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Volatility of Stock-Market Indexes—An Analysis Based on SEMIFAR Models

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By applying SEMIFAR models, we examine "long memory" in the volatility of worldwide stock-market indexes. Our analysis yields strong evidence of "long memory" in stock-market volatility, either in terms of stochastic long-range dependence or in the form of deterministic trends. In some cases, both components are detected in the data. Thus, at least partially, there appears to be even stronger and more systematic long memory than suggested by a stationary model with long-range dependence.

KEY WORDS: ARCH models; Bandwidth selection; Kernel estimation; Long-range dependence; Maximum likelihood estimation; Power transformation; Semiparametric model; Short-range dependence; Trend.

Modeling (conditional) variances has been one of the most important topics in the stochastic analysis of financial time series, as can be seen by the extensive amount of research on autoregressive conditional heteroscedastic (ARCH) models (Engle 1982) and their extensions, in particular generalized ARCH (GARCH), exponential GARCH (EGARCH), and integrated GARCH (IGARCH). These models are readily interpreted as autoregressive moving average (ARMA)- and autoregressive integrated moving average (ARIMA)-type models of the (conditional) variance (Bollerslev and Mikkelsen 1996). That is, the dependence of the conditional variance on the past decays exponentially with increasing lag. Moreover, long-range dependence in the (conditional) variance of financial time series, in particular stock-market indexes, has recently attracted considerable attention in the literature (e.g., see Ding, Granger, and Engle 1993; Crato and de Lima 1994; Bollerslev and Mikkelsen 1996; Ding and Granger 1996). The new approaches, such as fractionally integrated GARCH (FIGARCH) and long-memory ARCH (LM-ARCH), allow modeling a certain kind of volatility persistence, typically detected via a slow hyperbolical decay of the correlations of an appropriate volatility measure. In particular, Ding et al. (1993) found substantially high correlation between absolute returns and power-transformed absolute returns of some stock-market indexes for long lags. Independently Baillie, Bollerslev, and Mikkelsen (1996) arrived at similar results—namely, long memory in volatility series. Both studies appear to argue against short-range dependent ARCH-type specifications of the (conditional) variance based on squared return series.

In this article, the potential usefulness of SEMIFAR models (Beran 1998; Beran and Ocker 1999a,b; Beran, Feng, and Ocker 1999) is explained and their application to volatility series of worldwide nominal stock-market indexes is discussed. These models include a nonparametric trend function as well as a fractional differencing parameter. This allows for data-driven distinction of long-range dependence, difference stationarity, and deterministic trends.

The article is organized as follows. In Section 1, we give a brief description of Beran's (1998) SEMIFAR model. Most of this section is based on the results of Beran (1998); other preprints are also included. A data-driven algorithm is given in Section 2. The application of SEMIFAR models to volatility series of 19 nominal stock-market indexes is discussed in Section 3. Some final remarks are given in Section 4. The SEMIFAR algorithm is provided in the appendix.

1. THE SEMIFAR MODEL

A *SEMIFAR* model is a Gaussian process Y_i with an existing smallest integer $m \in \{0, 1\}$ such that

$$\phi(B)(1-B)^{\delta}\{(1-B)^m Y_i - g(t_i)\} = \epsilon_i, \tag{1}$$

where $t_i = (i/n)$, $\delta \in (-.5, .5)$, g is a smooth function on $[0, 1]$, B is the backshift operator, $\phi(x) = 1 - \sum_{j=1}^{p} \phi x^j$ is a polynomial with roots outside the unit circle, and ϵ_i $(i = \dots, -1, 0, 1, 2, \dots)$ are iid zero mean normal with $\mathrm{var}(\epsilon_i) = \sigma_\epsilon^2$. Here, the fractional difference $(1-B)^\delta$ introduced by Granger and Joyeux (1980) and Hosking (1981) is defined by

$$(1-B)^{\delta} = \sum_{k=0}^{\infty} a_k(\delta) B^k \tag{2}$$

with

$$a_k(\delta) = (-1)^k \frac{\Gamma(\delta+1)}{\Gamma(k+1)\Gamma(\delta-k+1)}. \tag{3}$$

Note that the assumption of Gaussianity can be replaced by suitable moment conditions on the innovations ϵ_i.

The motivation for this definition can be summarized as follows: We wish to have a model that may be decomposed into an arbitrary deterministic (possibly zero) trend and a random component that may be stationary or difference stationary. Moreover, short-range and long-range dependence as well as antipersistence should be included. Here, a stationary process Y_i with autocovariances $\gamma(k) = \text{cov}(Y_i, Y_{t+k})$ is said to have long-range dependence, if the the spectral density $f(\lambda) = (2\pi)^{-1} \sum_{k=-\infty}^{\infty} \exp(ik\lambda)\gamma(k)$ has a pole at the origin

$$f(\lambda) \quad c_f |\lambda|^{-\alpha} \ (|\lambda| \to 0) \tag{4}$$

for a constant $c_f > 0$ and $\alpha \in (0, 1)$, where means that the ratio of the left and right side converges to 1 (Mandelbrot 1983; Hampel 1987; Künsch 1986; Beran 1994 and references therein). In particular, this implies that, as $k \to \infty$, the autocovariances $\gamma(k)$ are proportional to $k^{\alpha-1}$ and hence they are not summable. On the other hand, a stationary process is called antipersistent if (4) holds with $\alpha \in (-1, 0)$. This implies that the sum of all autocovariances is 0. Note that for usual short-memory processes, such as stationary ARMA processes, (4) holds with $\alpha = 0$, and the autocovariances sum up to a nonzero finite value.

To model long-range dependence and to avoid overdifferencing, which is often encountered in the usual Box–Jenkins setting, Granger and Joyeux (1980) and Hosking (1981) introduced fractional ARIMA processes. There, the differencing parameter d is restricted to the stationary range $(-.5, .5)$. In a direct extension, Beran (1995) defined an arbitrary differencing parameter $d > -.5$ such that $(1 - B)^m Y_t$ is a stationary fractional ARIMA (p, δ, q) process, $m = [d + .5]$ is the integer part of $d + .5$, and $\delta = d - m$. This corresponds to Equation (1) with a constant function $g \equiv \mu$. Since the integer differencing parameter m assumes integer values only and the fractional differencing parameter δ is in $(-.5, .5)$, both differencing parameters can be recovered uniquely from the "overall differencing parameter" $d = m + \delta$. If $d > .5$, then we have a nonstantionary fractional ARIMA process. It should be noted, in particular, that this parameterization allows for maximum likelihood estimation of d. Thus not only δ but also m can be estimated from the data and confidence intervals can be given for both differencing parameters (see Beran 1995). Deterministic trends with stationary errors $(m = 0)$ and other than polynomial trends are excluded, however. SEMIFAR models extend the definition of fractional ARIMA models with arbitrary $d = m + \delta$ by including an arbitrary deterministic trend function g satisfying certain smoothness assumptions.

More specifically, for SEMIFAR models, $Z_i = \{(1 - B)^m Y_i - g(t_i)\}$ is a stationary (possibly) fractional autoregressive (AR) process. Thus, the spectral density of Z_i is proportional to $|\lambda|^{-2\delta}$ at the origin so that the process Z_i has long memory if $\delta > 0$, antipersistence if $\delta < 0$, and short memory if $\delta = 0$. Model (1) generalizes stationary fractional AR processes to the nonstationary case, including difference stationarity and deterministic trend. Four special cases of Model (1) are (1) $Y_i =$ no deterministic trend $+$ stationary process with short- or long-range dependence; (2) $Y_i =$ deterministic trend $+$ stationary process with short- or long-range dependence; (3) $Y_i =$ no deterministic trend $+$ difference-stationary process, whose first difference has short- or long-range dependence; and (4) $Y_i =$ deterministic trend $+$ difference-stationary process, whose first difference has short- or long-range dependence.

