Deterministic Optimization

Linear Optimization Modeling

Electricity Markets

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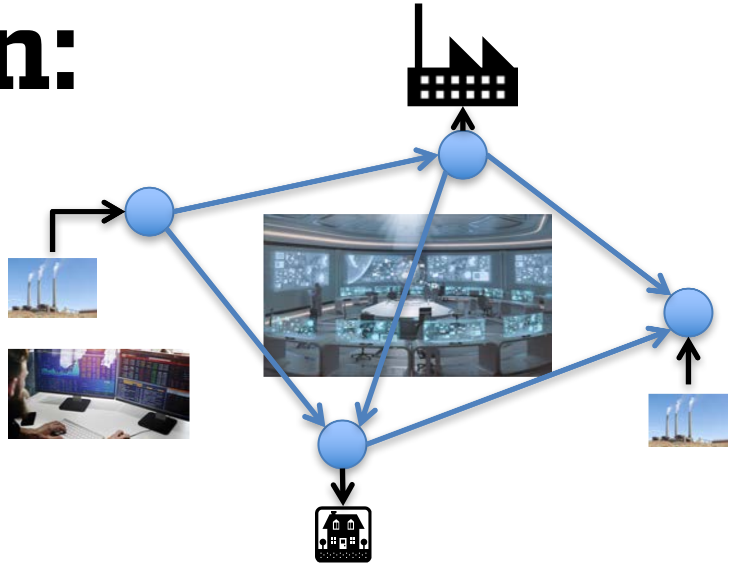
Market Clearing Mechanism

Modeling using Linear Programs

Learning Objectives for this Lesson

- Discover how electricity markets are cleared
- Discover how electricity prices are created

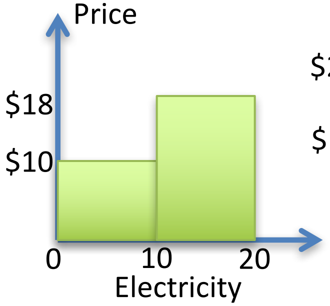
Electr. Market Clearing Problem: A Simplistic Version:



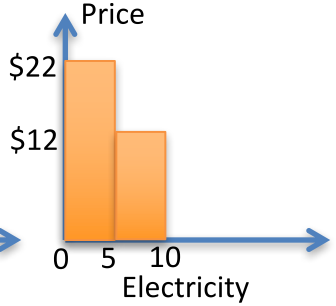
- An electricity market is composed of:
  - Generation companies **offer** $/amount of electricity they want to **sell at**
  - Customer serving companies **bid** $/amount of electricity they want to **buy at**
  - Independent system operator (ISO) operates the system and **clears the market**
- Objective:
  - Maximize **social welfare**
- Constraints:
  - Clear the market, i.e. supply equals demand

Offers and Bids and Clearing

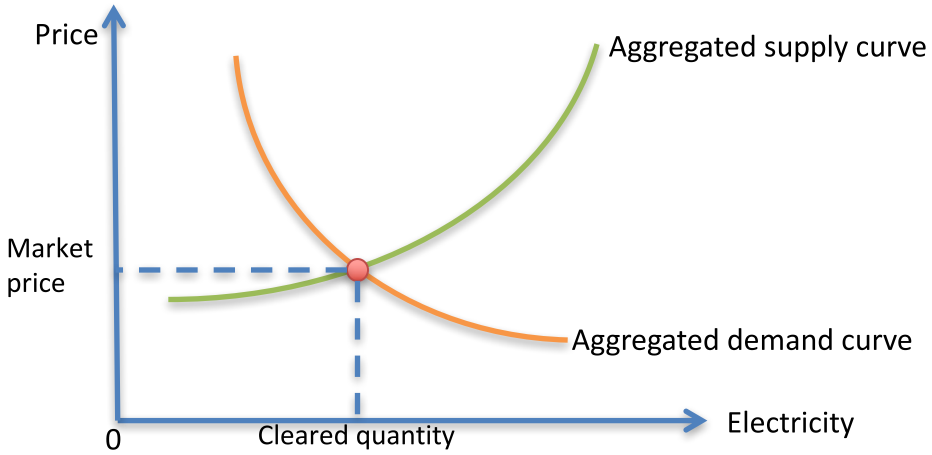
- A generator’s offer: How much electricity it wants to sell at what price
- A consumer’s bid: How much electricity it wants to buy at what price



