
c) $\neg P \vee Q$ d) $\neg \neg Q$

Solution

a) It is not summer.
b) It is summer and Leon is not playing tennis.
c) It is not summer or Leon is playing tennis.
d) Leon is playing tennis.

3. Let P be "She is a scientist" and Q be "She is intelligent". Express each of the following statements symbolically:

a) She is intelligent, but she is not a scientist.
b) She is a scientist, and she is intelligent.
c) She is a scientist, or she is not intelligent.
d) It is not true that she is a scientist or that she is intelligent.

Solution

a) $Q \wedge \neg P$ b) $P \wedge Q$

c) $P \vee \neg Q$ d) $\neg(P \vee Q)$

4. Construct a truth table for the following expression:

$$\neg(P \wedge Q) \vee \neg Q$$

Solution

P	Q	$P \wedge Q$	$\neg(P \wedge Q)$	$\neg Q$	$\neg(P \wedge Q) \vee \neg Q$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	F	T	F	T
F	F	F	T	T	T

5. Show that the expression $P \Rightarrow (P \vee Q)$ is a tautology by constructing a truth table

P	Q	$P \vee Q$	$P \Rightarrow (P \vee Q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

6. Consider the following statement:

$$P \Rightarrow \neg Q$$

What is a) the converse;
b) the inverse;
c) the contrapositive?

Solution

a) $\neg Q \Rightarrow P$

b) $\neg P \Rightarrow \neg \neg Q \equiv \neg P \Rightarrow Q$

c) $\neg \neg Q \Rightarrow \neg P \equiv Q \Rightarrow \neg P$

7. Use De Morgan's law to show that:

$$\neg(\neg P \wedge (P \vee Q)) \equiv P \vee (\neg P \wedge \neg Q)$$

Solution

$$\neg(\neg P \wedge (P \vee Q)) \equiv P \vee \neg(P \vee Q)$$

$$\equiv P \vee (\neg P \wedge \neg Q)$$

8. Use the distributive law to simplify the following expression:

$$\neg Q \wedge (\neg P \vee Q)$$

Solution

$$\begin{aligned} \neg Q \wedge (\neg P \vee Q) &\equiv (\neg Q \wedge \neg P) \vee (\neg Q \wedge Q) \\ &\equiv (\neg Q \wedge \neg P) \vee \mathsf{F} && \text{Complement Law} \\ &\equiv (\neg Q \wedge \neg P) && \text{Identity Law} \end{aligned}$$

9. Negate the following expression, and simplify your answer (hint: use De Morgan's Law):

$$(P \Rightarrow Q) \wedge Q$$

Solution

$$\begin{aligned} \neg((P \Rightarrow Q) \wedge Q)) &\equiv \neg(P \Rightarrow Q) \vee \neg Q) && \text{De Morgan's Law} \\ &\equiv (P \wedge \neg Q) \vee \neg Q && \text{Identity 2} \end{aligned}$$

10. In question 5 you drew a truth table to show that $P \Rightarrow (P \vee Q)$ is a tautology.

    Now do this using algebra

    Solution

$P \Rightarrow (P \vee Q) \equiv \neg P \vee (P \vee Q)$	Identity 1
$\equiv (\neg P \vee P) \vee Q$	$\vee$ is associative
$\equiv \mathrm{T} \vee Q$	Complement Law
$\equiv \mathrm{T}$	Identity Law

11. Draw a truth table for the following expression, using 3-valued logic: $P \vee \neg Q$

Solution

P	Q	$\neg Q$	$P \vee \neg Q$
T	T	F	**T**
T	F	T	**T**
T	UNDEFINED	UNDEFINED	**T**
F	T	F	**F**
F	F	T	**T**
F	UNDEFINED	UNDEFINED	**UNDEFINED**
UNDEFINED	T	F	**UNDEFINED**
UNDEFINED	F	T	**T**
UNDEFINED	UNDEFINED	UNDEFINED	**UNDEFINED**