Observe that alternative (3) includes stochastic trends that are typically generated by purely stochastic nonstationary processes $(m = 1)$ such as random walks or integrated ARIMA models. In addition to nonstationary models, stationary long-memory processes often exhibit local spurious trends that may be hard to distinguish from deterministic and/or purely stochastic trends in nonstationary time series. Here, alternative (1) allows the possibility of local spurious trends ($m = 0, \delta > 0$). Also, (3) allows a combination of stochastic and local spurious trends ($m = 1$ and $\delta > 0$), whereas (2) is a mixture of deterministic and local spurious trends. Alternative (4) includes a mixture of all three kinds of trends.

In practical applications, it is often very difficult to find the "right" model and, in particular, to decide whether a series is stationary or has a deterministic or stochastic trend or whether there may be long-range correlations. (In fact, often a combination of these may be present.) A possible approach to resolving the problem is given by the SEMIFAR model. The model provides a unified data-driven semiparametric approach that allows for simultaneous modeling of and distinction between deterministic trends, stochastic trends, and stationary short- and long-memory components. Within the given framework (1), the approach helps the data analyst to decide which components are present in the observed data.

Briefly speaking, a SEMIFAR model is a fractional stationary or nonstationary autoregressive model with a nonparametric trend. This extends Box–Jenkins ARIMA models (Box and Jenkins 1976) by using a fractional differencing parameter $d > .5$ and by including a nonparametric trend function g. The trend function can be estimated, for example, by kernel smoothing (see Beran 1998). The parameters may be estimated by an approximate maximum likelihood introduced by Beran (1995) (see also Beran, Bhansali, and Ocker 1999). Note in particular that with this method the integer differencing parameter is also estimated from the data. A data-driven algorithm for estimating SEMIFAR models, which is a mixture of these two approaches, is presented in Section 2. Confidence intervals and tests were given by Beran (1998).

2. A DATA–DRIVEN ALGORITHM

To fit a SEMIFAR model to observed data, three problems need to be solved simultaneously—(1) choice of optimal data-driven bandwidth for estimating the nonparametric trend; (2) data-driven choice of the autoregressive order p in the stochastic component; and (3) estimation of the unknown parameter vector θ that defines the dependence structure of the stochastic component. Note that (3) answers, in particular, the important question of whether the stochastic component is stationary (i.e., if $\hat{d} < 1/2$) or not (if $\hat{d} > 1/2$).

The algorithm used in our examples was proposed by Beran (1998). A detailed study of its properties as well as some alternative algorithms will be given in a forthcoming paper (Beran and Feng 1999). It is an adaptation of the algorithm of Beran (1995) in that $\hat{\mu}$ is replaced by a kernel estimate of g. The algorithm makes use of the fact that d is the only additional parameter, besides the autoregressive parameters, so that a systematic search with respect to d can be made. The optimal bandwidth is estimated by an iterative plug-in method similar to that of Herrmann, Gasser, and Kneip (1992) and Ray and Tsay (1997).

A detailed definition of the algorithm is given in the appendix. The general idea can be described as follows: For processes consisting of a deterministic trend plus a *short-memory* process, iterative plug-in methods for estimating the optimal bandwidth are known (e.g., see Herrmann et al. 1992). Suppose now, at first, that the autoregressive order p of the error process is known. For SEMIFAR models, the only additional difficulty, as compared to models with short-memory errors, is the presence of the unknown fractional differencing parameter $d = m + \delta$. The integer part m determines stationarity/nonstationarity ($m = 0$ and 1, respectively) and the fractional part δ models long memory ($\delta > 0$), short memory ($\delta = 0$), and antipersistence ($\delta < 0$), respectively. With respect to estimation, m determines whether the trend should be estimated from the original data or the first difference, whereas the value of δ determines the order of the optimal bandwidth. Now, if d were known, then a simple modification of the iterative algorithm by Hermann et al. (1992) could be applied, taking the first difference if $m = 1$ and using the appropriate formula (with δ) for the optimal bandwidth. To adapt this approach to the case in which d is unknown, note that d is a one-dimensional parameter. Thus, a full search with respect to d can be made. For each trial value of d, an iterative plug-in algorithm analogous to Herrmann et al. is applied. In each iteration, the short-memory parameters are estimated from the previously obtained error process by maximum likelihood, the corresponding optimal bandwidth is calculated, and a new estimated error process is obtained. Simultaneously, for each d, the algorithm yields a maximum likelihood estimate of θ for the given autoregressive order p. Finally, to estimate p, a model choice criterion is applied by using the corresponding likelihood values. In our calculations, the Bayesian information criterion (BIC) was used; that is, p was closed to minimize $\mathrm{BIC}(p) = -\text{log-likelihood} + \log(n)(p+2)$ with respect to p.

Note that the optimal bandwidth depends on the smoothness of the function g through the integral of $[g'']^2$. As in standard regression smoothing, it is assumed that the second derivative of g exists and the integral of $(g'')^2$ is not 0. The estimate $\hat{g}''$ is consistent, since it can be shown that the estimated bandwidth converges to the (asymptotically) optimal bandwidth. As a result, consistency and a central limit theorem of $\hat{\theta}$ can be derived. A detailed discussion of theoretical results was given by Beran (1998), Beran and Ocker (1999), and Beran and Feng (1999).

3. VOLATILITY OF STOCK–MARKET INDEXES

3.1 The Data

The data include 19 nominal stock-market closing indexes for the period January 1, 1992, to November 10, 1995. They are, according to the definition of the IFC (1997), indexes for 10 developed markets (DM's: Australia, Belgium, Canada, France, Germany, Hong Kong, Italy, Switzerland, United Kingdom, and United States) and 9 emerging markets (EM's: Brazil, Chile, Greece, Hungary, Malaysia, Mexico, Poland, South Korea, and Thailand). Table 1 presents the names and the exchanges for these indexes, together with global ranking by market capitalization in U.S.$ terms as of end-1995 (Euromoney 1996).

Table 1. Stock Indexes of Developed and Emerging Markets

Countries	*Exchange*	*Index*	*Ranking*
		DM series	
Australia	Sydney	All Ordinaries	10
Belgium	Brussels	Stock Index	20
Canada	Toronto	TSE 300	7
France	Paris	CAC 40	5
Germany	Frankfurt	DAX	4
Hong Kong	Hong kong	Hang Seng	9
Italy	Milan	DS General	12
Switzerland	Zurich	Swiss Bank Corporation	6
United Kingdom	London	FTSE 100	3
United States	New York	S&P 500	1
		EM series	
Brazil	Sao Paulo	BOVESPA	17
Chile	Santiago	IGPA	22
Greece	Athens	General Index	38
Hungary	Budapest	BUX	49
Malaysia	Kuala Lumpur	KLSE	11
Mexico	Mexico City	IPC	21
Poland	Warsaw	WIG	44
South Korea	Seoul	KOSPI	18
Thailand	Bangkok	Book Club	23

The indexes are expressed in local currencies and, overall, are neither adjusted for dividends nor for inflation. Figure 1 shows daily values of the indexes (weekdays only). Besides the large number of infrequent local spikes, which are often related to heteroscedasticity, most indexes exhibit an apparent high magnitude around the middle part of the period under consideration. Note the impact of the Mexican currency and banking crisis, beginning in the last quarter of 1994, and the corresponding (retarded) spillovers to Brazil and Chile. Observe also the low level of the East European indexes at the beginning of the period under consideration. The two stock markets in Hungary and Poland were reestablished in 1990 and 1991, respectively, resulting in a low degree of activity in 1992. Finally, observe the smooth sample path of the Brazilian index, which is due to several rebasements during the sample period.

