Deterministic Optimization

Linear Optimization Modeling

Electricity Markets

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A Real-World Example

Modeling using Linear Programs

Learning Objectives for this Lesson

- Solve some simple examples to gain intuition of locational marginal price
- Appreciate the power of optimization in real-world electricity markets

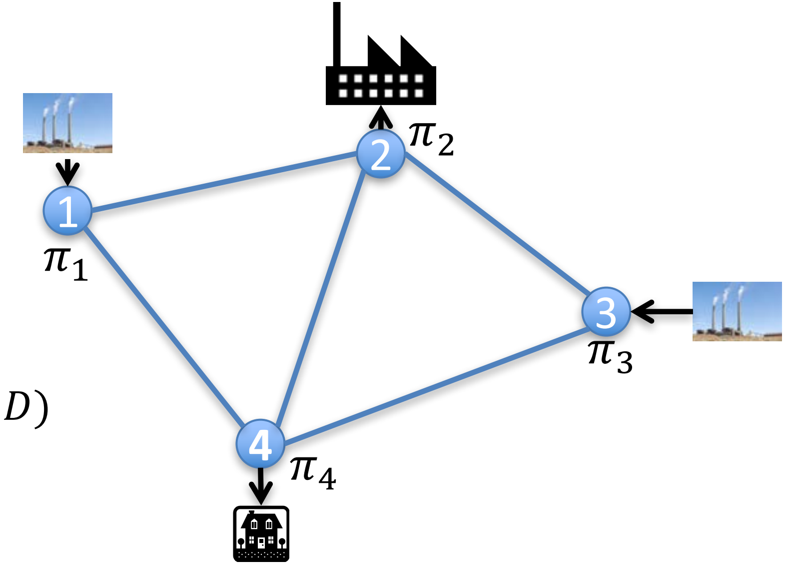
LP Model for Market Clearing

- $\max \sum_{i}^{|D|} v_i d_i - \sum_{i=1}^{|G|} c_i p_i$

subject to

π_i

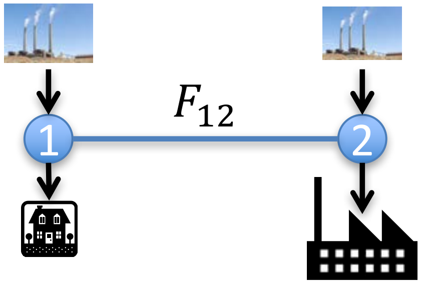
- $\sum_{j \in O(i)} f_{ij} - \sum_{j \in I(i)} f_{ij} = p_i \;\; \forall i \in G$
- $\sum_{j \in O(i)} f_{ij} - \sum_{j \in I(i)} f_{ij} = -d_i \;\; \forall i \in D$
- $\sum_{j \in O(i)} f_{ij} - \sum_{j \in I(i)} f_{ij} = 0 \;\; \forall i \in N \setminus (G \cup D)$
- $f_{ij} = B_{ij}(\theta_i - \theta_j) \;\; \forall (i,j) \in E$
- $-F_{ij} \le f_{ij} \le F_{ij} \;\; \forall (i,j) \in E$
- $p_i^m \le p_i \le p_i^M \;\; \forall i \in G$
- $d_i^m \le d_i \le d_i^M \;\; \forall i \in D$



<u>Locational Marginal Price</u>: π_i is the **optimal objective change per unit increase of demand at node *i***

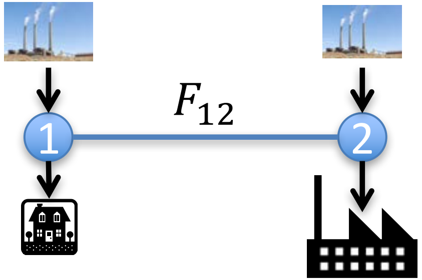
A Simple Example: No Congestion

- Two-node system:
  - Generator 1 offers \$20/MWh with capacity 150MW
  - Generator 2 offers \$10/MWh with capacity 300MW
  - Demand 1 is fixed at 150MW
  - Demand 2 is fixed at 100MW



- Case 1:
  - Transmission capacity $F_{12} = 300\text{MW}$
- Since transmission constraint is never binding $(\text{i.e.} -F_{12} < f_{12} < F_{12})$, we can ignore the network flow constraints
- Market clearing problem minimizes the total cost:
  - $p_1 = 0\text{MW}$, $p_2 = 250\text{MW}$, LMP: π_1 = \$10/MWh, π_2 = \$10/MWh
- When there is no congestion, price is uniform.

A Simple Example: Congestion



- Two-node system:
  - Generator 1 offers \$20/MWh with capacity 150MW
  - Generator 2 offers \$10/MWh with capacity 300MW
  - Demand 1 is fixed at 150MW
  - Demand 2 is fixed at 100MW
- Case 2:
  - Transmission capacity $F_{12} = 80\text{MW}$
- Transmission constraint will be binding, we cannot ignore them
- Market clearing problem minimizes the total cost:

$p_1 = 70$MW, $p_2 = 180$MW, $f_{12} = -80$MW, LMP: π_1 = \$20/MWh, π_2 = \$10/MWh

- When there is congestion, locational prices are different.

Real-World Examples

- [ISO New England Market]
- [PJM (Pennsylvania-New Jersey-Maryland) Market]
- [New York ISO Market]
- [MidContinental ISO Market]

Summary

- Explored the intuition of LMP by a simple two-node system
- Examined some real-world electricity markets' real time pricing