A TEMPLATE FOR ARXIV STYLE *

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ABSTRACT

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1 Introduction

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2 Contribution

3 Notation and Definitions

Definitions and assumptions

- We will use j when referring to a stabilizer, i when referring to a generator, k when referring to a bit
- We take $\boldsymbol{e}_Z$ to be the error vector
- We take $\boldsymbol{\sigma}_Z$
- Let $\mathcal{C}_Z$ be a code which can correct Z errors and $\mathcal{C}_X$ be a code a which can correct X errors.

* *Citation*: **Authors. Title. Pages.... DOI:000000/11111.**

- Let $H_X \in F_2^{M\times N}$ be the parity check matrix for $\mathcal{C}_X$ as well as the generator for $\mathcal{C}_Z$.
- Let $H_Z \in F_2^{M\times N}$ be the parity check matrix for $\mathcal{C}_Z$ as well as the generator for $\mathcal{C}_X$.
- Let M be the number of stabilizers. Let us also only consider the case where M is the number of generators. This results from when H_Z and H_X have the same number of rows.
- Let N be the number of bits.
- Assume H_X is the adjacency matrix of a (δ, γ)-`left-expander` bipartite graph where A is the set of bits and B is the set of stabilizers. Assume the graph is Δ_B, Δ_S regular.

For the rest of the paper, we will only consider Z errors. The algorithm and analysis remain the same if considering X errors, but H_Z must be used as the generator matrix and H_X must be used as the parity check matrix.

Notation

4 Background

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4.1 Small Set Flip

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$$\xi_{ij}(t) = P(x_t = i, x_{t+1} = j | y, v, w; \theta) = \frac{\alpha_i(t) a_{ij}^{w_t} \beta_j(t+1) b_j^{v_{t+1}}(y_{t+1})}{\sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i(t) a_{ij}^{w_t} \beta_j(t+1) b_j^{v_{t+1}}(y_{t+1})} \tag{1}$$

4.1.1 Headings: third level

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5 Algorithm: K-Top Probabilistic Flip Method (K-Top PFM)

5.1 A Moral Reason/ Intuition

The algorithm is essentially the same as Small Set Bit Flip [TODO: CITE] with a minor difference, only a constant number of generators are checked. The idea here is that given a syndrome σ and a parity check matrix, H_X, the ith row of $H_X^T\sigma$ equals the number of error-ed checks that a qubit touches. Then, the kth row of $H_Z H_X^T \sigma$ is roughly correlated to the number of error-ed checks that the qubits in the kth generator touch. This rough correlation comes from the fact that we are working with expander codes. So then, if you get the generators touching the most error-ed stabilizers, it

Algorithm 1: `sort-top-K(T)`

Data: A vector $T \in \mathbb{Z}_2^N$
Result: A set S of the top K indices in T

$S \leftarrow$ `indices of a descending radix sort of` T`'s rows`;
return A set of the top K indices in S;

Algorithm 2: `probabilistic-set-flip(`E`)`

Data: A syndrome $\sigma_0 \in \mathbb{F}_2^M$
Result: Deduced error $\widehat{E}$ if the algorithm converges and $\perp$ otherwise

$\widehat{E} \leftarrow 0^N$;
$\sigma \leftarrow \sigma_0$;
while $\exists F \in \mathscr{F} : |\sigma_Z| - |\sigma_Z \oplus H_Z \boldsymbol{k}| > 0$ **do**
  $T \leftarrow H_Z H_X^T \sigma$;
  `generators` $\leftarrow$ `sort-top-K`(T);
  `to-check` $\leftarrow \bigcup_{i \in \texttt{generators}} \mathcal{P}(\mathcal{C}_{Z\,i})$;
  $\boldsymbol{k} \leftarrow \arg\max_{\boldsymbol{k} \in \texttt{to-check}} \frac{|\sigma_Z| - |\sigma_Z \oplus H_Z \boldsymbol{k}|}{|\boldsymbol{k}|}$;
  $\widehat{E} \leftarrow \widehat{E} \oplus \boldsymbol{k}$;
  $\sigma \leftarrow \sigma \oplus \sigma_X(\boldsymbol{k})$;
end
return $\widehat{E}$ if $|\sigma| = 0$, $\perp$ otherwise.

would stand to reason that flipping some subset of qubits from a “highly error-ed generator" would result in decreasing the syndrome.