To study volatility, we analyze the power-transformed absolute differences $Y_t = |I_t - I_{t-1}|^{.25}$, where I_t denotes the original index. The corresponding series are shown in Figure 2 (weekdays only, excluding holidays). When evaluating multiple assets from different countries within a multivariate framework, the handling of holidays becomes an issue. Sophisticated statistical optimization methods may be required to specify stochastic models. In the current univariate study, we take a simple pragmatic approach. In a first step, missing values in the original index series are replaced by the closest previous closing value, resulting in zero increments. In a second step, zero values of Y_t were omitted and the series are treated as equidistant.

The reason for taking the fourth root of the increments is that the marginal distribution of the resulting series is very close to normal (see the normal probability plots in Fig. 3). A similar transformation approach was used, for instance, by

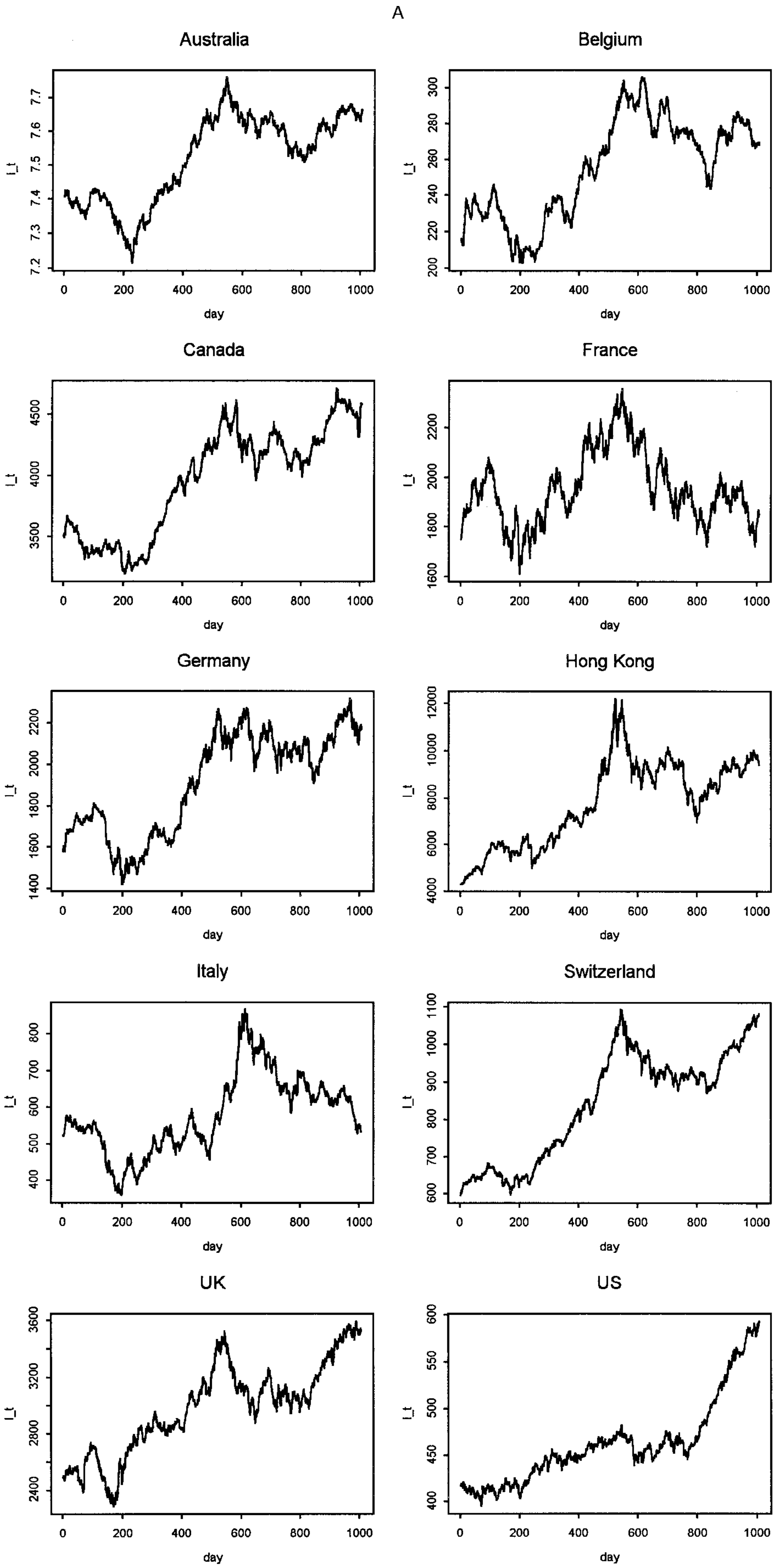

Figure 1. Daily Index Series: Panel A, Developed Markets; Panel B, Emerging Markets. (to be continued)

