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# A TEMPLATE FOR ARXIV STYLE \*

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**Author1, Author2**

Affiliation

Univ

City

{Author1, Author2}email@email

**Author3**

Affiliation

Univ

City

email@email

## ABSTRACT

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**Keywords** First keyword · Second keyword · More

## 1 Introduction

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## 2 Background

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### 2.1 Small Set Flip

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$$\xi_{ij}(t) = P(x_t = i, x_{t+1} = j | y, v, w; \theta) = \frac{\alpha_i(t) a_{ij}^{w_t} \beta_j(t+1) b_j^{v_{t+1}}(y_{t+1})}{\sum_{i=1}^N \sum_{j=1}^N \alpha_i(t) a_{ij}^{w_t} \beta_j(t+1) b_j^{v_{t+1}}(y_{t+1})} \quad (1)$$

### 2.1.1 Headings: third level

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## 3 Algorithm: K-Top Probabilistic Flip Method (K-Top PFM)

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### Algorithm 1: sort-top-K(T)

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**Data:** A vector  $T \in \mathbb{Z}_2^N$

**Result:** A set  $S$  of the top  $K$  indices in  $T$

- 1  $S \leftarrow$  indices of a descending radix sort of  $T$ 's rows;
  - 2 **return** A set of the top  $K$  indices in  $S$ ;
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### Algorithm 2: probabilistic-set-flip( $E$ )

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**Data:** A syndrome  $\sigma_0 \in \mathbb{F}_2^M$

**Result:** Deduced error  $\hat{E}$  if the algorithm converges and  $\perp$  otherwise

- 1  $\hat{E} \leftarrow 0^N$ ;
  - 2  $\sigma \leftarrow \sigma_0$ ;
  - 3 **while**  $\exists F \in \mathcal{F} : |\sigma| - |\sigma \oplus \sigma_X(\mathbf{k})| > 0$  **do**
  - 4      $T \leftarrow H_Z H_X^T \sigma$ ;
  - 5     generators  $\leftarrow$  sort-top-K( $T$ );
  - 6     to-check  $\leftarrow \bigcup_{i \in \text{generators}} \mathcal{P}(\mathcal{C}_{Z_i})$ ;
  - 7      $\mathbf{k} \leftarrow \arg \max_{\mathbf{k} \in \text{to-check}} \frac{|\sigma| - |\sigma \oplus \sigma_X(\mathbf{k})|}{|\mathbf{k}|}$ ;
  - 8      $\hat{E} \leftarrow \hat{E} \oplus \mathbf{k}$ ;
  - 9      $\sigma \leftarrow \sigma \oplus \sigma_X(\mathbf{k})$ ;
  - 10 **end**
  - 11 **return**  $\hat{E}$  if  $|\sigma| = 0$ ,  $\perp$  otherwise.
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### 3.1 The Intuition

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## 4 PFM Analysis

The following section assumes that we are working with syndrome  $\sigma_X$ , a generator matrix  $H_Z$ , and parity check matrix  $H_X$ . The analysis is the same for a syndrome,  $\sigma_Z$ , generator matrix  $H_X$ , and parity check matrix  $H_Z$ .

### Notation

Given a vector  $v$ , define  $v_i$  to be the value of the  $i$ th row of  $v$ .

### Definitions

Let  $\Delta_{\text{stabilizer}}$  equal to the degree of a stabilizer vertex. Note that due to the hypergraph's construction, all stabilizers have the same constant degree. Let  $\Delta_{\text{bit}}$  equal to the degree of a qubit vertex. As with the stabilizers, all qubits have the same constant degree. Also, for generator  $k$ , let  $\aleph$  be the set of stabilizers neighboring the generator. Note that  $|\aleph| \leq \Delta_{\text{stabilizer}} \Delta_{\text{bit}}$ .