6 PFM Analysis

Definitions

Let us start by defining a slew of random variables

- Let $S_j = 1$ if and only if $\boldsymbol{\sigma}_{Z\,j} = 1$ and 0 otherwise.
- Let $L_j = 1$ iff $S_j = 0 \wedge \exists k \in \Gamma(\text{stabilizer } j)$ such that $\boldsymbol{e}_{Z\,k} = 1$ and 0 otherwise. In other words, think of L_j as indicating whether stabilizer L_j is “lying" about not neighboring an error-ed bit.
- Let $\aleph_i$ be the number of unique stabilizers in the neighborhood of a generator. More formally,

$$\aleph_i = \bigcup_{k \in H_{X\,i}} \Gamma(k).$$

- Let G_i be the number of stabilizers in the neighborhood of a generator such that the stabilizer is flagged. More formally,

$$G_i = \sum_{j \in \aleph_i} S_j$$

- Let G'_i be T_i in the algorithm. Note that G'_i equals TODO: reference here

$$G'_i = \sum_{k \in H_{X\,i}} \sum_{j \in \Gamma(k)} S_j$$

- Define E_i to be the number of “lying" stabilizers in $\aleph$, or,

$$E_i = \sum_{j \in \aleph_i} L_j$$

- E'_i be equal to the sum of the number of neighboring “lying" stabilizers of each bit in generator i. Specifically,

$$E'_i = \sum_{k \in H_{X\,i}} \sum_{j \in \Gamma(k)} L_j$$

- Define $\mathcal{O}_i^G = G'_i - G_i$. $\mathcal{O}_i^G$ can be thought of as the number of “over-counted" flagged stabilizers which contribute to G'_i.
- Define $\mathcal{O}_i^E = E'_i - E_i$. $\mathcal{O}_i^E$ can be thought of as the number of “over-counted" lying stabilizers which contribute to E'_i.

Observations and Lemmas

Lemma 6.1. *For a generator, given E_i and G_i, we can find some correction vector $\boldsymbol{k} \in \mathbb{F}_2^N$ such that $|\sigma_Z| - |\sigma_Z \oplus H_Z \boldsymbol{k}| \geq G_i - E_i$ and $|k| \leq \frac{1-\delta}{\Delta_B}(G_i + E_i)$ for $G_i > E_i$.*

Proof. TODO: part 1 is that there are 3 types of stabilizers in neighbourhood. Those from G’, those from E, and those in neither. If those in neither, there is no neighbourhood in their error, so you can leave those bits alone. Then, by flipping bits connected to G’ you decrease syndrome by G’, but you add in at most E’

part 2: each flipping bit effects at least (1-delta) $\Delta_B$stables □

Lemma 6.2. *As $M, N \to \infty$ and for stabilizers $j_1, j_2, ..., j_C$ where C is less than some constant, then $\Gamma(j_1), \Gamma(j_2), ..., \Gamma(j_C)$ are independent.*

Proof. Let $A \subsetneq [C]$ and $B_{A_1}, B_{A_2}, ..., B_{A_{|A|}} \subseteq [N]$ such that $|B_{A_i}| = \Delta_S$. Then, to show independence, we want to show that for all $j \in C \setminus A$ and some set $B_j \subseteq [N], |B_j| = \Delta_S$,

$$\mathbf{Pr}[\Gamma(j) = B_j] = \mathbf{Pr}\Big[\Gamma(j = B_j) \mid \Gamma(A_1) = B_{A_1}, ..., \Gamma(A_{|A|}) = B_{A_{|A|}})\Big].$$

The above can easily be seen as $N, M \to \infty$ as

$$\mathop{\mathbf{Pr}}_{B_j \subseteq [N], |B_j| = \Delta_S}[\Gamma(j) = B_j] = \Delta_S! \prod_{i=1}^{|B_j|} \mathbf{Pr}\Big[B_{j_i} \in \Gamma(j) \mid B_{j_{i-1}}, ..., B_{j_1} \in \Gamma(j)\Big] = \Delta_S! \cdot \frac{\Delta_S!}{N^{\Delta_S}}.$$