B

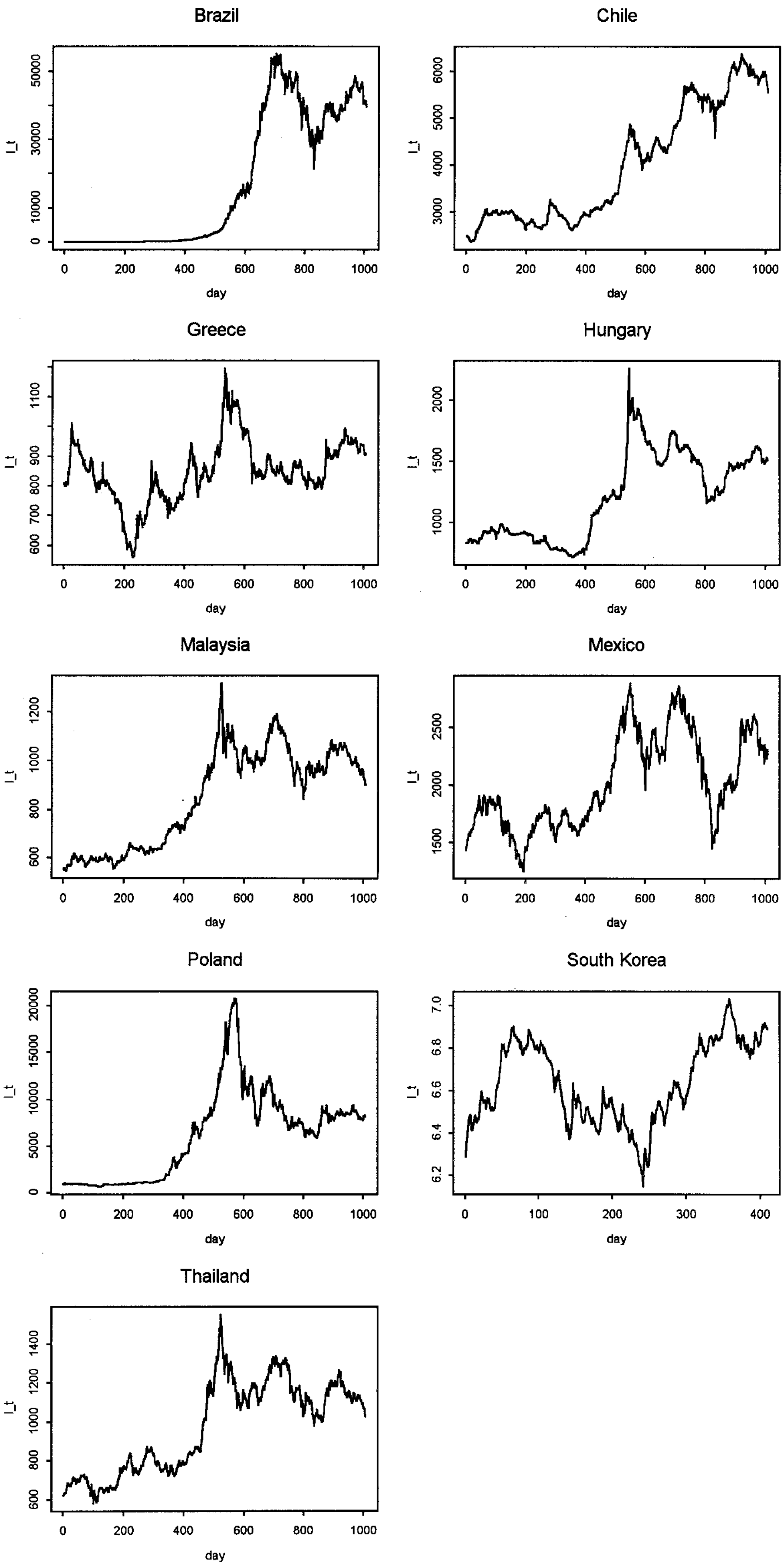

Figure 1. (continued)

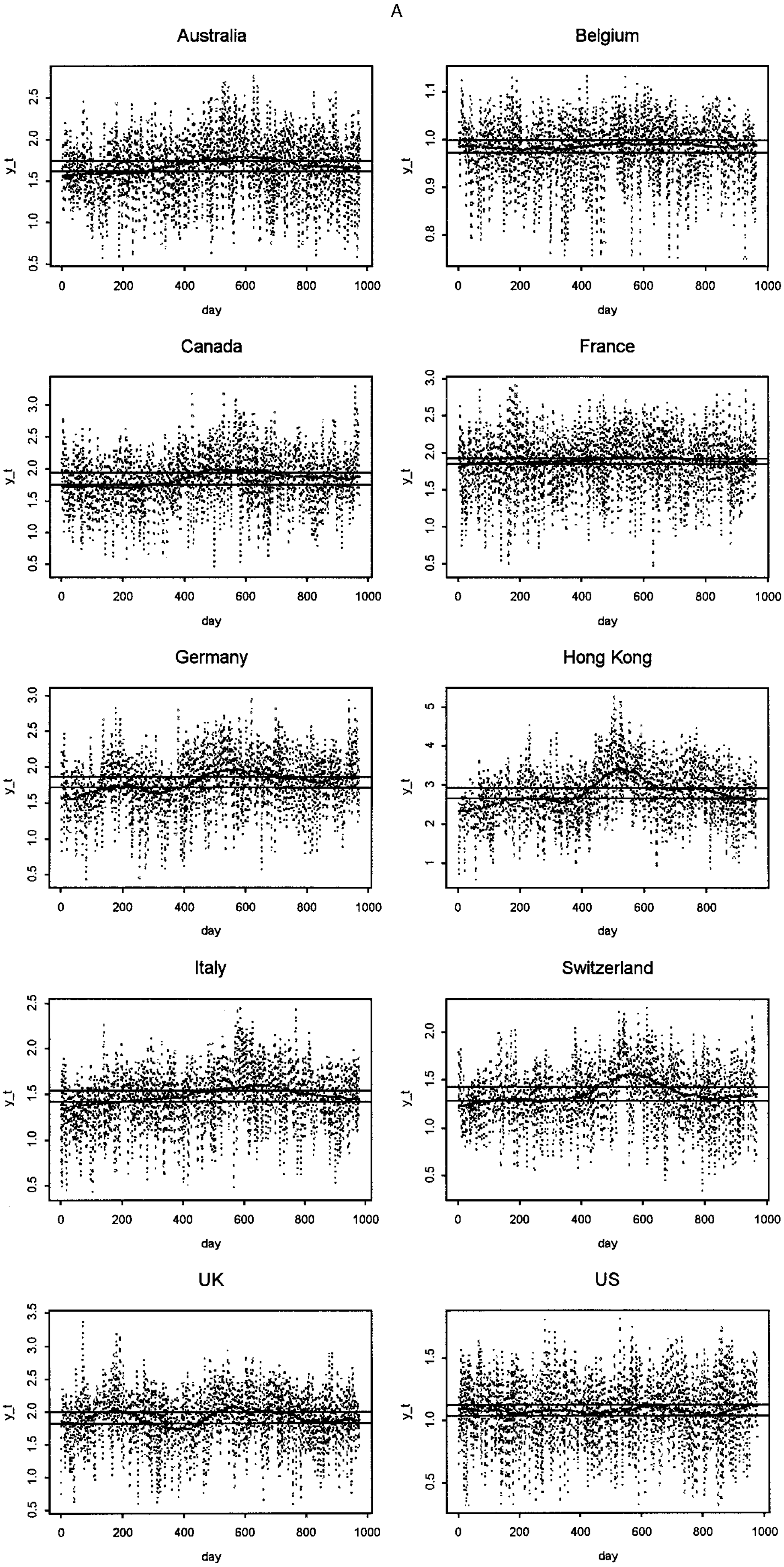

Figure 2. Trends in Daily Volatility With 5% Rejection Limits: Panel A, Developed Markets; Panel B, Emerging Markets. (to be continued)