A single generator offer



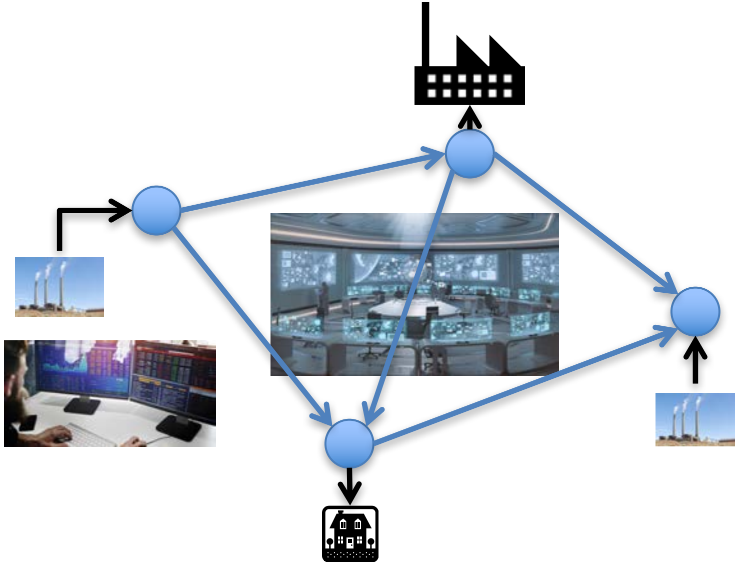
A single consumer bid



Market cleared: supply=demand;
Social welfare is maximized;
Electricity price is determined.

Electr. Market Clearing Problem: Real-Life Version:

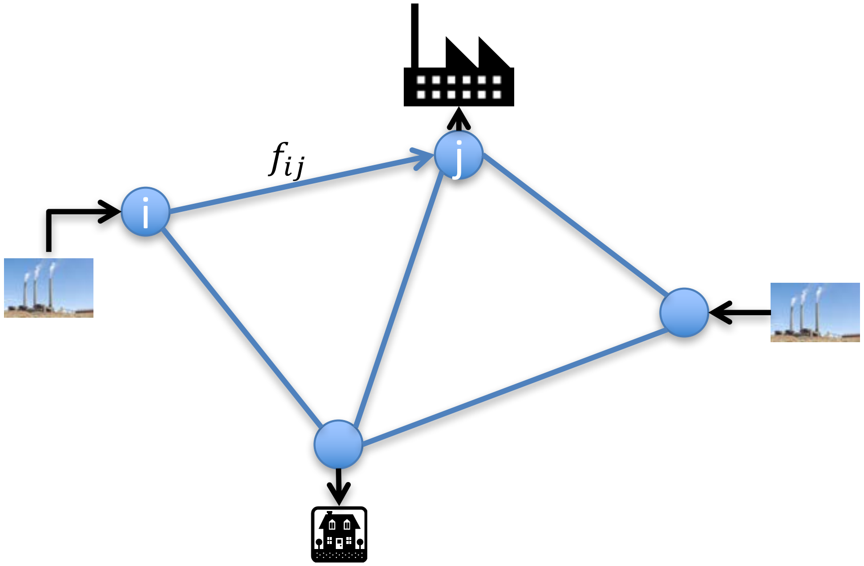
- An electricity market is composed of:
  - Generation companies offer $/amount
  - Customer serving companies bid $/amount
  - Independent system operator (ISO) operates the system and clears the market
- Objective:
  - Maximize social welfare
- Constraints:
  - Clear the market, i.e. supply equals demand
  - **Physical constraints: Network flow constraints, generation limit, etc**



Simply intersecting supply and demand curves won’t work! Need Optimization.

