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# A TEMPLATE FOR ARXIV STYLE \*

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## ABSTRACT

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**Keywords** First keyword · Second keyword · More

## 1 Introduction

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## 2 Background

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### 2.1 Small Set Flip

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$$\xi_{ij}(t) = P(x_t = i, x_{t+1} = j | y, v, w; \theta) = \frac{\alpha_i(t) a_{ij}^{w_t} \beta_j(t+1) b_j^{v_{t+1}}(y_{t+1})}{\sum_{i=1}^N \sum_{j=1}^N \alpha_i(t) a_{ij}^{w_t} \beta_j(t+1) b_j^{v_{t+1}}(y_{t+1})} \quad (1)$$

### 2.1.1 Headings: third level

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## 3 Algorithm: K-Top Probabilistic Flip Method (K-Top PFM)

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### Algorithm 1: sort-top-K(T)

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**Data:** A vector  $T \in \mathbb{Z}_2^N$

**Result:** A set  $S$  of the top  $K$  indices in  $T$

- 1  $S \leftarrow$  indices of a descending radix sort of  $T$ 's rows;
  - 2 **return** A set of the top  $K$  indices in  $S$ ;
- 

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### Algorithm 2: probabilistic-set-flip( $E$ )

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**Data:** A syndrome  $\sigma_0 \in \mathbb{F}_2^M$

**Result:** Deduced error  $\hat{E}$  if the algorithm converges and  $\perp$  otherwise

- 1  $\hat{E} \leftarrow 0^N$ ;
  - 2  $\sigma \leftarrow \sigma_0$ ;
  - 3 **while**  $\exists F \in \mathcal{F} : |\sigma| - |\sigma \oplus \sigma_X(\mathbf{k})| > 0$  **do**
  - 4      $T \leftarrow H_Z H_X^T \sigma$ ;
  - 5      $\text{generators} \leftarrow \text{sort-top-K}(T)$ ;
  - 6      $\text{to-check} \leftarrow \bigcup_{i \in \text{generators}} \mathcal{P}(C_{Z_i})$ ;
  - 7      $\mathbf{k} \leftarrow \arg \max_{\mathbf{k} \in \text{to-check}} \frac{|\sigma| - |\sigma \oplus \sigma_X(\mathbf{k})|}{|\mathbf{k}|}$ ;
  - 8      $\hat{E} \leftarrow \hat{E} \oplus \mathbf{k}$ ;
  - 9      $\sigma \leftarrow \sigma \oplus \sigma_X(\mathbf{k})$ ;
  - 10 **end**
  - 11 **return**  $\hat{E}$  if  $|\sigma| = 0$ ,  $\perp$  otherwise.
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### 3.1 A Moral Reason/ Intuition

The algorithm is essentially the same as Small Set Bit Flip [TODO: CITE] with a minor difference, only a constant number of generators are checked. The idea here is that given a syndrome  $\sigma$  and a parity check matrix,  $H_X$ , the  $i$ th row of  $H_X^T \sigma$  equals the number of error-ed checks that a qubit touches. Then, the  $k$ th row of  $H_Z H_X^T \sigma$  is roughly correlated

to the number of error-ed checks that the qubits in the  $k$ th generator touch. This rough correlation comes from the fact that we are working with expander codes. So then, if you get the generators touching the most error-ed stabilizers, it would stand to reason that flipping some subset of qubits from a “highly error-ed generator” would result in decreasing the syndrome.

## 4 PFM Analysis

The following section assumes that we are working with syndrome  $\sigma_X$ , a generator matrix  $H_Z$ , and parity check matrix  $H_X$ . The analysis is the same for a syndrome,  $\sigma_Z$ , generator matrix  $H_X$ , and parity check matrix  $H_Z$ .

### Notation

Given a vector  $v$ , define  $v_i$  to be the value of the  $i$ th row of  $v$ .

### Definitions

Let  $\Delta_{\text{stabilizer}}$  equal to the degree of a stabilizer vertex. Note that due to the hypergraph’s construction, all stabilizers have the same constant degree. Let  $\Delta_{\text{bit}}$  equal to the degree of a qubit vertex. As with the stabilizers, all qubits have the same constant degree. Also, for generator  $k$ , let  $\aleph$  be the set of stabilizers neighboring the generator. Note that  $|\aleph| \leq \Delta_{\text{stabilizer}} \Delta_{\text{bit}}$ .