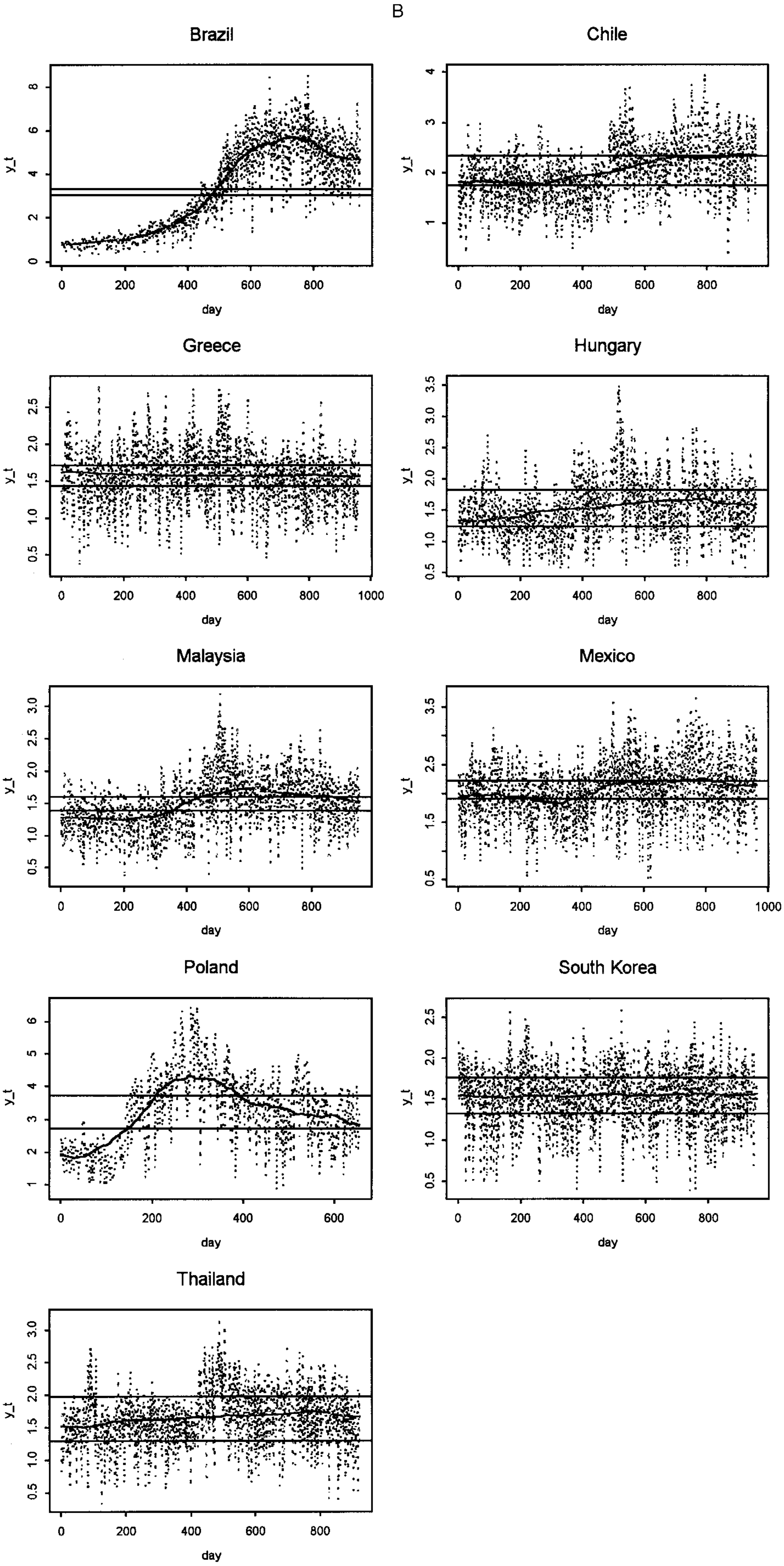

Figure 2. (continued)

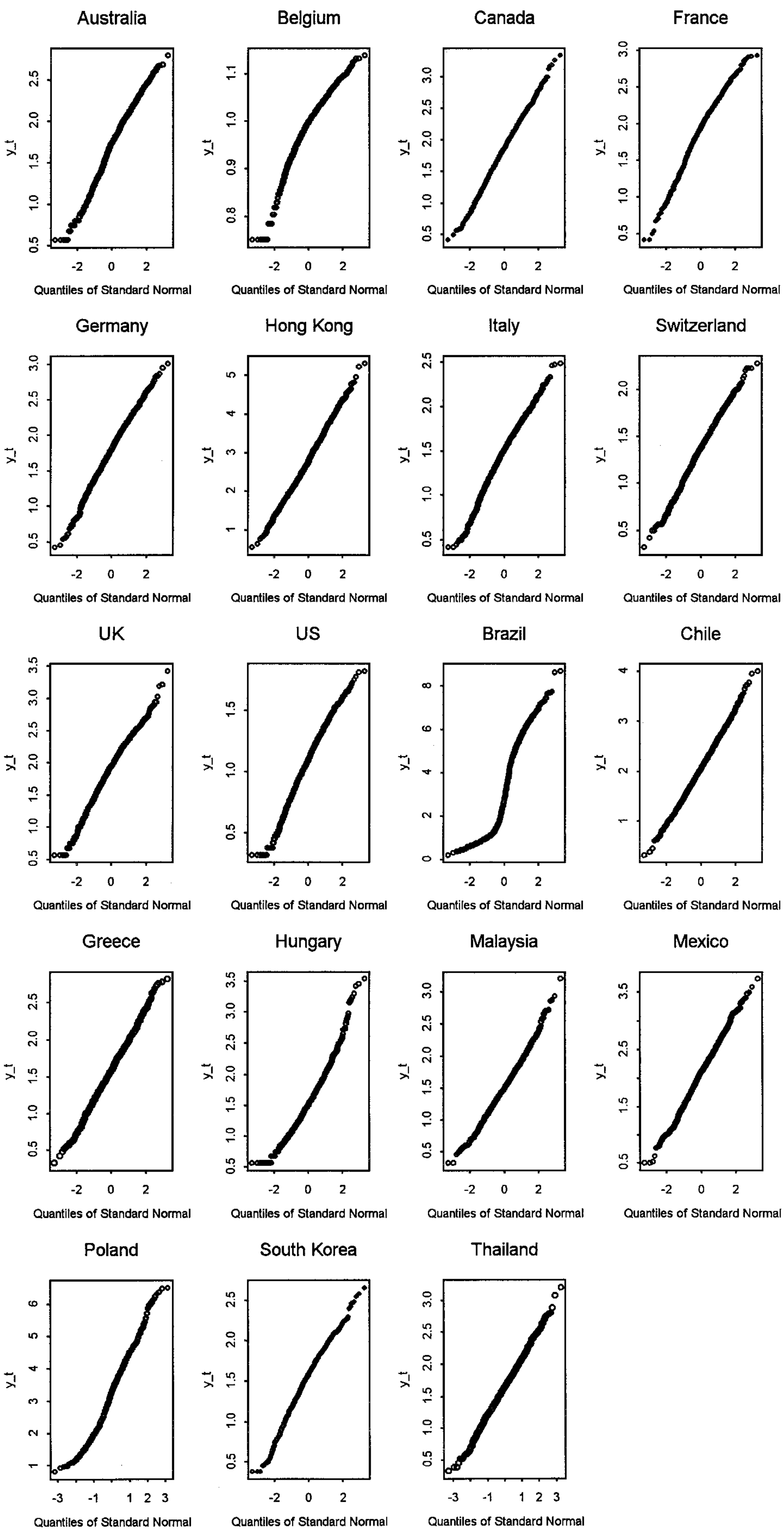

Figure 3. Normal Probability Plots of Daily Volatilities.

Ding et al. (1993). As in their study, the correlograms of Y_t in Figure 4 do indeed indicate slowly decaying autocorrelations (with the exception of the three DM's Belgium, France, and the United States). The question arises whether this behavior may be explained by long-range dependence in the stochastic component and/or a (nonparametric) deterministic trend. A nonparametric deterministic (and essentially arbitrary) trend function as an additional building block can, apart form d, explain long-term fluctuations. A smooth deterministic function can be interpreted as an even stronger (and more systematic) degree of temporal dependence than stationarity with slowly decaying correlations.

3.2 Empirical Results

Table 2 summarizes the essential features of the fitted SEMIFAR models for the daily volatility series. The corresponding 95% confidence intervals are given in brackets. The models were selected using the BIC.

The estimated value of d and the confidence intervals suggest that the stochastic part of all series is stationary ($d < .5$). This is not very surprising in view of the general visual impression given in Figure 2. For the EM's (except Brazil) and the two small DM's Belgium and Italy, $d = 0$ is not, or almost not (for Belgium), contained in the 95% confidence interval. Thus, the estimates indicate that there is long-range dependence in the stochastic component of daily volatility series of EM's and small DM's. For these stock markets, as a finding, the degree of persistence becomes stronger the smaller the market (see Fig. 7). Applying Spearman's rank correlation, we found that $\hat{\rho} = .77$ (p value $= .022$).

Substantial short-term dependence, which is typically assumed in traditional ARCH specifications, was only found in one series (i.e., for Thailand) in the form of a small AR(1) term.

