

Matrices – Answers to Tutorial Questions

1. Consider the following matrices:

$$A = \begin{pmatrix} -2 & 3 & 5 \\ 1 & -2 & 9 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 2 & 7 \\ 3 & 8 & 4 \end{pmatrix}$$

Find the value of:

- a) $A + B$ b) $A - B$ c) $2A + 3B$ d) A^T

Solution

$$\text{a) } A + B = \begin{pmatrix} -2+1 & 3+2 & 5+7 \\ 1+3 & -2+8 & 9+4 \end{pmatrix} = \begin{pmatrix} -1 & 5 & 12 \\ 4 & 6 & 13 \end{pmatrix}$$

$$\text{b) } A - B = \begin{pmatrix} -2-1 & 3-2 & 5-7 \\ 1-3 & -2-8 & 9-4 \end{pmatrix} = \begin{pmatrix} -3 & 1 & -2 \\ -2 & -10 & 5 \end{pmatrix}$$

$$\text{c) } 2A + 3B = \begin{pmatrix} -4 & 6 & 10 \\ 2 & -4 & 18 \end{pmatrix} + \begin{pmatrix} 3 & 6 & 21 \\ 9 & 24 & 12 \end{pmatrix} = \begin{pmatrix} -1 & 12 & 31 \\ 11 & 20 & 30 \end{pmatrix}$$

$$\text{d) } A^T = \begin{pmatrix} -2 & 1 \\ 3 & -2 \\ 5 & 9 \end{pmatrix}$$

2. Consider the following matrices:

$$A = \begin{pmatrix} -2 & 3 \end{pmatrix} \quad B = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$$

Calculate $A \times B$.

Solution

$$\begin{aligned} \begin{pmatrix} -2 & 3 \end{pmatrix} \times \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix} &= (-2 \times 4 + 3 \times 2 \quad -2 \times 1 + 3 \times 3) \\ &= (-2 \quad 7) \end{aligned}$$

3. Consider the following matrices:

$$A = \begin{pmatrix} 2 & 7 \\ 1 & 3 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix}$$

Calculate $A \times B$.

Solution $A \times B = \begin{pmatrix} 2 \times 1 + 7 \times 0 & 2 \times 5 + 7 \times 2 \\ 1 \times 1 + 3 \times 0 & 1 \times 5 + 3 \times 2 \end{pmatrix} = \begin{pmatrix} 2 & 24 \\ 1 & 11 \end{pmatrix}$

4. Find the determinant of the following matrix, A :

$$A = \begin{pmatrix} 1 & 3 \\ 4 & -2 \end{pmatrix}$$

Solution

$$\det(A) = (1 \times -2) - (4 \times 3)$$

$$= -2 - 12 = -14$$

5. Where possible, find the inverse of the following matrices:

$$\text{a) } A = \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix} \quad \text{b) } B = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix} \quad \text{c) } C = \begin{pmatrix} 3 & 6 \\ 2 & 4 \end{pmatrix}$$

Solution

$$\text{a) } \det(A) = 4 \times 3 - 1 \times 2 = 10$$

$$A^{-1} = \frac{1}{10} \begin{pmatrix} 3 & -2 \\ -1 & 4 \end{pmatrix} = \begin{pmatrix} 0.3 & -0.2 \\ -0.1 & 0.4 \end{pmatrix}$$

b) There is no inverse because B is not a square matrix.

c) There is no inverse because $\det(C) = 0$.

6. Consider the matrix A from the previous question: $A = \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$

Now consider the following matrix D . $D = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

If $A \times X = D$, find the value of X .

Solution

$$A \times X = D$$

$$\therefore X = A^{-1} \times D$$

From the previous question we know that

$$A^{-1} = \begin{pmatrix} 0.3 & -0.2 \\ -0.1 & 0.4 \end{pmatrix}$$

$$\therefore X = \begin{pmatrix} 0.3 & -0.2 \\ -0.1 & 0.4 \end{pmatrix} \times \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 0.3 \times 2 + -0.2 \times 2 \\ -0.1 \times 2 + 0.4 \times 2 \end{pmatrix}$$

$$= \begin{pmatrix} 0.2 \\ 0.6 \end{pmatrix}$$

7. Use matrices to solve the following equations:

$$\begin{array}{rcrcrcrcrcl} 2x & + & y & + & 4z & = & 7 \\ & & 5y & + & z & = & 3 \\ -x & + & 4y & + & 2z & = & -2 \end{array}$$

Solution

$$\begin{array}{c} \text{A} \rightarrow \begin{pmatrix} 2 & 1 & 4 \\ 0 & 5 & 1 \\ -1 & 4 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ 3 \\ -2 \end{pmatrix} \leftarrow \begin{array}{c} X \\ B \end{array} \end{array}$$

$$A \times X = B$$

$$X = A^{-1} \times B$$

Using an online calculator:

$$A^{-1} = \begin{pmatrix} \frac{6}{31} & \frac{14}{31} & \frac{-19}{31} \\ \frac{-1}{31} & \frac{8}{31} & \frac{-2}{31} \\ \frac{5}{31} & \frac{-9}{31} & \frac{10}{31} \end{pmatrix}$$

$$X = A^{-1}B = \begin{pmatrix} \frac{122}{31} \\ \frac{21}{31} \\ \frac{-12}{31} \end{pmatrix}$$

$$x = \frac{122}{31} \qquad y = \frac{21}{31} \qquad z = \frac{-12}{31}$$