Then, let $f(k, B_j, A)$ equal to the total number of instances some $k \in B_j$ is in B_{A_i} for all $i \in |A|$. Note that

$$\mathop{\mathbf{Pr}}_{B_j \subseteq [N], |B_j| = \Delta_S}[f(k, B_j, A) > 0] \to 0$$

as $N \to \infty$. Then,

$$\begin{aligned}
&\mathop{\mathbf{Pr}}_{B_j \subseteq [N], |B_j| = \Delta_S}\big[\Gamma(j = B_j) \mid \Gamma(A_1) = B_{A_1}, ..., \Gamma(A_{|A|}) = B_{A_{|A|}})\big] \\
=&\Delta_S! \prod_{i=1}^{|B_j|} \mathbf{Pr}\Big[B_{j_i} \in \Gamma(j) \mid B_{j_{i-1}}, ..., B_{j_1} \in \Gamma(j), \Gamma(A_1) = B_{A_1}, ..., \Gamma(A_{|A|}) = B_{A_{|A|}})\Big] \\
=&\Delta_S! \prod_{i=1}^{|B_j|} \frac{(\Delta_S + 1 - i)\big(\Delta_B - f(B_{j_i}, B_j, A)\big)}{\Delta_B N}
\end{aligned}$$

as $N \to \infty$, $\Delta_S! \cdot \dfrac{\Delta_S!}{N^{\Delta_S}}$.

□

Lemma 6.3. *Assuming that the error rate is independent, then $S_1, S_2, ..., S_C$ are independent where $C \leq \Delta_S \Delta_B$ and*

$$\mathbf{Pr}[S_j = 1] = \tfrac{1}{2} - \tfrac{1}{2}(1 - 2p)^{\Delta_S}$$

Proof. The $\mathbf{Pr}[S_j = 1]$ is then just equal to the probability that

$$\mathbf{Pr}[|\{\, \boldsymbol{e}_{Zk} = 1 : k \in \Gamma(j) \,\}| \texttt{ is odd}].$$

Note we are assuming by lemma 6.2 that for $j \in [C]$, all the $\Gamma(j)$ are independent. Thus, we have that $\mathbf{Pr}[|\{\, \boldsymbol{e}_{Zk} = 1 : k \in \Gamma(j) \,\}| \texttt{ is odd}]$ equals to the probability that a sample from $\mathrm{binomial}(\Delta_S, p)$ is odd. This is then equal to $\frac{1}{2} - \frac{1}{2}(1-2p)^{\Delta_S}$. Note also that we can assume $S_1, S_2, ..., S_C$ to be independent by lemma 6.2.

□

Lemma 6.4. *Assuming that the error rate is independent, $L_1, L_2, ..., L_C$ are independent for $C \leq \Delta_S \Delta_B$ and*

$$\mathbf{Pr}[L_j = 1] = \frac{1}{2} + \frac{1}{2}(1-2p)^{\Delta_S} - (1-p)^{\Delta_S}.$$

Proof. Let $s = |\{\, \boldsymbol{e}_{Zk} = 1 : k \in \Gamma(j) \,\}|$. Note that L_j is 1 iff $s > 0$ and s is even. Then,

$$\mathbf{Pr}[s > 0, s \text{ is even}] = \mathbf{Pr}[s \text{ is even}] - \mathbf{Pr}[s = 0] = \frac{1}{2} + \frac{1}{2}(1-2p)^{\Delta_S} - (1-p)^{\Delta_S}.$$

We can also take that $L_1, ..., L_C$ are independent by lemma 6.2. □

Distributions on the random variables

Probability of single generator error

Probability of error

Error Probability Graphs

7 PFM Numerical Simulations

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8 PFM Future Outlook

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9 Conclusion

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10 Acknowledgments

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11 Examples of citations, figures, tables, references

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The documentation for `natbib` may be found at

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Of note is the command `\citet`, which produces citations appropriate for use in inline text. For example,

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> Hasselmo, et al. (1995) investigated...

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11.1 Figures

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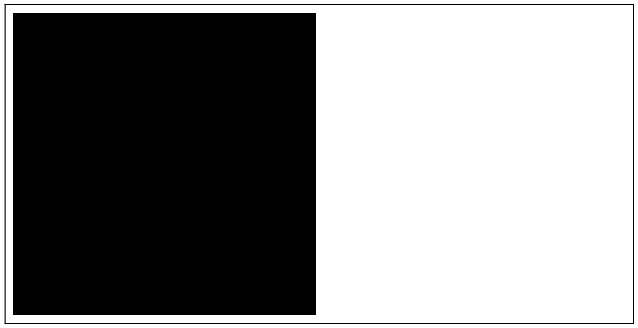

Figure 1: Sample figure caption.

Table 1: Sample table title

Part		
Name	Description	Size (μm)
Dendrite	Input terminal	$\sim$100
Axon	Output terminal	$\sim$10
Soma	Cell body	up to 10^6

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11.2 Tables

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11.3 Lists

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12 Conclusion

Your conclusion here

Acknowledgments

This was was supported in part by......

[2] Sample of the first footnote.

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