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# A TEMPLATE FOR ARXIV STYLE \*

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## ABSTRACT

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**Keywords** First keyword · Second keyword · More

## 1 Introduction

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## 2 Contribution

## 3 Notation and Definitions

### Definitions and assumptions

- We will use  $j$  when referring to a stabilizer,  $i$  when referring to a generator,  $k$  when referring to a bit
- We take  $e_Z$  to be the error vector
- We take  $\sigma_Z$
- Let  $\mathcal{C}_Z$  be a code which can correct  $Z$  errors and  $\mathcal{C}_X$  be a code which can correct  $X$  errors.

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- Let  $H_X \in F_2^{M \times N}$  be the parity check matrix for  $\mathcal{C}_X$  as well as the generator for  $\mathcal{C}_Z$ .
- Let  $H_Z \in F_2^{M \times N}$  be the parity check matrix for  $\mathcal{C}_Z$  as well as the generator for  $\mathcal{C}_X$ .
- Let  $M$  be the number of stabilizers. Let us also only consider the case where  $M$  is the number of generators. This results from when  $H_Z$  and  $H_X$  have the same number of rows.
- Let  $N$  be the number of bits.
- Assume  $H_X$  is the adjacency matrix of a  $(\delta, \gamma)$ -left-expander bipartite graph where  $A$  is the set of bits and  $B$  is the set of stabilizers. Assume the graph is  $\Delta_B, \Delta_S$  regular.

For the rest of the paper, we will only consider  $Z$  errors. The algorithm and analysis remain the same if considering  $X$  errors, but  $H_Z$  must be used as the generator matrix and  $H_X$  must be used as the parity check matrix.

## Notation

## 4 Background

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### 4.1 Small Set Flip

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$$\xi_{ij}(t) = P(x_t = i, x_{t+1} = j | y, v, w; \theta) = \frac{\alpha_i(t) a_{ij}^{w_t} \beta_j(t+1) b_j^{v_{t+1}}(y_{t+1})}{\sum_{i=1}^N \sum_{j=1}^N \alpha_i(t) a_{ij}^{w_t} \beta_j(t+1) b_j^{v_{t+1}}(y_{t+1})} \quad (1)$$

#### 4.1.1 Headings: third level

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## 5 Algorithm: K-Top Probabilistic Flip Method (K-Top PFM)

### 5.1 A Moral Reason/ Intuition

The algorithm is essentially the same as Small Set Bit Flip [TODO: CITE] with a minor difference, only a constant number of generators are checked. The idea here is that given a syndrome  $\sigma$  and a parity check matrix,  $H_X$ , the  $i$ th row of  $H_X^T \sigma$  equals the number of error-ed checks that a qubit touches. Then, the  $k$ th row of  $H_Z H_X^T \sigma$  is roughly correlated to the number of error-ed checks that the qubits in the  $k$ th generator touch. This rough correlation comes from the fact that we are working with expander codes. So then, if you get the generators touching the most error-ed stabilizers, it

**Algorithm 1:** sort-top-K( $T$ )**Data:** A vector  $T \in \mathbb{Z}_2^N$ **Result:** A set  $S$  of the top  $K$  indices in  $T$ 

- 1  $S \leftarrow$  indices of a descending radix sort of  $T$ 's rows;
- 2 **return** A set of the top  $K$  indices in  $S$ ;

**Algorithm 2:** probabilistic-set-flip( $E$ )**Data:** A syndrome  $\sigma_0 \in \mathbb{F}_2^M$ **Result:** Deduced error  $\hat{E}$  if the algorithm converges and  $\perp$  otherwise

- 1  $\hat{E} \leftarrow 0^N$ ;
- 2  $\sigma \leftarrow \sigma_0$ ;
- 3 **while**  $\exists F \in \mathcal{F} : |\sigma_Z| - |\sigma_Z \oplus H_Z \mathbf{k}| > 0$  **do**
- 4      $T \leftarrow H_Z H_X^T \sigma$ ;
- 5      $\text{generators} \leftarrow \text{sort-top-K}(T)$ ;
- 6      $\text{to-check} \leftarrow \bigcup_{i \in \text{generators}} \mathcal{P}(C_{Z_i})$ ;
- 7      $\mathbf{k} \leftarrow \arg \max_{\mathbf{k} \in \text{to-check}} \frac{|\sigma_Z| - |\sigma_Z \oplus H_Z \mathbf{k}|}{|\mathbf{k}|}$ ;
- 8      $\hat{E} \leftarrow \hat{E} \oplus \mathbf{k}$ ;
- 9      $\sigma \leftarrow \sigma \oplus \sigma_X(\mathbf{k})$ ;
- 10 **end**
- 11 **return**  $\hat{E}$  if  $|\sigma| = 0$ ,  $\perp$  otherwise.

would stand to reason that flipping some subset of qubits from a “highly error-ed generator” would result in decreasing the syndrome.