For all DM's (except Belgium), a significant deterministic trend is found. For the EM's, only five out of nine markets have a significant trend. However, for them, stochastic long-range dependence is found (except Brazil). Figure 2 shows the volatilities Y_t with the fitted trends and upper and lower 5% critical limits for testing significance of the trends. Note that, for $\hat{m} = 1$, the critical limits are obtained from the asymptotic normal distribution of $\hat{g}$, under the assumption that g is identically equal to 0. The results indicate that there are relatively long periods in which volatility is high/low systematically for the DM series. This is, in particular, apparent for the DM's Australia, Canada, Germany, Hong Kong, Italy, Switzerland, and the United Kingdom, where a significant trend is detected due (at least a posteriori) to the relatively long period of high volatility around the middle part of the considered time period. Observe in particular the similarity between the trends for Hong Kong and Switzerland. These findings are less evident for the United States and France (which also corresponds to their correlograms; see Fig. 4). Some EM's (Brazil, Chile, Malaysia, Mexico, and Poland) also exhibit periods with high/low volatility in the form of a (local) significant deteministic trend. In particular, the stock markets of Brazil, Malaysia, and Poland show highly deterministic volatility patterns. Note the extreme behavior of the Brazilian series, which may be due to several rebasements during the period under consideration. For the other EM's, apparent local trends do not persist long enough and can therefore be explained as spurious.

The satisfactory fits of the models are demonstrated by the normal probability plots and correlograms of the residuals in Figures 5 and 6. Slight departures from normality can be observed for Belgium and Brazil. Note, however, that normality of the residuals is not required in order that the theoretical results hold (Beran 1998).

Overall, the estimates indicate that there is long memory in the volatility of stock-market indexes, either in the form of local deterministic trend (for the DM's and some EM's) or in the form of long-range dependence in the stochastic component (for the EM's and small DM's) resulting in local spurious trends. In some cases, both components are present in the data. In contrast, there is almost no evidence for short memory as it is typically assumed in traditional ARCH specifications. Moreover, the significant trends fitted to the volatility series indicate that there may be even stronger and more systematic long memory in volatility than suggested by a stationary model with long-range dependence.

4. FINAL REMARKS

In this article, we illustrated the potential usefulness of SEMIFAR models for volatility analysis by several data examples. We found strong evidence of long memory in power-transformed absolute-return series. Long memory is understood here as stochastic long-range dependence and/or deterministic trends. A deterministic trend as an additional building block can, apart from d, explain long-term fluctuations.

Long memory in the volatility of stock-market indexes has some important implications:

1. If long memory is indeed present in the data, statistical inferences concerning asset-pricing models based on traditional testing procedures may no longer be valid (e.g., see Mandelbrot 1971; Bollerslev and Mikkelsen 1996).
2. In addition, the discovery of long memory suggests possibilities for improved volatility forecasting performance, especially over longer forecasting horizons (e.g., see Granger and Joyeux 1980; Geweke and Porter-Hudak 1983; Beran and Ocker 1999; Ocker 1999).
3. Moreover, many financial time series are available in temporarily aggregated form. Long-range dependence is, in contrast to traditional short memory, robust with respect to temporal aggregation (e.g., see Ocker 1999; Beran and Ocker 1999). Realistic models should therefore include the possibility of long memory (stochastic and deterministic).

Our results indicate that traditional short-memory ARCH-type specifications may not be appropriate for modeling volatility of stock-market indexes. Our findings suggest that there may be even stronger and more systematic temporal dependence in volatility than suggested by a stationary ARCH model with stochastic long-range dependence. A more sophisticated analysis of volatility may be obtained by applying

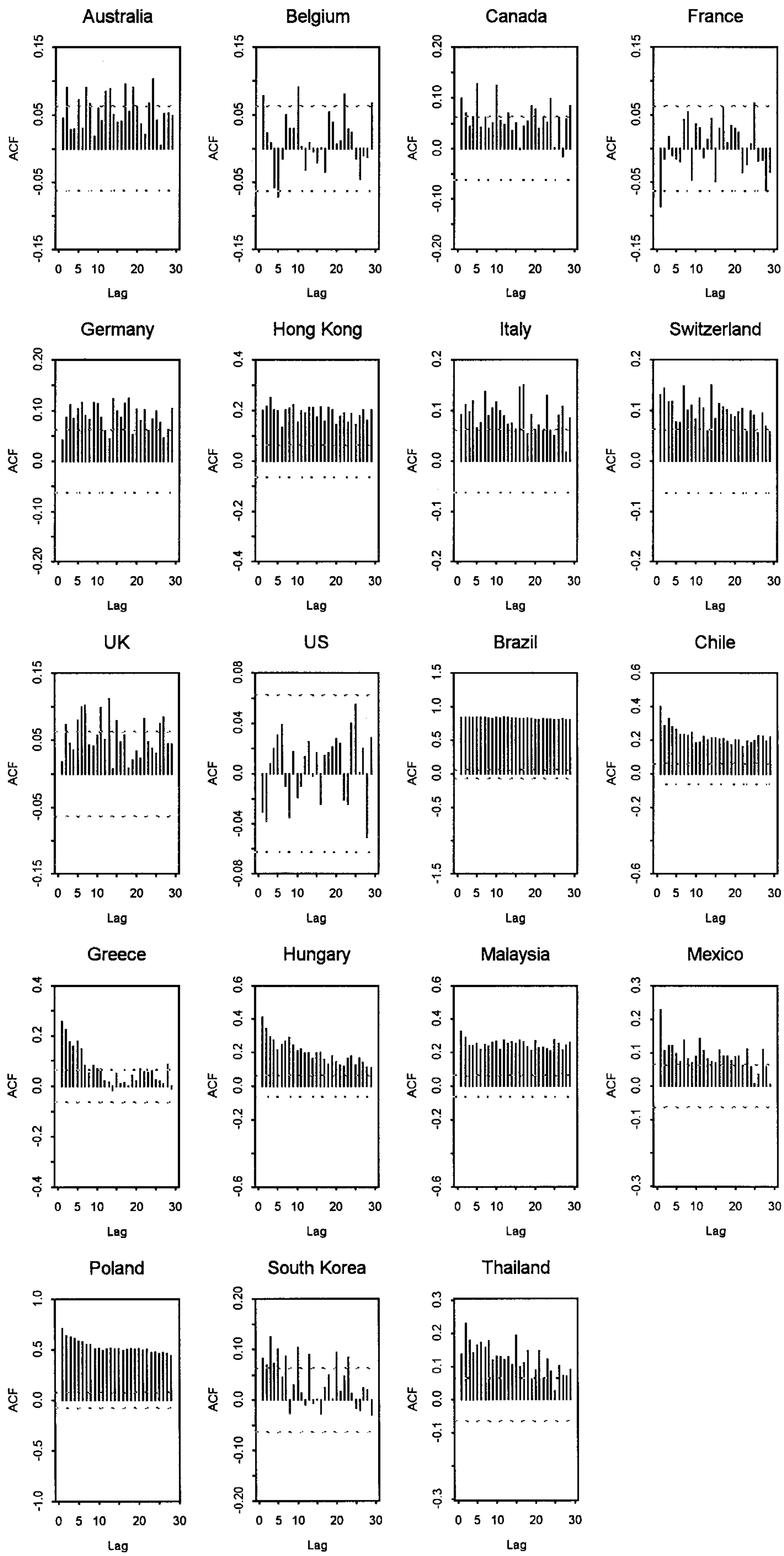

Figure 4. Autocorrelations of Daily Volatilities.

