CN4004 Maths for Computing: Tutorial

Mathematical Logic (Part 2)

1. $D(x)$ is the predicate *x is a duck*, defined over the domain of animals.

   a) Write the following statements in words:

   i) $D(\text{BASIL})$
   ii) $\forall x \bullet \neg D(x)$
   iii) $\exists x \bullet D(x)$
   iv) $\exists! x \bullet \neg D(x)$
   v) $\neg \forall x \bullet D(x)$

   b) If we had not stated the domain of discourse in advance, how would we have written part (ii), using A to represent the set of animals?

2. $B(x)$ is the predicate *x is a bird*, and $F(x)$ is the predicate *x can fly*, both defined over the domain of animals.

   a) Write the following statements in words:

   i) $\forall x \bullet (B(x) \Rightarrow F(x))$
   ii) $\exists x \bullet (B(x) \wedge F(x))$
   iii) $B(\text{JACK}) \vee \exists x \bullet F(x)$

   b) Write the following statements in symbols:

   i) If Mary is a bird, then no animal can fly.
   ii) Only birds can fly (every animal is a bird or it cannot fly).
   iii) There is one and only one bird that cannot fly.

3. Negate the predicate $\exists x \bullet (\neg P(x) \wedge \neg \mathrm{Q}(x))$ using the universal quantifier ($\forall$) instead of the existential quantifier ($\exists$).

4. Simplify your answer to question 3 by using De Morgan's law.

5. Show that if the following statements are true:

- if I do the ironing I have a cup of tea in the afternoon.
- if I have a cup of tea in the afternoon it is Thursday.
- I do the ironing.

then it follows that:

- it is Thursday.

6. Show that if the following statements are true:

- Bernard is a cat and Susan is a cat;
- Bernard likes watching television.
- If there is at least one cat that likes watching television then the moon is made of cheese.

then it follows that:

- The moon is made of cheese.

7. Show that if the following statements are true:

- Sam is a snake;
- If Sam can bite then Paris is in France;
- All snakes can bite;

then it follows that: Paris is in France.

Advanced question

8. Prove that the following statement holds for all $n \geq 1$:

$$2 + 2^2 + 2^3 + 2^4 + \ldots + 2^n = 2^{n+1} - 2$$