Given a syndrome,  $\sigma_X$ , define a “bit-score vector”,  $\mathbf{b} = H_X^T \sigma_X$  where  $\mathbf{b} \in \mathbb{Z}^N$ . Then, define a “generator-score vector” as  $\mathbf{g} = H_Z \mathbf{b}$  where  $\mathbf{g} \in \mathbb{Z}^M$ . Moreover, assume that for error  $e \in \mathbb{F}_2^N$ ,  $\Pr[e_i = 1] = p$  for all  $i \in [N]$  (i.e. the error is modeled as independent). Let  $q = 1 - p$ . Let  $s_1, s_2, \dots, s_M$  denote the set of stabilizer vertices. Let  $N_i = \sum_{j \in \Gamma(s_i)} e_j$  where  $N_i \in \mathbb{Z}$ .  $N_i$  can be thought of as the number of qubits with an error in the neighborhood of stabilizer  $i$ .

Also, let random variable  $S_i \in \mathbb{F}_2$  correspond to  $\sigma_{X_i}$ . Then we know that

$$\Pr[S_i = 1] = \Pr[N_i \text{ is odd}] = \frac{1}{2} - \frac{1}{2}(1 - 2p)^{\Delta_{\text{stabilizer}}}.$$

Next, define indicator random variable,  $L_j$  to be 1 if  $\sigma_{X_i} = 0$  and  $N_i > 0$ . Basically,  $L_j$  indicates whether a stabilizer check succeeds, but an error is in its neighborhood. I.e. stabilizer  $j$  is “lying.”

So then,

$$\begin{aligned} \Pr[L_j = 1] &= \Pr[\sigma_{X_i} = 0 \mid S_i > 0] \\ &= \Pr[S_i \text{ is even}] - \Pr[S_i = 0] \\ &= \frac{1}{2} + \frac{1}{2}(1 - 2p)^{\Delta_{\text{stabilizer}}} - (1 - p)^{\Delta_{\text{stabilizer}}}. \end{aligned}$$

Then define random variable,  $E_k$  to be

$$E_k = \sum_{\text{Stabilizer } j \in \aleph_k} L_j.$$

$E_k$  is basically the number of time a generator  $k$ , lies for all stabilizers neighboring the generator.

We can then say that

$$E_k \sim \text{Binom}(\aleph_k, \frac{1}{2} + \frac{1}{2}(1 - 2p)^{\Delta_{\text{stabilizer}}} - (1 - p)^{\Delta_{\text{stabilizer}}}).$$

Then, let random variable  $B_i \in \mathbb{Z}$  correspond to  $\mathbf{b}_i$  and random variable  $G_i \in \mathbb{Z}$  correspond to  $\mathbf{g}_i$ .

So,

$$S_i \sim \text{Bernoulli}\left(\frac{1}{2} - \frac{1}{2}(1 - 2p)^{\Delta_{\text{stabilizer}}}\right).$$

Then,

$$B_i = \sum_{\text{Stabilizer } j \in \Gamma(\text{Bit } i)} S_j$$

So,

$$B_i \sim \text{Binom}(\Delta_{\text{bit}}, \frac{1}{2} - \frac{1}{2}(1-2p)^{\Delta_{\text{stabilizer}}}).$$

And then,

$$G_k = \sum_{\text{Bit } i \in \Gamma(\text{generator } k)} \sum_{\text{Stabilizer } j \in \Gamma(\text{Bit } i)} S_j$$

So then,

$$G_k \sim \text{Binom}(\Delta_{\text{bit}}\Delta_{\text{stabilizer}}, \frac{1}{2} - \frac{1}{2}(1-2p)^{\Delta_{\text{stabilizer}}}).$$

The pfm algorithm (Algorithm 2) on line 5 gets the  $K$  generators, indexed by  $g_1, g_2, \dots, g_K$ , with the top values of  $g_i$  for  $i \in [M]$ . WLOG, assume  $g_{g_1} \geq g_{g_2} \geq \dots \geq g_{g_K}$ . Then, we can think of  $\mathbf{E}[G_{g_i}]$  as the expected value of the  $i$ th top sample from  $M$  samples of the distribution defining  $G_i$ .