## 6 Time Bounds

TODO: basically each round is  $O(\sqrt{n})$  b/c matrix includes  $O(\sqrt{n})$  rows, and is sparse, so mm occur in that time. Then radix on  $O(\sqrt{n})$  elems

## 7 PFM Analysis

### Definitions

Let us start by defining a slew of random variables

- Let  $S_j = 1$  if and only if  $\sigma_{Z_j} = 1$  and 0 otherwise.
- Let  $L_j = 1$  iff  $S_j = 0 \wedge \exists k \in \Gamma(\text{stabilizer } j)$  such that  $e_{Z_k} = 1$  and 0 otherwise. In other words, think of  $L_j$  as indicating whether stabilizer  $L_j$  is “lying” about not neighboring an error-ed bit.
- Let  $\aleph_i$  be the number of unique stabilizers in the neighborhood of a generator. More formally,

$$\aleph_i = \bigcup_{k \in H_{X_i}} \Gamma(k).$$

- Let  $G_i$  be the number of stabilizers in the neighborhood of a generator such that the stabilizer is flagged. More formally,

$$G_i = \sum_{j \in \aleph_i} S_j$$

- Let  $G'_i$  be  $T_i$  in the algorithm. Note that  $G'_i$  equals TODO: reference here

$$G'_i = \sum_{k \in H_{X_i}} \sum_{j \in \Gamma(k)} S_j$$

- Define  $E_i$  to be the number of “lying” stabilizers in  $\mathbb{N}$ , or,

$$E_i = \sum_{j \in \mathbb{N}_i} L_j$$

- $E'_i$  be equal to the sum of the number of neighboring “lying” stabilizers of each bit in generator  $i$ . Specifically,

$$E'_i = \sum_{k \in H_{X_i}} \sum_{j \in \Gamma(k)} L_j$$

- Define  $\mathcal{O}_i^G = G'_i - G_i$ .  $\mathcal{O}_i^G$  can be thought of as the number of “over-counted” flagged stabilizers which contribute to  $G'_i$ .
- Define  $\mathcal{O}_i^E = E'_i - E_i$ .  $\mathcal{O}_i^E$  can be thought of as the number of “over-counted” lying stabilizers which contribute to  $E'_i$ .

### Observations and Lemmas

**Lemma 7.1.** For a generator, given  $E_i$  and  $G_i$ , we can find some correction vector  $\mathbf{k} \in \mathbb{F}_2^N$  such that  $|\sigma_Z| - |\sigma_Z \oplus H_Z \mathbf{k}| \geq G_i - E_i$  and  $|\mathbf{k}| \leq \frac{1-\delta}{\Delta_B} (G_i + E_i)$  for  $G_i > E_i$ .

*Proof.* TODO: part 1 is that there are 3 types of stabilizers in neighbourhood. Those from  $G'$ , those from  $E$ , and those in neither. If those in neither, there is no neighbourhood in their error, so you can leave those bits alone. Then, by flipping bits connected to  $G'$  you decrease syndrome by  $G'$ , but you add in at most  $E'$

part 2: each flipping bit effects at least  $(1-\delta) \Delta_B$  stabilizers □

**Lemma 7.2.** As  $M, N \rightarrow \infty$  and for stabilizers  $j_1, j_2, \dots, j_C$  where  $C$  is less than some constant, then  $\Gamma(j_1), \Gamma(j_2), \dots, \Gamma(j_C)$  are independent.

*Proof.* Let  $A \subsetneq [C]$  and  $B_{A_1}, B_{A_2}, \dots, B_{A_{|A|}} \subseteq [N]$  such that  $|B_{A_i}| = \Delta_S$ . Then, to show independence, we want to show that for all  $j \in C \setminus A$  and some set  $B_j \subseteq [N]$ ,  $|B_j| = \Delta_S$ ,

$$\Pr[\Gamma(j) = B_j] = \Pr[\Gamma(j) = B_j \mid \Gamma(A_1) = B_{A_1}, \dots, \Gamma(A_{|A|}) = B_{A_{|A|}}].$$

The above can easily be seen as  $N, M \rightarrow \infty$  as

$$\Pr_{B_j \subseteq [N], |B_j| = \Delta_S}[\Gamma(j) = B_j] = \Delta_S! \prod_{i=1}^{|B_j|} \Pr[B_{j_i} \in \Gamma(j) \mid B_{j_{i-1}}, \dots, B_{j_1} \in \Gamma(j)] = \Delta_S! \cdot \frac{\Delta_S!}{N^{\Delta_S}}.$$

Then, let  $f(k, B_j, A)$  equal to the total number of instances some  $k \in B_j$  is in  $B_{A_i}$  for all  $i \in |A|$ . Note that

$$\Pr_{B_j \subseteq [N], |B_j| = \Delta_S}[f(k, B_j, A) > 0] \rightarrow 0$$

as  $N \rightarrow \infty$ . Then,

$$\begin{aligned} & \Pr_{B_j \subseteq [N], |B_j| = \Delta_S}[\Gamma(j) = B_j \mid \Gamma(A_1) = B_{A_1}, \dots, \Gamma(A_{|A|}) = B_{A_{|A|}}] \\ &= \Delta_S! \prod_{i=1}^{|B_j|} \Pr[B_{j_i} \in \Gamma(j) \mid B_{j_{i-1}}, \dots, B_{j_1} \in \Gamma(j), \Gamma(A_1) = B_{A_1}, \dots, \Gamma(A_{|A|}) = B_{A_{|A|}}] \\ &= \Delta_S! \prod_{i=1}^{|B_j|} \frac{(\Delta_S + 1 - i)(\Delta_B - f(B_{j_i}, B_j, A))}{\Delta_B N} \\ & \text{as } N \rightarrow \infty, \Delta_S! \cdot \frac{\Delta_S!}{N^{\Delta_S}}. \end{aligned}$$