Given a syndrome,  $\sigma_X$ , define a "bit-score vector",  $\mathbf{b} = H_X^T \sigma_X$  where  $\mathbf{b} \in \mathbb{Z}^N$ . Then, define a "generator-score vector" as  $\mathbf{g} = H_Z \mathbf{b}$  where  $\mathbf{g} \in \mathbb{Z}^M$ . Moreover, assume that for error  $e \in \mathbb{F}_2^N$ ,  $\Pr[e_i = 1] = p$  for all  $i \in [N]$  (i.e. the error is modeled as independent). Let  $q = 1 - p$ . Let  $s_1, s_2, \dots, s_M$  denote the set of stabilizer vertices. Let  $N_i = \sum_{j \in \Gamma(s_i)} e_j$  where  $N_i \in \mathbb{Z}$ .  $N_i$  can be thought of as the number of qubits with an error in the neighborhood of stabilizer  $i$ .

Then, let random variable  $B_i \in \mathbb{Z}$  correspond to  $\mathbf{b}_i$  and random variable  $G_i \in \mathbb{Z}$  correspond to  $\mathbf{g}_i$ . Also, let random variable  $S_i \in F_2$  correspond to  $\sigma_{X_i}$ .

$$\Pr[S_i = 1] = \Pr[N_i \text{ is odd}] = \frac{1}{2} - \frac{1}{2}(1 - 2p)^{\Delta_{\text{stabilizer}}}$$

So,

$$S_i \sim \text{Bernoulli}\left(\frac{1}{2} - \frac{1}{2}(1 - 2p)^{\Delta_{\text{stabilizer}}}\right).$$

Then,

$$B_i = \sum_{\text{Stabilizer } j \in \Gamma(\text{Bit } i)} S_j$$

So,

$$B_i \sim \text{Binom}(\Delta_{\text{bit}}, \frac{1}{2} - \frac{1}{2}(1 - 2p)^{\Delta_{\text{stabilizer}}}).$$

And then,

$$G_k = \sum_{\text{Bit } i \in \Gamma(\text{generator } k)} \sum_{\text{Stabilizer } j \in \Gamma(\text{Bit } i)} S_j$$

So then,

$$G_k \sim \text{Binom}(\Delta_{\text{bit}} \Delta_{\text{stabilizer}}, \frac{1}{2} - \frac{1}{2}(1 - 2p)^{\Delta_{\text{stabilizer}}}).$$

The pfm algorithm (Algorithm 2) on line 5 gets the  $K$  generators, indexed by  $g_1, g_2, \dots, g_K$ , with the top values of  $\mathbf{g}_i$  for  $i \in [M]$ . WLOG, assume  $\mathbf{g}_{g_1} \geq \mathbf{g}_{g_2} \geq \dots \geq \mathbf{g}_{g_K}$ . Then, we can think of  $\mathbf{E}[G_{g_i}]$  as the expected value of the  $i$ th top sample from  $M$  samples of the distribution defining  $G_i$ .

Let random variable  $G'_k$  then equal

$$G'_k = \sum_{\text{Stabilizer } j \in \aleph_k} S_j.$$

Note that  $|\aleph| \geq (1 - \delta) \Delta_{\text{stabilizer}} \Delta_{\text{bit}}$  because we are working with expander codes. So then,

$$\begin{aligned} \mathbf{E}[G'_k] &= \sum_{\text{Stabilizer } j \in \aleph_k} \mathbf{E}[S_j] \\ &\geq (1 - \delta) \Delta_{\text{stabilizer}} \Delta_{\text{bit}} \mathbf{E}_{j \in [M]}[S_j] \\ &\geq (1 - \delta) \mathbf{E}[G_k]. \end{aligned}$$

Next, define indicator random variable,  $L_j$  to be 1 if  $\sigma_{\mathbf{x}_i} = 0$  and  $N_i > 0$ . Basically,  $L_j$  indicates whether a stabilizer check succeeds, but an error is in its neighborhood. I.e. stabilizer  $j$  is “lying.”

So then,

$$\begin{aligned}\Pr[L_j = 1] &= \Pr[\sigma_{\mathbf{x}_i} = 0 \mid S_i > 0] \\ &= \Pr[S_i \text{ is even}] - \Pr[S_i = 0] \\ &= \frac{1}{2} + \frac{1}{2}(1 - 2p)^{\Delta_{\text{stabilizer}}} - (1 - p)^{\Delta_{\text{stabilizer}}}\end{aligned}$$

Then define random variable,  $E_k$  to be

$$E_k = \sum_{\text{Stabilizer } j \in \mathbb{N}} L_j.$$

$E_k$  is basically the number of time a generator  $k$ , lies for all stabilizers neighboring the generator.