Table 2. Estimation Results

Countries	$\hat{d}$	*95% c.i. d*	$\hat{\phi}_1$	*95% c.i.* ϕ_1	*Signicant trend*
		DM series			
Australia	.025	[−.024, .074]	—	—	yes
Belgium	.048	[−.022, .097]	—	—	no
Canada	.035	[−.014, .084]	—	—	yes
France	−.085	[−.135, −.036]	—	—	yes
Germany	−.020	[−.069, .029]	—	—	yes
Hong Kong	−.004	[−.054, .046]	—	—	yes
Italy	.052	[.003, .101]	—	—	yes
Switzerland	.036	[−.013, .085]	—	—	yes
U.K.	−.025	[−.074, .024]	—	—	yes
U.S.	−.041	[−.090, .008]	—	—	yes
		EM series			
Brazil	−.062	[−.112, −.012]	—	—	yes
Chile	.248	[.198, .298]	—	—	yes
Greece	.221	[.172, .271]	—	—	no
Hungary	.283	[.233, .333]	—	—	no
Malaysia	.108	[.058, .158]	—	—	yes
Mexico	.104	[.055, .153]	—	—	yes
Poland	.233	[.173, 293]	—	—	yes
South Korea	.142	[.067, .217]	−.085	[−.180, .011]	no
Thailand	.208	[.136, .280]	−.179	[−.269, −.089]	no

GARCH-type extensions of SEMIFAR models to the original index series I_t. The mathematical theory necessary for such extensions is subject to current research. For fractional models that do not include deterministic trend functions, Ling and Li (1997) extended the maximum likelihood method of Beran (1995) to fractional GARCH models. Moreover, an extension to moving average terms (which may be called "SEMIFARIMA models") is obvious.

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APPENDIX: ALGORITHM

The following notations will be used in this appendix: $I(g'') = \int_0^1 [g''(x)]^2 dx$, $V = \lim_{n\to\infty} (nb_n)^{1-2\delta} \operatorname{var}(\hat{g}(t))$ (asymptotic variance of $\hat{g}$ as a function of δ), and $C_{\text{opt}} = C_{\text{opt}}(\theta)$ such that the asymptotically optimal bandwidth (as a function of θ) is $b_{\text{opt}} = C_{\text{opt}} n^{(2\delta-1)/(5-2\delta)}$.

The steps of the SEMIFAR algorithm are as follows:

Step 1: Define $L =$ maximal order of $\phi(B)$ that will be tried, and a sufficiently fine grid G $(-.5, 1.5)$. Then, for each $p \in \{0, 1, \ldots, L\}$, carry out steps 2 through 4.

Step 2: For each $d \in G$, set $m = [d + .5]$, $\delta = d - m$, and $U_i(m) = (1-B)^m Y_i$ and carry out step 3.

Step 3: Carry out the following iteration:

Step 3a: Let $b_o = C_o \min(n^{(2\delta-1)/(5-2\delta)}, .5)$ with C_o such that $C_o n^{(2\delta-1)/(5-2\delta)}$ is the asymptotically optimal bandwidth under the assumption that $\phi_i = 0$ for all $i = 1, \ldots, p$, $\sigma_\epsilon^2 = 1$, and $I(g'') = 1$. Set $j = 1$.

Step 3b: Set $b = b_{j-1}$.

Step 3c: Calculate $\hat{g}(t_i; m)$ using the bandwidth b. Set $\widehat{X}_i = U_i(m) - \hat{g})t_i; m)$.

Step 3d: Set $\tilde{e}_i(d) = \sum_{s-0}^{i-1} a_s(\delta)\widehat{X}_{i-s}$, where the coefficients a_s are defined by (3).

Step 3e: Estimate the autoregressive parameters $\phi_1, \ldots, \phi_p$ from $\tilde{d}_i(d)$ and obtain the estimates $\hat{\sigma}_\epsilon^2 = \hat{\sigma}_\epsilon^2(d; j)$ and $\hat{c}_f = \hat{c}_f(j)$. Estimation of the parameters can be done, for instance, by using the S-PLUS functions *ar.burg* or *arima.mle*. If $p = 0$, set $\hat{\sigma}_\epsilon^2$ equal to $n^{-1} \sum \tilde{e}_i^2(d)$ and $\hat{c}_f$ equal to $\hat{\sigma}_\epsilon^2/(2\pi)$.

Step 3f: Set $h = b^{(5-2\delta)/(9-2\delta)}$ and estimate g'' by

$$\hat{g}''(t) = \frac{1}{nh^3} \sum_{s=1}^{n} \widetilde{K} \left(\frac{t_s - t}{h}\right) U_s(m),$$

where $\widetilde{K}: R \to R$ is a polynomial symmetric kernel such that $\widetilde{K}(x) = 0$ for $|x| > 1$, $\int \widetilde{K}(x)dx = 0$, and $\int \widetilde{K}(x)x^2 dx = 2$. Calculate $I(\hat{g}'')$.

Step 3g: Calculate the new optimal bandwidth $b_j = C_{\text{opt}} \times n^{(2\delta-1)/(5-2\delta)}$ from δ and the estimated parameters obtained in Step 3f.

Step 3f: Increase j by 1 and repeat steps 3b through 3g until $b_j - b_{j-1}$ is small (or a maximal number of iterations is reached). This yields, for each $d \in G$ separately, the ultimate value of $\hat{\sigma}_\epsilon^2(d)$, as a function of d.

