Lez 14. stabilità, oscillazioni, divergenze.

1. Reminder trasformate di Laplace

$$\frac{1}{s-p_0} \to e^{p_0 t} \qquad p_0 = \text{polo} = \sigma_0 + j\omega_0$$

$$\downarrow$$

$$e^{\sigma_0 t} e^{j\omega_0 t} \to \begin{cases} \sigma_0 > 0 & \text{divergente} \\ \sigma_0 = 0 & \text{oscillatorio} \\ \sigma_0 < 0 & \text{smorzato} \end{cases}$$

2. Polinomio doppio polo $\quad P_2(s) = s^2 + \frac{\omega_0}{Q} s + \omega_0^2$

ω_0 = freq. oscillazione

Q = "Q valore"

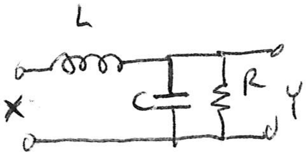
Nasce da vari sistemi fisici

(a.) Nostro oscillatore ponte di Wien;

risposta al rumore se $A \neq 3$: $\quad G(s) = \frac{A}{1-\beta A} =$

$$= \frac{A}{1 - \frac{A s\omega_0}{s^2+3s\omega_0+\omega_0^2}} = \frac{A(s^2+3\omega_0+\omega_0^2)}{s^2+(3-A)\omega_0 s+\omega_0^2} \to Q = \frac{1}{3-A}$$

(b.) LCR

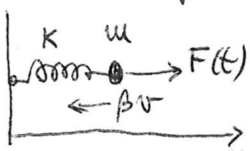


$$G(s) = Y/X = \frac{1}{1+sL\left(sC+\frac{1}{R}\right)}$$

$$G(s) = \frac{1/LC}{s^2+\frac{\omega_0}{RC}+\frac{1}{LC}} \to \omega_0^2 = 1/LC$$

$$Q = \omega_0 RC = \sqrt{\frac{R^2 C}{L}} = \sqrt{\frac{RC}{L/R}}$$

(c.) Oscillatore forzato/smorzato



$$\left(s^2+\frac{\beta}{m}s+\frac{K}{m}\right)\tilde{x} = \frac{\tilde{F}}{m} \to \tilde{x} = \frac{\tilde{F}/m}{s^2+\frac{\beta}{m}s+\frac{K}{m}}$$

$$m\ddot{x} = -Kx - \beta\dot{x} + F(t)$$

$$\ddot{x} + \frac{\beta}{m}\dot{x} + \frac{K}{m} = \frac{F}{m}$$

$$\omega_0^2 = K/m$$

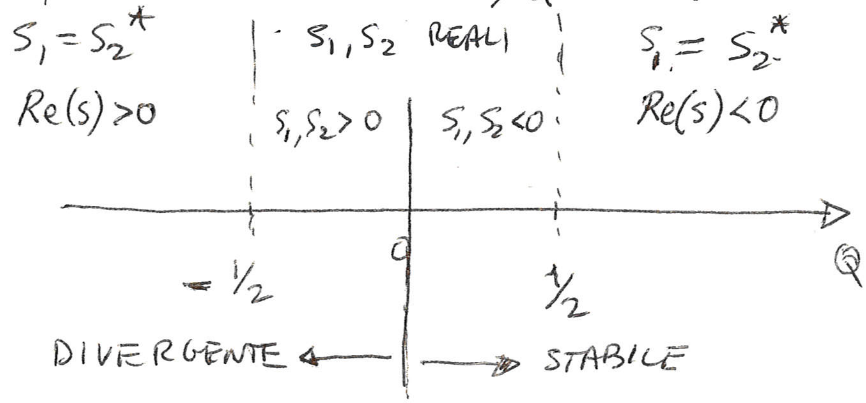
$$Q = \frac{\omega_0}{\beta/m} = \frac{\sqrt{Km}}{\beta}$$

3. Discussione poli di $P_2(s) = s^2 + \frac{\omega_0}{Q} s + \omega_0^2$

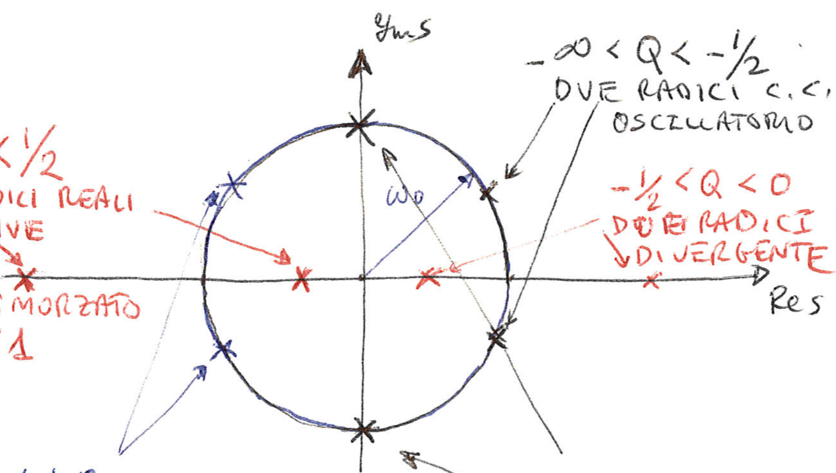
$$\Delta = \omega_0^2\left(\frac{1}{Q^2} - 4\right) = \frac{\omega_0^2}{Q^2}\left(1 - 4Q^2\right)$$

$$s_{1,2} = -\frac{\omega_0}{2Q} \pm \frac{\omega_0}{2Q}\sqrt{1-4Q^2} = -\frac{\omega_0}{2Q}\left(1 \mp \sqrt{1-4Q^2}\right)$$

Proprietà : $s_1 + s_2 = -\omega_0/Q$ $\quad s_1 s_2 = \omega_0^2$ $\quad (\omega_0 > 0)$



$Re(s_1)$ e $Re(s_2)$ hanno sempre lo stesso segno



a.) $3 < A < 5$

$-\infty < Q < -1/2$
DUE RADICI C.C.
OSCILLATORIO DIVERGENTE

$0 < Q < 1/2$
DUE RADICI REALI NEGATIVE
SOVRASMORZATO
a.) $A < 1$

$-1/2 < Q < 0$
DUE RADICI REALI POSITIVE
DIVERGENTE
a.) $A > 5$

$1/2 < Q < +\infty$
DUE RADICI COMPLESSE CONIUGATE

$$|s_1|^2 = s_1 \cdot s_1^* = s_1 s_2 = \omega_0^2$$

OSCILLATORIO SMORZATO
a.) $1 < A < 3$

$Q = \infty$ $\quad s_{1,2} = \pm j\omega_0$
OSCILLATORIO STABILE
$(Re(s) = 0)$

a.) $A = 3$ GUADAGNO SPECIALE
b.) $R = \infty$ NO RESISTENZA
c.) $\beta = 0$ NO VISCOSITÀ