# Deterministic Optimization

## Linear Optimization Modeling

## Network Flow Problems

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Modeling using Linear Optimization

# Modeling using Linear Programs

## Learning Objectives

- Construct linear programming models for a wide range of applications
- Solve LP models with CVX
- Recognize nonlinear problems that can be modeled as LPs

# This lesson:
# LP models via examples Part 1

- Ingredients of a linear optimization model
- A simple example

# Ingredients of a linear program

A Linear program (or a linear optimization model) is composed of:

- Variables:

$$\boldsymbol{x} = (x_1, x_2, \ldots, x_n)$$

- A linear objective function:

$$f(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{n} c_i x_i = \boldsymbol{c}^\top \boldsymbol{x}.$$

- Linear constraints:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &\le b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &\ge b_2 \\ a_{31}x_1 + a_{32}x_2 + \cdots + a_{3n}x_n &= b_3 \end{aligned}$$

$$\begin{aligned} \boldsymbol{a}_1^\top \boldsymbol{x} &\le b_1 \\ \boldsymbol{a}_2^\top \boldsymbol{x} &\ge b_2 \\ \boldsymbol{a}_3^\top \boldsymbol{x} &= b_3 \end{aligned}$$

# A Simple Example of LP

A simple example of a linear program is given below:

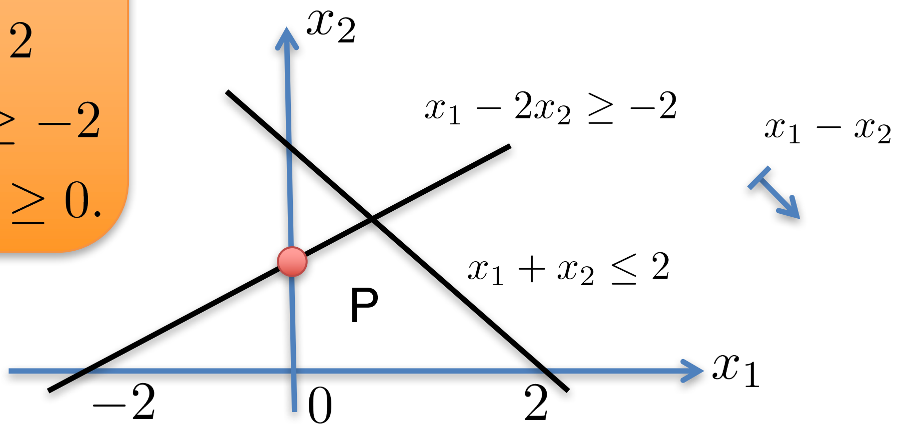
$$\begin{aligned} \min \quad & x_1 - x_2 \\ \text{s.t.} \quad & x_1 + x_2 \leq 2 \\ & x_1 - 2x_2 \geq -2 \\ & x_1 \geq 0, x_2 \geq 0. \end{aligned}$$

$$\begin{aligned} \min \quad & \begin{bmatrix} 1 \\ -1 \end{bmatrix}^\top \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ \text{s.t.} \quad & x_1 + x_2 \leq 2 \\ & x_1 - 2x_2 \geq -2 \\ & x_1 \geq 0, x_2 \geq 0. \end{aligned}$$

Optimal solution:

$$\boldsymbol{x}^* = \begin{bmatrix} \mathbf{0} \\ \mathbf{1} \end{bmatrix}$$

To visualize the feasible region of this LP, we can draw the following picture:




P: Polyhedron

# Summary

- To construct a linear program, we need to define variables, objective function, and constraints.
- The objective function must be a linear function of the variables.
- The constraints must be linear inequality or equality constraints.
- Simple LPs in 2-D can be drawn in pictures.