# Ep. 10: Jordan Normal Form

LetsSolveMathProblems: Navigating Linear Algebra

**Problem 1.** *If $A$ and $B$ are $3\times 3$ complex matrices that have the same minimal polynomial, as well as the same characteristic polynomial, are $A$ and $B$ necessarily similar? Does the answer change for the $4 \times 4$ complex matrices?*

**Problem 2.** *Given that a $5 \times 5$ complex matrix $A$ has the minimal polynomial of $\lambda^3$, what are all possible Jordan normal forms of $A$? (Write these Jordan forms as partitions of 5.)*

**Problem 3.** *If a complex matrix's minimal polynomial and characteristic polynomial are the same, what can we say about the matrix's Jordan normal form?*

**Problem 4.** *Show that if $A$ is an $n \times n$ complex matrix, then $A$ and $A^T$ are similar.*[1]

**Problem 5.** *Do there exist $5\times 5$ nilpotent complex matrices $A$ and $B$ such that $\operatorname{rank}(A^i) = \operatorname{rank}(B^i)$ for $i = 2, 3, 4$, yet $A$ and $B$ are not similar?*

[1] I learned this fascinating fact from a StackExchange post, like `https://math.stackexchange.com/questions/62497/matrix-is-conjugate-to-its-own-transpose`, which argues that the given statement is true for matrices over any field!