Let random variable  $G'_k$  then equal

$$G'_k = \sum_{\text{Stabilizer } j \in \mathbb{N}_k} S_j.$$

Note that  $|\mathbb{N}| \geq (1-\delta)\Delta_{\text{stabilizer}}\Delta_{\text{bit}}$  because we are working with expander codes. So then,

$$\begin{aligned} \mathbf{E}[G'_k] &= \sum_{\text{Stabilizer } j \in \mathbb{N}_k} \mathbf{E}[S_j] \\ &\geq (1-\delta)\Delta_{\text{stabilizer}}\Delta_{\text{bit}} \mathbf{E}_{j \in [M]}[S_j] \\ &\geq (1-\delta) \mathbf{E}[G_k]. \end{aligned}$$

**Lemma 4.1.** For a generator, given  $E_k$  and  $G'$ , we can find some correction vector  $\mathbf{k} \in \mathbb{F}_2^N$  such that  $|\sigma| - |\sigma \oplus \sigma_X(\mathbf{k})| \geq G'_k - E_k$  and  $|\mathbf{k}| \leq \frac{1-\delta}{\Delta_{\text{bit}}}(G'_k + E_k)$  for  $G' > E_k$ .

*Proof.* TODO: part 1 is that there are 3 types of stabilizers in neighbourhood. Those from  $G'$ , those from  $E$ , and those in neither. If those in neither, there is no neighbourhood in their error, so you can leave those bits alone. Then, by flipping bits connected to  $G'$  you decrease syndrome by  $G'$ , but you add in at most  $E$

part 2: each flipping bit effects at least  $(1-\delta)\Delta_{\text{bit}}$  stabilizers □

So then for any  $e$  where  $|e| < \min(\gamma_A n_A, \gamma_B n_B)$  by TODO: cite hypergraph prod paper, we know that we can always successfully correct errors if we can find a  $\mathbf{k}$  such that  $\mathbf{k}$  is a subset of a generator and

$$\frac{|\sigma| - |\sigma \oplus \sigma_X(\mathbf{k})|}{|\mathbf{k}|} \geq \frac{1}{3}.$$

**Lemma 4.2.** We claim the following holds for an  $i \in [K]$  and for  $p_S = \Pr[S_j = 1]$  for any stabilizer  $j$

$$\begin{aligned} &\Pr\left[\frac{\Delta_{\text{bit}}(G'_i g_i - E_{g_i})}{(1-\delta)(G'_i g_i + E_{g_k})} < \frac{1}{3}\right] \\ &\leq \sum_{e=0}^{\Delta_{\text{bit}}-1} \text{orderprob}\left(\Delta_{\text{bit}}\Delta_{\text{stabilizer}} - \Delta_{\text{stabilizer}}e, p_S, i, \frac{(3\Delta_{\text{bit}} + 1 - \delta)e}{3\Delta_{\text{bit}} - 1 + \delta}\right) \cdot \Pr[E_{g_i} = e] + \sum_{e=\Delta_{\text{bit}}}^{\Delta_{\text{bit}}\Delta_{\text{stabilizer}}} \Pr[E_{g_i} = e] \end{aligned}$$

where

$$\text{orderprob}(n, p, i, v) = \Pr[W_i \leq v]$$

and  $W_i$  is the  $i$ th largest order statistic from  $M$  samples of  $\text{Binomial}(n, p)$ .

See appendix TODO: cite for details

So then,

$$\Pr[\text{loop cannot find a correcting error}] \leq \prod_{i \in [K]} \Pr\left[\frac{\Delta_{\text{bit}}(G'_i g_i - E_{g_i})}{(1-\delta)(G'_i g_i + E_{g_k})} < \frac{1}{3}\right]$$

lemma 4.1,

## 5 PFM Numerical Simulations

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## 6 PFM Future Outlook

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## 7 Conclusion

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## 8 Acknowledgments

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## 9 Examples of citations, figures, tables, references

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The documentation for natbib may be found at

<http://mirrors.ctan.org/macros/latex/contrib/natbib/natnotes.pdf>

Of note is the command `\citet`, which produces citations appropriate for use in inline text. For example,

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Hasselmo, et al. (1995) investigated...