□

**Lemma 7.3.** Assuming that the error rate is independent, then  $S_1, S_2, \dots, S_C$  are independent where  $C \leq \Delta_S \Delta_B$  and

$$\Pr[S_j = 1] = \frac{1}{2} - \frac{1}{2}(1 - 2p)^{\Delta_S}$$

*Proof.* The  $\Pr[S_j = 1]$  is then just equal to the probability that

$$\Pr[|\{e_{Z_k} = 1 : k \in \Gamma(j)\}| \text{ is odd}].$$

Note we are assuming by lemma 7.2 that for  $j \in [C]$ , all the  $\Gamma(j)$  are independent. Thus, we have that  $\Pr[|\{e_{Z_k} = 1 : k \in \Gamma(j)\}| \text{ is odd}]$  equals to the probability that a sample from  $\text{binomial}(\Delta_S, p)$  is odd. This is then equal to  $\frac{1}{2} - \frac{1}{2}(1 - 2p)^{\Delta_S}$ . Note also that we can assume  $S_1, S_2, \dots, S_C$  to be independent by lemma 7.2. □

**Lemma 7.4.** *Assuming that the error rate is independent,  $L_1, L_2, \dots, L_C$  are independent for  $C \leq \Delta_S \Delta_B$  and*

$$\Pr[L_j = 1] = \frac{1}{2} + \frac{1}{2}(1 - 2p)^{\Delta_S} - (1 - p)^{\Delta_S}.$$

*Proof.* Let  $s = |\{e_{Z_k} = 1 : k \in \Gamma(j)\}|$ . Note that  $L_j$  is 1 iff  $s > 0$  and  $s$  is even. Then,

$$\Pr[s > 0, s \text{ is even}] = \Pr[s \text{ is even}] - \Pr[s = 0] = \frac{1}{2} + \frac{1}{2}(1 - 2p)^{\Delta_S} - (1 - p)^{\Delta_S}.$$

We can also take that  $L_1, \dots, L_C$  are independent by lemma 7.2. □

## Distributions on the random variables

### Probability of single generator error

### Probability of error

### Error Probability Graphs

## 8 PFM Numerical Simulations

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## 9 PFM Future Outlook

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## 10 Conclusion

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## 11 Acknowledgments

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## 12 Examples of citations, figures, tables, references

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The documentation for natbib may be found at

<http://mirrors.ctan.org/macros/latex/contrib/natbib/natnotes.pdf>

Of note is the command `\citet`, which produces citations appropriate for use in inline text. For example,

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Hasselmo, et al. (1995) investigated...

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### 12.1 Figures

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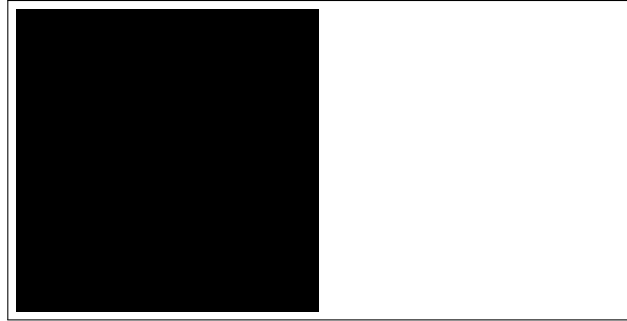


Figure 1: Sample figure caption.

Table 1: Sample table title

Part		
Name	Description	Size ( $\mu\text{m}$ )
Dendrite	Input terminal	$\sim 100$
Axon	Output terminal	$\sim 10$
Soma	Cell body	up to $10^6$

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## 12.2 Tables

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## 12.3 Lists

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## 13 Conclusion

Your conclusion here

## Acknowledgments

This was supported in part by.....

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<sup>2</sup>Sample of the first footnote.

## References

- [1] George Kour and Raid Saabne. Real-time segmentation of on-line handwritten arabic script. In *Frontiers in Handwriting Recognition (ICFHR), 2014 14th International Conference on*, pages 417–422. IEEE, 2014. 12
- [2] George Kour and Raid Saabne. Fast classification of handwritten on-line arabic characters. In *Soft Computing and Pattern Recognition (SoCPaR), 2014 6th International Conference of*, pages 312–318. IEEE, 2014. 12
- [3] Guy Hadash, Einat Kermany, Boaz Carmeli, Ofer Lavi, George Kour, and Alon Jacovi. Estimate and replace: A novel approach to integrating deep neural networks with existing applications. *arXiv preprint arXiv:1804.09028*, 2018. 12