We can then say that

$$E_k \sim \text{Binom}(\Delta_{\text{bit}} \Delta_{\text{stabilizer}}, \frac{1}{2} + \frac{1}{2}(1 - 2p)^{\Delta_{\text{stabilizer}}} - (1 - p)^{\Delta_{\text{stabilizer}}}).$$

**Lemma 4.1.** *For a generator, given  $E_k$  and  $G'$ , we can find some correction vector  $\mathbf{k} \in \mathbb{F}_2^N$  such that  $|\sigma| - |\sigma \oplus \sigma_X(\mathbf{k})| \geq G'_k - E_k$  and  $|\mathbf{k}| \leq \frac{1-\delta}{\Delta_{\text{bit}}}(G'_k + E_k)$  for  $G' > E_k$ .*

*Proof.* TODO: part 1 is that there are 3 types of stabilizers in neighbourhood. Those from  $G'$ , those from  $E$ , and those in neither. If those in neither, there is no neighbourhood in their error, so you can leave those bits alone. Then, by flipping bits connected to  $G'$  you decrease syndrome by  $G'$ , but you add in at most  $E$

part 2: each flipping bit effects at least  $(1-\delta) \Delta_{\text{bit}}$  stables □

So then, by TODO: cite hypergraph prod paper, we know that we can always successfully correct errors if, for each round of the while loop,

$$\frac{|\sigma| - |\sigma \oplus \sigma_X(\mathbf{k})|}{|\mathbf{k}|} \geq \frac{1}{3}.$$

So then,

$$\Pr[\text{loop cannot find a correcting error}] \leq \prod_{i \in [K]} \left( 1 - \Pr \left[ \frac{\Delta_{\text{bit}}(G'_{g_i} - E_{g_i})}{(1 - \delta)(G'_{g_i} + E_{g_i})} \geq \frac{1}{3} \right] \right)$$

lemma 4.1,

## 5 PFM Numerical Simulations

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## 6 PFM Future Outlook

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## 7 Conclusion

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## 8 Acknowledgments

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## 9 Examples of citations, figures, tables, references

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The documentation for natbib may be found at

<http://mirrors.ctan.org/macros/latex/contrib/natbib/natnotes.pdf>

Of note is the command `\citet`, which produces citations appropriate for use in inline text. For example,

```
\citet{hasselmo} investigated\dots
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produces

Hasselmo, et al. (1995) investigated...

<https://www.ctan.org/pkg/booktabs>

### 9.1 Figures

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### 9.2 Tables

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<sup>2</sup>Sample of the first footnote.

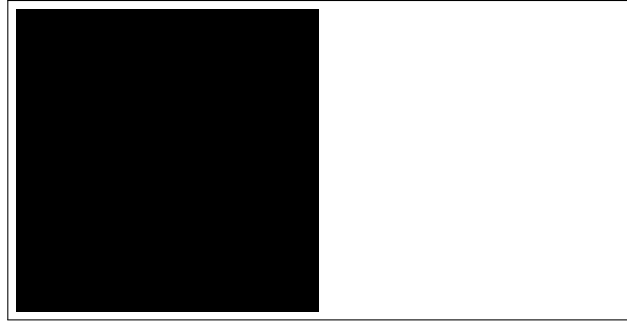


Figure 1: Sample figure caption.

Table 1: Sample table title		
Part		
Name	Description	Size ( $\mu\text{m}$ )
Dendrite	Input terminal	$\sim 100$
Axon	Output terminal	$\sim 10$
Soma	Cell body	up to $10^6$

### 9.3 Lists

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- Aliquam dignissim blandit est, in dictum tortor gravida eget. In ac rutrum magna.

## 10 Conclusion

Your conclusion here

## Acknowledgments

This was supported in part by.....

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