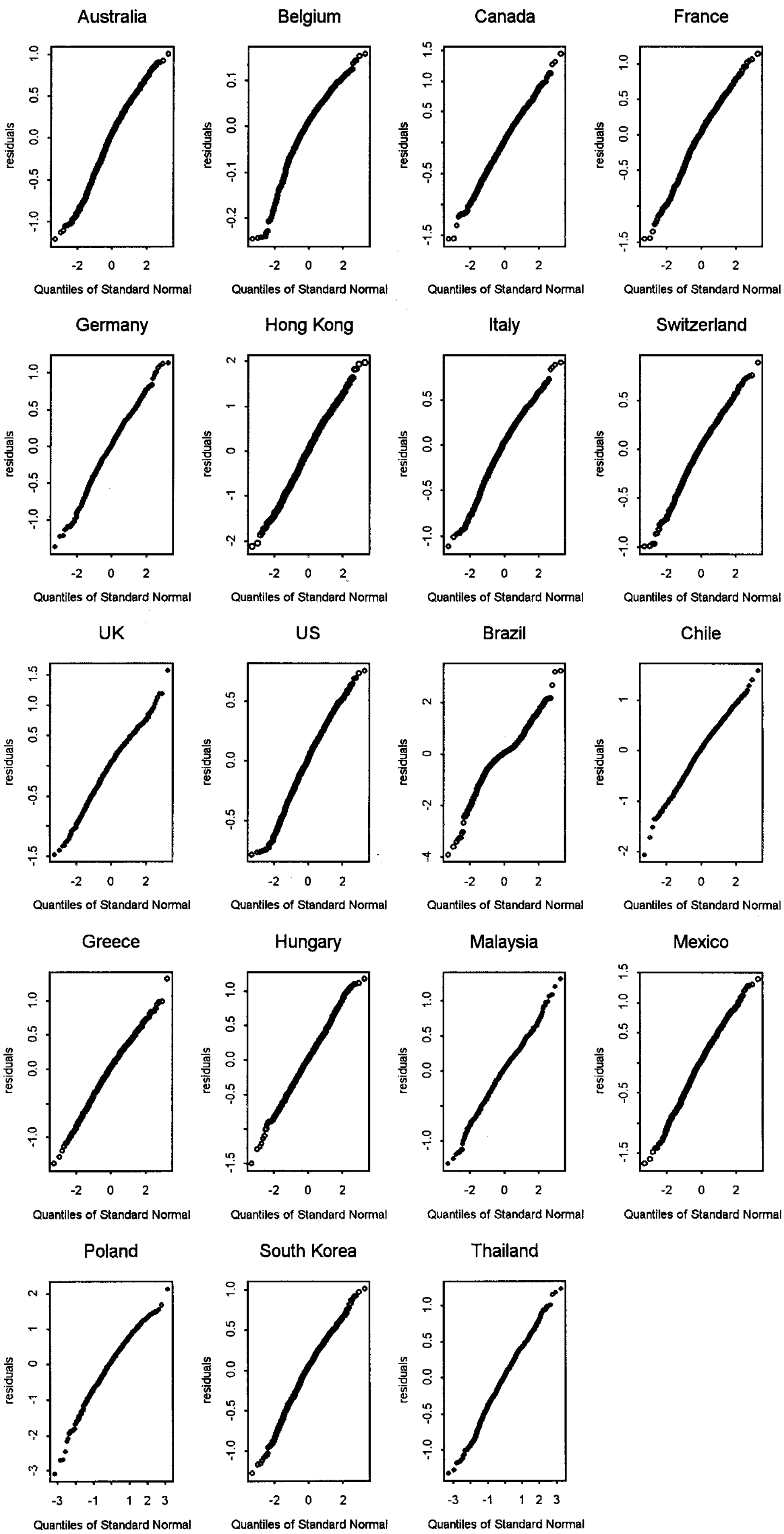

Figure 5. Normal Probability Plots for the Residuals.

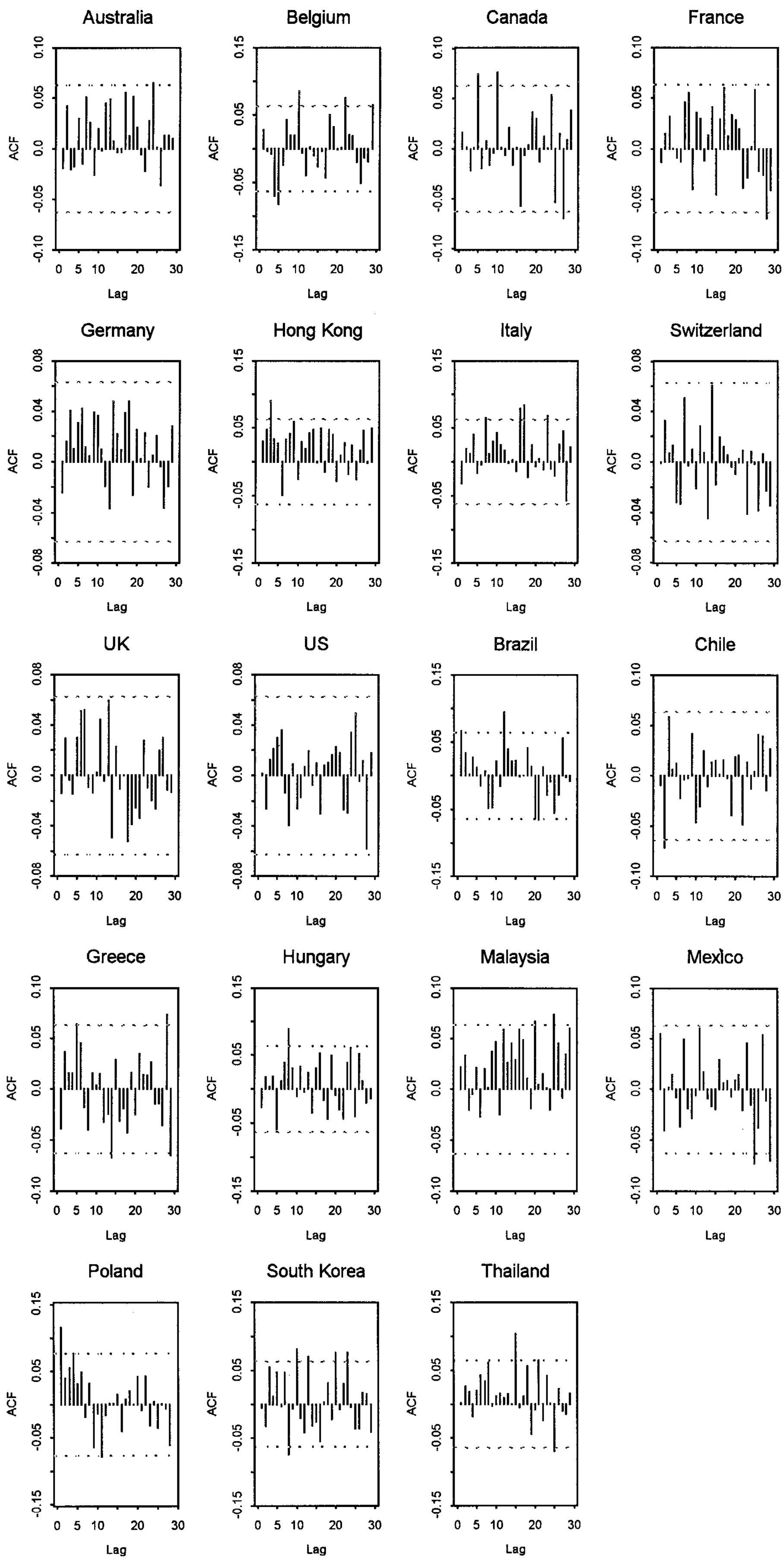

Figure 6. Autocorrelations of the Residuals.

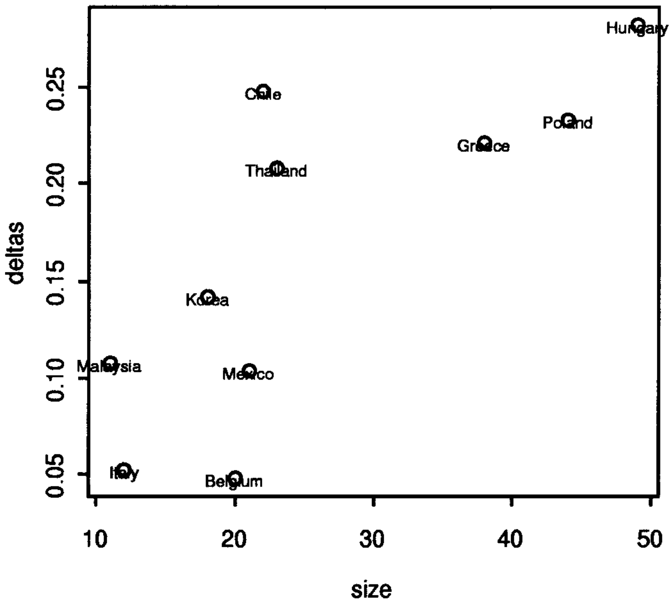

Figure 7. Long-memory Parameter as a Function of Market Ranking.

Step 4: Define $\hat{d}$ to be the value of d for which $\hat{\sigma}_\epsilon^2(d)$ is minimal. This, together with the corresponding estimates of the AR parameters, yields an automatic model-selection criteria such as the BIC(p) (as a function of p) and the corresponding values of $\hat{\theta}$ and $\hat{g}$ for the given order p.

Step 5: Select the order p that minimizes BIC(p). This yields the final estimates of θ and g.

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