Deterministic Optimization

Linear Optimization Modeling

Electricity Markets

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Modeling Power Plant Generation Using LP

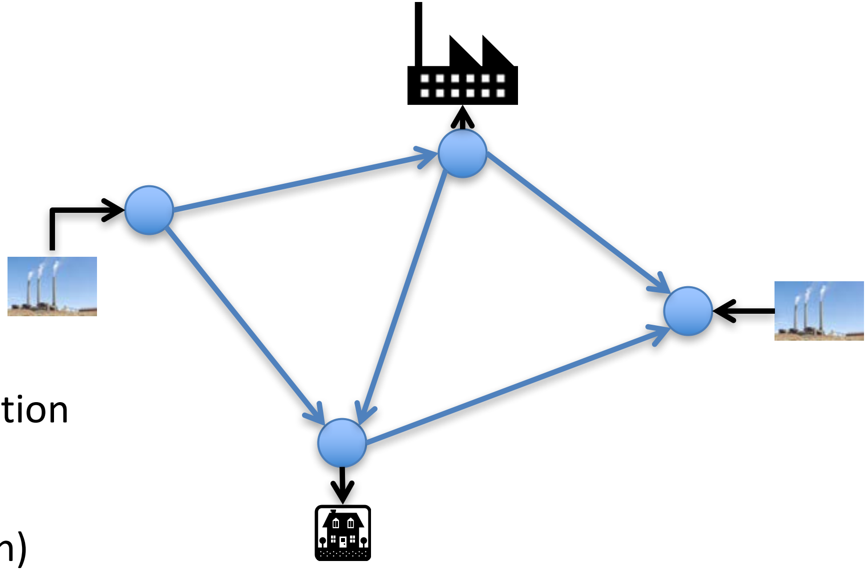
Modeling using Linear Programs

Learning Objectives for this Lesson

- Create LP models for scheduling generation in power systems

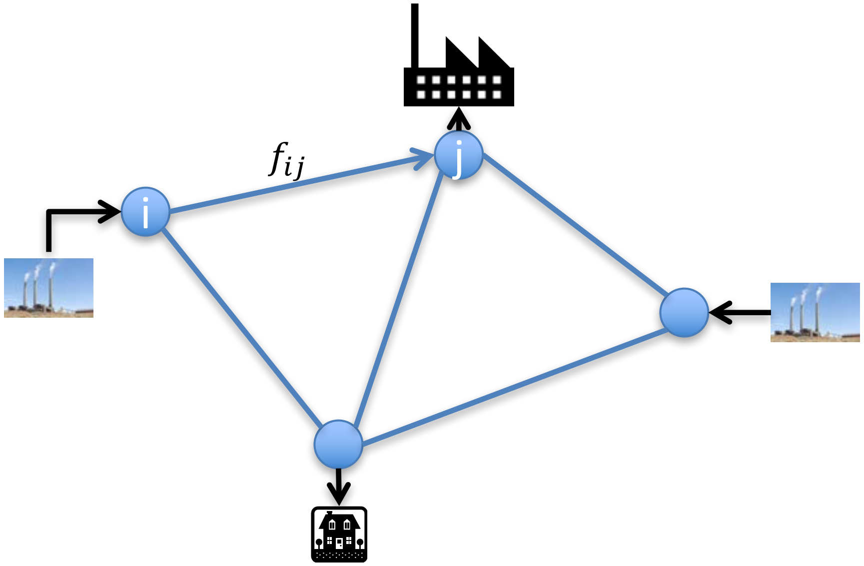
Power System Scheduling Problem

- A power system is composed of:
  - A network of transmission lines
  - Generators (power suppliers)
  - Demand (power consumers)
- Objective:
  - Satisfy demand with minimum cost generation
- Constraints:
  - Network flow constraint (flow conservation)
  - Special constraint linking branch flow and nodal potential
  - Flow limit constraints
  - Power plant physical limits on production



Formulating LP Model

- Decision variables:
  - Generator output: p_i for each generator $i \in G$
  - Power flow: f_{ij} on each edge $(i, j) \in E$
  - Nodal potential: θ_i on each node $i \in N$
- Objective: a linear cost function
  - $\sum_{i=1}^{|G|} c_i p_i$
- Constraints:
  - Flow conservation:
    - $\sum_{j \in O(i)} f_{ij} - \sum_{j \in I(i)} f_{ij} = p_i$ for $i \in G$
    - $\sum_{j \in O(i)} f_{ij} - \sum_{j \in I(i)} f_{ij} = -d_i$ for $i \in D$
    - $\sum_{j \in O(i)} f_{ij} - \sum_{j \in I(i)} f_{ij} = 0$ for $i \in N \backslash (G \cup D)$
  - Constraint linking branch flow and nodal potential:
    - $f_{ij} = B_{ij}(\theta_i - \theta_j)$ for all $(i, j) \in E$
  - Flow limit constraint:
    - $-F_{ij} \le f_{ij} \le F_{ij}$ for all $(i, j) \in E$



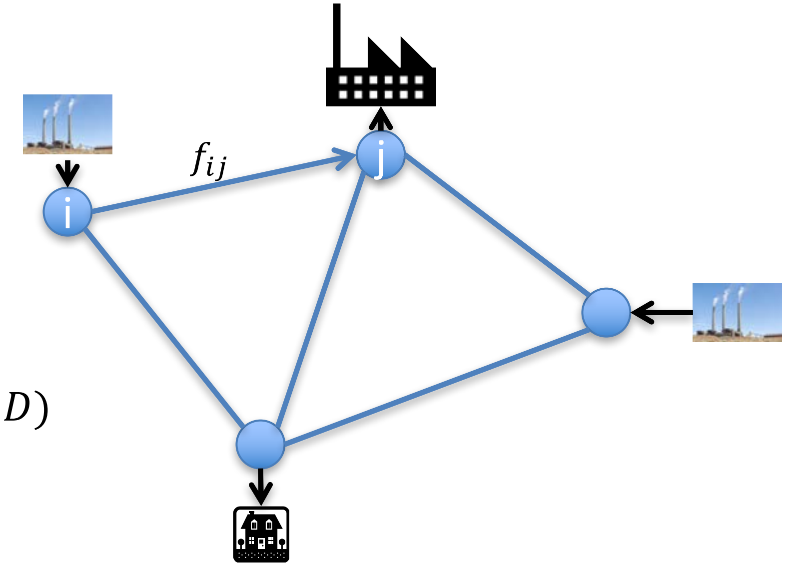
- Generator physical limit constraints:
  - $p_i^m \le p_i \le p_i^M$ for all $i \in G$

LP Model for Power Scheduling

- $\min \sum_{i=1}^{|G|} c_i p_i$

  subject to

  - $\sum_{j \in O(i)} f_{ij} - \sum_{j \in I(i)} f_{ij} = p_i \quad \forall i \in G$
  - $\sum_{j \in O(i)} f_{ij} - \sum_{j \in I(i)} f_{ij} = -d_i \quad \forall i \in D$
  - $\sum_{j \in O(i)} f_{ij} - \sum_{j \in I(i)} f_{ij} = 0 \quad \forall i \in N \backslash (G \cup D)$
  - $f_{ij} = B_{ij}(\theta_i - \theta_j) \quad \forall (i,j) \in E$
  - $-F_{ij} \leq f_{ij} \leq F_{ij} \quad \forall (i,j) \in E$
  - $p_i^m \leq p_i \leq p_i^M \quad \forall i \in G$



Summary

- Power systems are operated by optimization software

- Formulated LP for Power System Scheduling
    - Network flow constraint appeared again
    - Nodal potentials are needed
    - Other physical constraints