Deterministic Optimization

Introduction

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Classification of optimization problems

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Classification

Learning Objectives

- Discover the various types of optimization problems
- Recognize the type of a given problem

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Review: Generic Formulation

Generic formulation: $\begin{array}{ll} \min & f(\mathbf{x}) \\ \text{s.t.} & \mathbf{x} \in X \end{array}$ or $\min_{\mathbf{x}} \{ f(\mathbf{x}) : \ \mathbf{x} \in X \}$

where $X = \{ \mathbf{x} \in \mathbb{R}^{n-p} \times \mathbb{Z}^p : g_i(\mathbf{x}) \leq b_i \ i = 1, \ldots, m \}$

- Finite number of variables
- Single objective
- Finite number of constraints defined by inequalities/equalities
- All functions are algebraic
- Possible domain restrictions

Program vs. Optimization Problem

- A “program” or “mathematical program” is an optimization problem with a finite number of variables and constraints written out using explicit mathematical (algebraic) expressions
- The word “program” / “programming” means “plan” / “planning”
- Early applications of optimization arose in planning resource allocations (esp. in defense) and gave rise to “programming” to mean optimization (predates computer programming)
- We will use “program” / “programming” and “optimization problem” / “optimization” interchangeably

Problem Classification

The tractability of a large-scale optimization problem depends on the structure of the functions that make up the objective and constraints, and the domain restrictions on the variables.

Functions	Variable domains	Problem Type	Difficulty
All linear	Continuous variables	Linear Program (LP) or Linear Optimization problem	Easy
Some nonlinear	Continuous variables	Nonlinear Program (NLP) or Nonlinear Optimization Problem	Easy/Difficult
Linear/nonlinear	Some discrete	Integer Program (IP) or Discrete Optimization Problem	Difficult

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Subclasses of NLP

- Unconstrained optimization: No constraints or simple bound constraints on the variables
  - Example: Box design example in Lesson 1: $$\begin{array}{ll} \max & x(1-2x)^2 \\ \text{s.t.} & 0 \le x \le 1/2 \end{array}$$

- Quadratic Programming: Objectives and constraints involve quadratic functions
  - Example: Data fitting example in Lesson 1:

$$\begin{array}{ll} \min & \displaystyle\sum_{i=1}^{N} (y_i - a^\top x_i - b)^2 \\ \text{s.t.} & a \in \mathbb{R}^n,\ b \in \mathbb{R} \end{array}$$

Subclasses of IP

- Mixed Integer Linear Program (MILP):
    - All linear functions
    - Some variables are continuous and some are discrete
- Mixed Integer Nonlinear Program (MINLP)
    - Some nonlinear functions
    - Some variables are continuous and some are discrete
- Mixed Integer Quadratic Program (MIQLP)
    - Nonlinear functions are quadratic
    - Some variables are continuous and some are discrete

Why and how to classify?

- Important to recognize the type of an optimization problem
    - to formulate problems to be amenable to certain solution methods
    - to anticipate the difficulty of solving the problem
    - to know which solution methods to use
    - to design customized solution methods

- How to classify?
    - Check domain restrictions on variables
    - Check the structure of the functions involved

Summary

- Type of an optimization problem depends on the domain restrictions on the variables and the structure of the constraints involved
- Important to recognize problem classification for modeling and solving optimization problems.