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### 9.1 Figures

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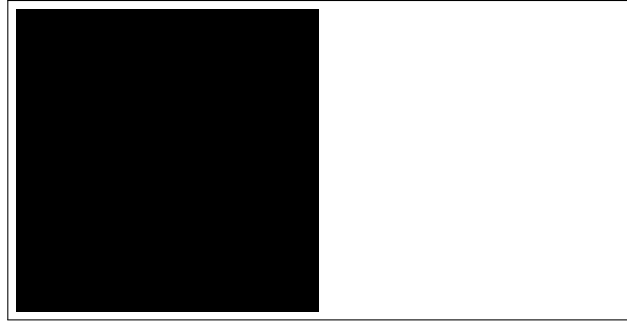


Figure 1: Sample figure caption.

Table 1: Sample table title

Part		
Name	Description	Size ( $\mu\text{m}$ )
Dendrite	Input terminal	$\sim 100$
Axon	Output terminal	$\sim 10$
Soma	Cell body	up to $10^6$

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## 9.2 Tables

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## 9.3 Lists

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## 10 Conclusion

Your conclusion here

## Acknowledgments

This was supported in part by.....

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<sup>2</sup>Sample of the first footnote.

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## A Proof of Lemma 4.1

First, to just restate the lemma. We claim the following holds for an  $i \in [K]$  and for  $p_S = \Pr[S_j = 1]$  for any stabilizer  $j$

$$\Pr\left[\frac{\Delta_{\text{bit}}(G'g_i - E_{g_i})}{(1-\delta)(G'g_i + E_{g_k})} < \frac{1}{3}\right] \leq \sum_{e=0}^{\Delta_{\text{bit}}-1} \text{orderprob}\left(\Delta_{\text{bit}}\Delta_{\text{stabilizer}} - \Delta_{\text{stabilizer}}e, p_S, i, \frac{(3\Delta_{\text{bit}} + 1 - \delta)e}{3\Delta_{\text{bit}} - 1 + \delta}\right) \cdot \Pr[E_{g_i} = e] + \sum_{e=\Delta_{\text{bit}}}^{\Delta_{\text{bit}}\Delta_{\text{stabilizer}}} \Pr[E_{g_i} = e]$$

where

$$\text{orderprob}(n, p, i, v) = \Pr[v < (1-\delta)W_i]$$

and  $W_i$  is the  $i$ th largest order statistic from  $M$  samples of  $\text{Binomial}(n, p)$ .

First, because  $0 \leq E_{g_i} \leq \Gamma(\text{generator } i) \leq \Delta_{\text{bit}}\Delta_{\text{stabilizer}}$ ,

$$\begin{aligned} \Pr\left[\frac{\Delta_{\text{bit}}(G'g_i - E_{g_i})}{(1-\delta)(G'g_i + E_{g_k})} < \frac{1}{3}\right] &= \sum_{e=0}^{\Delta_{\text{bit}}\Delta_{\text{stabilizer}}} \Pr\left[\frac{\Delta_{\text{bit}}(G'g_i - e)}{(1-\delta)(G'g_i + e)} < \frac{1}{3}\right] \Pr[E_{g_i} = e] \\ &\leq \sum_{e=0}^{\Delta_{\text{bit}}-1} \Pr\left[\frac{\Delta_{\text{bit}}(G'g_i - e)}{(1-\delta)(G'g_i + e)} < \frac{1}{3}\right] \Pr[E_{g_i} = e] + \sum_{e=\Delta_{\text{bit}}}^{\Delta_{\text{bit}}\Delta_{\text{stabilizer}}} 1 \cdot \Pr[E_{g_i} = e] \\ &= \sum_{e=0}^{\Delta_{\text{bit}}-1} \Pr\left[G'g_i < \frac{(3\Delta_{\text{bit}} + 1 - \delta)e}{3\Delta_{\text{bit}} - 1 + \delta}\right] \Pr[E_{g_i} = e] + \sum_{e=\Delta_{\text{bit}}}^{\Delta_{\text{bit}}\Delta_{\text{stabilizer}}} 1 \cdot \Pr[E_{g_i} = e]. \end{aligned}$$

