

If there were two bases A and B with different sizes then one can be increased by adding the elements that differ to make an acceptable set. But a base can't have a superset. This would cause a contradiction. Therefore all bases have the same size.