Problem 8.2

Leul Shiferaw

23-11-16

Q1.

$$
\begin{aligned}
f &= n^4 \\
g &= n^3 log(n) \\
n &>_a log(n) \\
n^3 * n &>_a n^3 * log(n) \\
n^4 &>_a n^3 log(n) \\
\therefore f &\in \Omega(g)
\end{aligned}
$$

Q2.

$$
\begin{aligned}
& f = n + n^2 \\
& g = n + 3n^3 \\
& \lim_{n\to\infty} (\frac{f}{g}) \\
& \lim_{n\to\infty} \frac{n^2 + n}{3n^3 + n} = 0 \\
& \therefore f \in O(g)
\end{aligned}
$$

Q3.

$$
\begin{aligned}
& f = e^{n^2} \\
& g = 2^n \\
& \lim_{n\to\infty} \frac{e^{n^2}}{2^n} = \infty
\end{aligned}
$$

Since e is greater than 2 and $n^2 > n$ the limit goes to infinity

$$\therefore f \in \Omega(g)$$

Q4.

$$
\begin{aligned}
& f = n \\
& g = sin(n)
\end{aligned}
$$

sin(n) is bounded between 0 and 1.

$$\therefore f \in \Omega(g)$$

Q5.

$$
\begin{aligned}
f &= 12 \\
g &= 24 \\
g &= 2f \\
\therefore f &\in \Theta(g)
\end{aligned}
$$

Q6.

$$
\begin{aligned}
& f = \frac{n^2}{log(n)} \\
& g = n^4 + log(n) \\
& \lim_{n\to\infty} f/g \\
& \lim_{n\to\infty} \frac{n^2}{n^4 log(n)} = 0 \\
& \therefore f \in O(g)
\end{aligned}
$$

Q7.

$$
\begin{aligned}
f &= ln(n) \\
g &= log(n) \\
f &= \frac{log(n)}{log(e)} \\
f &= \frac{g}{log(e)} \\
\therefore f &\in \Theta(g)
\end{aligned}
$$

Q8.

$$
\begin{aligned}
f &= 5^n \\
g &= n!
\end{aligned}
$$

5^n is always multiplying 5, but n! starts from n and goes down. Therefore asymptotically n! will be greater than 5^n

$$\therefore f \in O(n!)$$

Q9.

$$f = 9^n$$
$$g = n^{log_n(8^n)}$$
$$g = 8^n$$
$$\lim_{n\to\infty} \frac{9^n}{8^n}$$
$$\lim_{n\to\infty} 1.125^n = \infty$$
$$\therefore f \in \Omega(g)$$

Q10.

$$f = (n^3 + 3)^2$$
$$g = (3n^3 + 2)^2$$
$$f = n^6 + ...$$
$$g = 9n^6 + ...$$
$$\lim_{n\to\infty} \frac{f}{g} = 1/9$$
$$\therefore f \in \Theta(g)$$