LP Model for Market Clearing

- Decision variables:
    - Generator output: p_i for each generator $i \in G$
    - Consumer demand: d_j for each consumer $j \in D$
    - Power flow: f_{ij} on each edge $(i, j) \in E$
- Objective: Social welfare
    - $\max \sum_i^{|D|} v_i d_i - \sum_{i=1}^{|G|} c_i p_i$
- Constraints:
    - Flow conservation:
        - $\sum_{j \in O(i)} f_{ij} - \sum_{j \in I(i)} f_{ij} = p_i$ for $i \in G$
        - $\sum_{j \in O(i)} f_{ij} - \sum_{j \in I(i)} f_{ij} = -d_i$ for $i \in D$
        - $\sum_{j \in O(i)} f_{ij} - \sum_{j \in I(i)} f_{ij} = 0$ for $i \in N \backslash (G \cup D)$
    - Constraint linking branch flow and nodal potential:
        - $f_{ij} = B_{ij}(\theta_i - \theta_j)$ for all $(i, j) \in E$
    - Flow limit constraint:
        - $-F_{ij} \leq f_{ij} \leq F_{ij}$ for all $(i, j) \in E$



- Generator physical limit constraints:
    - $p_i^m \leq p_i \leq p_i^M$ for all $i \in G$
- Demand limit constraints:
    - $d_i^m \leq d_i \leq d_i^M$ for all $i \in D$

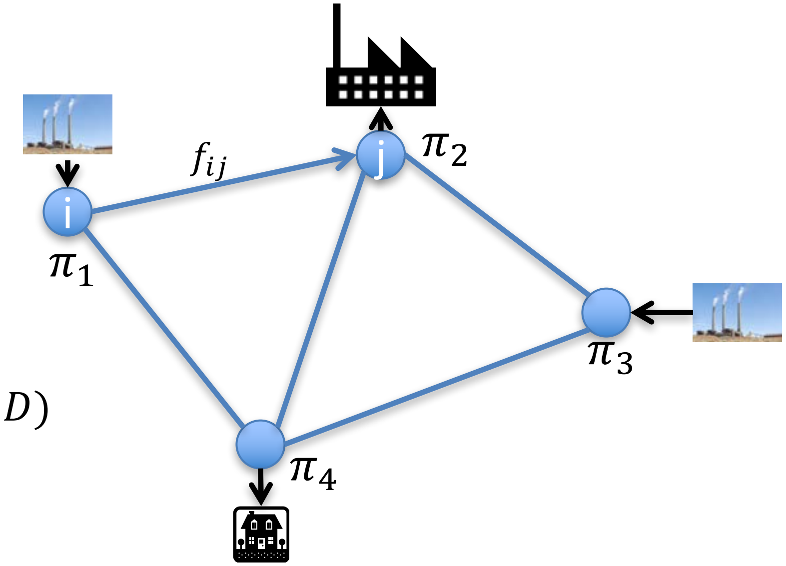
LP Model for Market Clearing

- $\max \sum_{i}^{|D|} v_i d_i - \sum_{i=1}^{|G|} c_i p_i$

subject to

π_i

- $\sum_{j \in O(i)} f_{ij} - \sum_{j \in I(i)} f_{ij} = p_i \ \ \forall i \in G$
- $\sum_{j \in O(i)} f_{ij} - \sum_{j \in I(i)} f_{ij} = -d_i \ \ \forall i \in D$
- $\sum_{j \in O(i)} f_{ij} - \sum_{j \in I(i)} f_{ij} = 0 \ \ \forall i \in N \setminus (G \cup D)$
- $f_{ij} = B_{ij}\left(\theta_i - \theta_j\right) \ \ \forall (i,j) \in E$
- $-F_{ij} \le f_{ij} \le F_{ij} \ \ \forall (i,j) \in E$
- $p_i^m \le p_i \le p_i^M \ \ \forall i \in G$
- $d_i^m \le d_i \le d_i^M \ \ \forall i \in D$



π_i: Dual variable gives market price at **each node** in network, called ***Locational Marginal Price***

Summary

- How trading is done in electricity market
- Simplified market clearing model (Econ101)
- How electricity market is cleared in real life
- How electricity price is determined