COMMENT ON TELSER

Assuming that the underlying data are generated by an autoregressive process, Telser (1967) considered the effects of temporal aggregation on the time-series properties of observed data when the observed data are "an equi-spaced sample of moving sums of the basic [underlying] data for non-overlapping discrete intervals" (p. 484). As an illustration, he derived the "true" and "aggregate" autocorrelation functions when the underlying process is first-order autoregressive and then tabulated those functions for certain values of the autoregressive parameter. Because of the influence of Telser's work on subsequent literature on temporal aggregation, it is of more than passing interest that some of his numerical results are incorrect. This note provides the correct values.

It is helpful to summarize Telser's AR(1) example. Suppose that the underlying series $\{y_t; t = 0, 1, 2, \ldots\}$ is generated by $y_t = ay_{t-1} + u_t$, $|a| < 1$, and $u_t \sim D(0, \sigma^2)$, where the u_t's are mutually uncorrelated random variables distributed with a mean of zero and a variance of σ^2, and $E(y_{t-1}u_t) = 0$. The observed (aggregate) series is $\{Y_{\tau n}; \tau = 0, 1, 2, \ldots\}$, where $Y_t = \sum_{j=0}^{n-1} y_{t-j}$, and n is the number of y_t's summed to make Y_t ($n = m + 1$ in Telser's notation). The variance and autocovariances of y_t are

$$\begin{aligned}\gamma_i \equiv E(y_t y_{t-i}) &= \sigma^2/(1 - a^2), \qquad i = 0\\ &= a^i\sigma^2/(1 - a^2), \qquad i = 1, 2, \ldots .\end{aligned}$$

Hence, the autocorrelation function for y_t is a^i ($i > 0$). Comparable formulas for Y_t are

$$\begin{aligned}\bar{\gamma}_{jn} \equiv E(Y_t Y_{t-jn}) &= \frac{n\sigma^2}{(1 - a)^2}\left[1 - \frac{2a(1 - a^n)}{n(1 - a^2)}\right], \qquad j = 0\\ &= \frac{a^{n(j-1)}a(1 - a^n)^2\sigma^2}{(1 - a^2)(1 - a)^2}, \qquad j = 1, 2, \ldots .\end{aligned}$$

Telser (1967, Table 1) obtained parallel results for $n = 3$ (i.e., his $m = 2$), where his $c(k)$ is our $\bar{\gamma}_k(1 - a^2)$, with k a multiple of n.

From his Table 1, Telser (1967, Table 2) calculated the true autocorrelation γ_i/γ_0 and the aggregate autocorrelation $\bar{\gamma}_i/\bar{\gamma}_{0n}$, for various values of the lag i and autoregressive parameter a. Some of the aggregate autocorrelations in his Table 2 are incorrect: Our Table 2* lists the correct values, with Telser's values in parentheses where discrepancies exist. Two discrepancies are minor and attributable to rounding errors. The rest are substantial, however, and appear to have arisen from miscalculating the aggregate autocorrelation at the second lag, with subsequent values being calculated as a function of the value at the second (or previous) lag. Two such autocorrelations are clearly inadmissible because they are greater than unity.

Table 2. Comparison of the True and Summed Autocorrelations for a First-Order Autoregression and [n = 3]*

Lag	True	Aggregate	True	Aggregate	True	Aggregate
	$a = .25$		$a = .5$		$a = .9$	
0	1	1	1	1	1	1
1	.25	.761	.5	.841	.9	.967
2	.062	.418	.25	.557	.81	.893 (.926)
3	.016	.104	.125	.278	.729	.804 (.833)
4	.004	.026	.062	.139	.656	.724 (.750)
5	.001	.007 (.006)	.031	.070 (.069)	.590	.651 (.675)
	$a = -.25$		$a = -.5$		$a = -.9$	
0	1	1	1	1	1	1
1	−.25	.522	−.5	.25	−.9	−.695
2	.062	.311 (1.236)	.25	.375	.81	.812 (1.07)
3	−.016	−.078 (−.309)	−.125	−.187	−.729	−.731 (−.96)
4	.004	.019 (.077)	.062	.094	.656	.658 (.87)
5	−.001	−.005 (−.019)	−.031	−.047	−.590	−.592 (−.78)

Telser's Equation (3) contained a typographical error: U_{t-1} should be U_t.

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REFERENCE

Telser, L. G. (1967), "Discrete Samples and Moving Sums in Stationary Stochastic Processes," *Journal of the American Statistical Association*, 62, 484–499.

CORRECTIONS

Douglas M. Hawkins, "On the Bounds of the Range of Order Statistics," 66, No. 335 (September 1971), 644–645.

H. S. Konijn has pointed out two errors: (a) the definition of s^2 in Equation (1.3) should not have the divisor n, and (b) the $1/\sqrt{2}$ just above Theorem 2 should be $-1/\sqrt{2}$. Neither error affects the results given. Readers interested in these inequalities should consult Wolkowicz and Styan (1988) for interesting generalizations and additional references.

REFERENCE

Wolkowicz, H., and Styan, G. P. H. (1988), "Samuelson–Nair Inequality," in *Encyclopedia of Statistical Sciences* (Vol. 8), eds. S. Kotz, N. L. Johnson, and C. B. Read, New York: John Wiley, pp. 258–259.

J. M. Chambers, C. L. Mallows, and B. W. Stuck, "A Method for Simulating Stable Random Variables," 71, No. 354 (June 1976), 340–344; Correction, 82, No. 398 (June 1987), 704.

On page 344, line 7 should read as follows:

&,.18001 33704 07390 023 D3

This also applies to line 5 of the correction.

Hans-Georg Müller, "Weighted Local Regression and Kernel Methods for Nonparametric Curve Fitting," 82, No. 397 (March 1987), 231–238.

The references to Laurent (1984) should be to Lejeune (1984), and Lejeune (1985) contains a related discussion about the connection between kernel estimates and local least squares. The author regrets the omission of this reference. Related results are also found in work by Müller (1983).

REFERENCES

Lejeune, M. (1984), "Optimization in Nonparametric Regression," in *Compstat 1984* (*Proceedings in Computational Statistics*), eds. T. Havranek, Z. Sidak, and M. Novak, Vienna: Physica-Verlag, pp. 421–426.

——— (1985), "Estimation Non-Paramétrique par Noyaux: Régression Polynomiale Mobile," *Revue de Statistiques Appliquées*, 33, 43–67.

Müller, H. G. (1983), "Beiträge zur Nichtparametrischen Kurvenschätzung," unpublished Ph.D. thesis, University of Ulm, West Germany.