So then, for a given  $e$ , we just need to show that

$$\Pr\left[G'g_i < \frac{(3\Delta_{\text{bit}} + 1 - \delta)e}{3\Delta_{\text{bit}} - 1 + \delta}\right] \leq \text{orderprob}\left(\Delta_{\text{bit}}\Delta_{\text{stabilizer}} - \Delta_{\text{stabilizer}}e, p_S, i, \frac{(3\Delta_{\text{bit}} + 1 - \delta)e}{3\Delta_{\text{bit}} - 1 + \delta}\right).$$

Then, also observe that, for a fixed  $E_{g_k}j$ ,

$$\Pr\left[\frac{\Delta_{\text{bit}}(G''g_i - E_{g_i})}{(1-\delta)(G''g_i + E_{g_k})} < \frac{1}{3}\right] \geq \Pr\left[\frac{\Delta_{\text{bit}}(G'g_i - E_{g_i})}{(1-\delta)(G'g_i + E_{g_k})} < \frac{1}{3}\right]$$

if  $\mathbf{E}[G''_{g_i}] < \text{TODO} : \text{thethingwewant } \mathbf{E}[G''_{g_i}] \leq \mathbf{E}[G'_{g_i}]$ ,  $\frac{\mathbf{Var}[G''_{g_i}]}{\mathbf{Var}[G'_{g_i}]} \geq \frac{\mathbf{E}[G'_{g_i}]}{\mathbf{E}[G''_{g_i}]}$ , TODO: AND ARE CALCULATED VIA CHEBYSHEVS INEQUALITY. TODO: the above comes from some magic having to do with Chebyshev's bounds and square root and the way we calculate thing huh.

We then know that  $\mathbf{E}[G'_{g_i}] \geq (1-\delta)\mathbf{E}[G_{g_i}]$  and that TODO: var. And we also know that  $G_{g_i}$  is the  $i$ th largest of  $M$  samples from  $\text{Binomial}(\Delta_{\text{stabilizer}}\Delta_{\text{bit}}, p_S)$ . If  $E_{g_i} = e$ , we also know that for some  $s_1, s_2, \dots, s_e \in \mathbb{N}$ ,  $S_{s_j} = 0$  for  $j \in [e]$ .

So then, given  $e$ ,  $G_{g_i}$  is the  $i$ th largest sample of  $M$  from  $\text{Binomial}(\Delta_{\text{stabilizer}}\Delta_{\text{bit}} - c\Delta_{\text{stabilizer}}e, p_S)$  for some  $0 < c \leq 1$ . This is true because each stabilizer,  $s_j$ , that must be 0 neighbors at most  $\Delta_{\text{stabilizer}}$  bits in generator  $g_i$  and neighbors at least one bit in  $g_i$ .

So then, we can see that

$$\mathbf{E}[(1-\delta)G_{g_i} \mid E_{g_i} = e] \geq \mathbf{E}[(1-\delta)G''_{k_i}]$$

where

$$G'' \sim \text{Binomial}(\Delta_{\text{stabilizer}}\Delta_{\text{bit}} - \Delta_{\text{stabilizer}}e, p_S)$$

and  $k_1, k_2, \dots, k_K$  are the order statistics with  $G''_{k_1} \geq G''_{k_2} \geq \dots \geq G''_{k_K}$ . Finally, because

$$\Pr\left[\frac{\Delta_{\text{bit}}(G''g_i - E_{g_i})}{(1-\delta)(G''g_i + E_{g_k})} < \frac{1}{3} \mid E_{g_i} = e\right] = \text{orderprob}\left(\Delta_{\text{bit}}\Delta_{\text{stabilizer}} - \Delta_{\text{stabilizer}}e, p_S, i, \frac{(3\Delta_{\text{bit}} + 1 - \delta)e}{3\Delta_{\text{bit}} - 1 + \delta}\right)$$

the lemma holds.