Multiple Secret Leaders

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1 Preliminaries

1.1 Threshold FHE

[JRS17, BGG$^+$18] defines threshold FHE encryption. For the sake of completeness, we will define it here.

Definition 1.1 (TFHE [JRS17])**.** Let $P = \{ P_1, ..., P_N \}$ be a set of N parties and $\mathbb{S}$ be a class of access structures on P. A TFHE scheme for $\mathbb{S}$ is a tuple of PPT algorithms

$$(\texttt{TFHE.Setup}, \texttt{TFHE.Encrypt}, \texttt{TFHE.Eval}, \texttt{TFHE.PartDec}, \texttt{TFHE.FinDec})$$

such that the following specifications are met

- $(pk, sk_1, ..., sk_N) \leftarrow \texttt{TFHE.Setup}(1^\lambda, 1^s, \mathbb{A})$: Takes as input a security parameter λ, a depth bound on the circuit, and a structure $\mathbb{A} \in \mathbb{S}$. Outputs a public key pk and a secret key sk_i for each party P_i.

- $\texttt{ct} \leftarrow \texttt{TFHE.Encrypt}(pk, \mu)$: Takes as input a public key and a message $\mu \in \{ 0, 1 \}$ and outputs a ciphertext $\texttt{ct}$.

- $\hat{\texttt{ct}} \leftarrow \texttt{TFHE.Eval}(C, \texttt{ct}_1, ..., \texttt{ct}_k)$: Takes as input a circuit C of depth at most d and k ciphertexts $\texttt{ct}_1, ..., \texttt{ct}_k$. Outputs a ciphertext $\hat{\texttt{ct}} = C(\texttt{ct}_1, ..., \texttt{ct}_k)$.

- $p_i \leftarrow \texttt{TFHE.PartDec}(\texttt{ct}, sk_i)$: Takes as input a ciphertext $\texttt{ct}$ and a secret key sk_i and outputs a partial decryption p_i.

- $\hat{\mu} \leftarrow \texttt{TFHE.FinDec}(B)$: Takes as input a set $B = \{ p_i \}_{i \in S}$ for some $S \subseteq [N]$ and deterministically outputs a message $\hat{\mu} \in \{ 0, 1, \perp \}$.

Further, we remember the definitions of evaluation correctness, semantic security, and simulation security as outlined in, [BEHG20].

Definition 1.2 (Evaluation Correctness [JRS17])**.** . We have that TFHE scheme is correct if for all λ, depth bounds d, access structure $\mathbb{A}$, circuit $C : \{ 0, 1 \}^k \to \{ 0, 1 \}$ of depth at most d, $S \in \mathbb{A}$, and $\mu_i \in \{ 0, 1 \}$, we have the following. For $(pk, sk_1, ..., sk_N) \leftarrow \texttt{TFHE.Setup}(1^\lambda, 1^d, \mathbb{A})$, $\texttt{ct}_i \leftarrow \texttt{TFHE.Encrypt}(pk, \mu_i)$ for $i \in [k]$, $\hat{\texttt{ct}} \leftarrow \texttt{TFHE.Eval}(pk, C, \texttt{ct}_1, ..., \texttt{ct}_k)$,

$$\mathbf{Pr} \left[\texttt{TFHE.FinDec}(pk, \{ \texttt{TFHE.PartDec}(pk, \hat{\texttt{ct}}, sk_i) \}_{i \in S}) = C(\mu_1, ..., \mu_k) \right] = 1 - \texttt{negl}(\lambda).$$

Definition 1.3 (Semantic Security [JRS17])**.** We have that a `TFHE` scheme satisfies semantic security for for all λ, and depth bound d if the following holds. There is a stateful PPT algorithm $\mathcal{S} = (\mathcal{S}_1, \mathcal{S}_2)$ such that for any PPT adversary $\mathcal{A}$, the following experiment outputs 1 with negligible probability in λ:

1. On input 1^λ and depth 1^d, the adversary outputs $\mathbb{A} \in \mathbb{S}$

2. The challenger runs $(pk, sk_1, ..., sk_N) \leftarrow \text{TFHE.Setup}(1^\lambda, 1^d, \mathbb{A})$ and provides pk to $\mathcal{A}$.

3. $\mathcal{A}$ outputs a set $S \subseteq \{P_1, ..., P_N\}$ such that $S \notin \mathbb{A}$.

4. The challenger provides $\{sk_i\}_{i \in S}$ and $\text{TFHE.Encrypt}(pk, \mu)$ to $\mathcal{A}$ where $\mu \xleftarrow{\$} \{0, 1\}$.

5. $\mathcal{A}$ outputs a guess μ'. The experiment outputs 1 if $\mu' = \mu$.

Definition 1.4 (Simulation Security [JRS17])**.** We say that a `TFHE` scheme is simulation secure if for all λ, depth bound d, and access structure $\mathbb{A}$ if there exists a stateful PPT simulator, $\mathcal{S}$, such that for any PPT adversary $\mathcal{A}$, we have that the experiments $\text{Expt}_{\mathcal{A},\text{Real}}(1^\lambda, 1^d)$ and $\text{Expt}_{\mathcal{A},\text{Sim}}(1^\lambda, 1^d)$ are statisically close as a function of λ. The experiments are defined as follows:

- $\text{Expt}_{\mathcal{A},\text{Real}}(1^\lambda, 1^d)$:

    1. On input the security parameter 1^λ and depth bound d, the adversary outputs $\mathbb{A} \in \mathbb{S}$.
    2. Run $\text{TFHE.Setup}(1^\lambda, 1^d, \mathbb{A})$ to obtain $(pk, sk_1, ..., sk_N)$. The adversary is given pk.
    3. The adversary outputs a set $S \subseteq \{P_1, ..., P_N\}$ such that $S \notin \mathbb{A}$ together with plaintest messages $\mu_1, ..., \mu_k \in \{0, 1\}$. The adversary is handed over $\{sk_i\}_{i \in S}$
    4. For each μ_i, the adversary is given $\text{TFHE.Encrypt}(pk, \mu_i) \rightarrow \text{ct}_i$.
    5. The adversary issues a polynomial number of queries, $(S_i \subseteq \{P_1, ..., P_N\}, C_i)$. for circuites $C_i : \{0, 1\}^k \rightarrow \{0, 1\}$. After each query the adversary receuves for $l \in S_i$ the value
    $$\text{TFHE.PartDec}(\text{TFHE.Eval}(C_i, \text{ct}_1, ..., \text{ct}_k), sk_l) \rightarrow p_l$$
    6. $\mathcal{A}$ outputs out, the experiment's output.

- $\text{Expt}_{\mathcal{A},\text{Sim}}(1^\lambda, 1^d)$:

    1. On input the security parameter 1^λ and depth bound d, the adversary outputs $\mathbb{A} \in \mathbb{S}$.
    2. Run $\text{TFHE.Setup}(1^\lambda, 1^d, \mathbb{A})$ to obtain $(pk, sk_1, ..., sk_N)$. The adversary is given pk.
    3. $\mathcal{A}$ outputs a set $S^* \subseteq \{P_1, ..., P_N\}$ such that $S \notin \mathbb{A}$ and plaintexts $\mu_1, ..., \mu_k \in \{0, 1\}$. The simulator is given $pk, \mathbb{A}, S^*$ as input and outputs $\{sk_i\}_{i \in S^*}$ and the state state. The adversary is given $\{sk_i\}_{i \in S^*}$
    4. For each μ_i, the adversay is given $\text{TFHE.Encrypt}(pk, \mu_i) \rightarrow \text{ct}_i$.
    5. $\mathcal{A}$ issues a polynomial number of queries of the form $(S_i \subseteq \{P_1, ..., P_N\}, C_i)$
    6. for circuits $C_i : \{0, 1\}^k \rightarrow \{0, 1\}$. After each query, the simulator computes
    $$\text{Sim}_{\text{TFHE}}(C_i, \{\text{ct}_l\}_{l=1}^k, C_i(\mu_1, ..., \mu_k), \text{state}) \rightarrow \{p_l\}_{l \in S_i}$$
    and sends $\{p_l\}_{l \in S_i}$ to the adversary.
    7. $\mathcal{A}$ outputs out, the experiment's output.

1.2 Non Interactive Zero-Knowledge

The following definition is taken almost verbatim from [CFG22]. A non-Interactive Zero-Knowledge (NZIK) proof system for relationship Rel is a tuple of PPT algorithms $(\texttt{NZIK.G}, \texttt{NZIK.P}, \texttt{NZIK.V})$ such that $\texttt{NZIK.G}$ generates a common reference string, $\texttt{NZIK.crs}$, $\texttt{NZIK.P}(\texttt{NZIK.crs}, x, w)$ given $(x, w) \in Rel$, outputs a proof π, and $\texttt{NZIK.V}(\texttt{NZIK.crs}, x, \pi)$ outputs 1 if $(x, w) \in Rel$ and 0 otherwise. A $\texttt{NZIK}$ is correct is for every $\texttt{NZIK.crs}$ and all $(x, w) \in Rel$, we have that $\texttt{NZIK.V}(\texttt{NZIK.crs}, x, \texttt{NZIK.P}(\texttt{NZIK.crs}, x, w)) = 1$ holds with probability 1. We also require that our NZIKs satisfy the notions of weak simulation extractability [Sah99] and zero-knowledge [FLS90].

Weak simulation extractability guarentees the extractability of proofs produced by the adversary that are not equal to proofs previously observed. Thus, we make each proof "unique" by implicitly adding a session ID to the statement. For more of a commentary, see section 2.7 of [CFG22]. We will not detail how to handle these session IDs.

We recall the notion of simulation security from [Sah99]:

Definition 1.5 (NZIK simulation security). We say that a proof system for relationship Rel is simulation secure if proof system $(\texttt{NZIK.G}, \texttt{NZIK.P}, \texttt{NZIK.V})$ is a non-interactive proof system and $\text{Sim}_1, \text{Sim}_2$ are PPT algorithms such that for all PPT adversaries $\mathcal{A}_1, \mathcal{A}_2$ we have that $\left| \mathbf{Pr}[\texttt{Expt}_{\mathcal{A}, \text{Real}}(\lambda) = 1] - \mathbf{Pr}[\texttt{Expt}_{\mathcal{A}, \text{Sim}}(\lambda) = 1] \right|$ is negligible in λ. The experiments are defined as follows:

- $\texttt{Expt}_{\mathcal{A}, \text{Real}}$:
    1. $\texttt{NZIK.crs} \leftarrow \texttt{NZIK.G}(1^\lambda)$
    2. $\mathcal{A}_1$ outputs $(x, w, \texttt{state}_1)$
    3. $\pi \leftarrow \texttt{NZIK.P}(\texttt{NZIK.crs}, x, w)$
    4. return $\mathcal{A}_2(\pi, \texttt{state}_1)$

- $\texttt{Expt}_{\mathcal{A}, \text{Sim}}$:
    1. $\texttt{NZIK.crs} \leftarrow \text{Sim}_1(1^\lambda)$
    2. $\mathcal{A}_1$ outputs $(x, w, \texttt{state})$
    3. $\pi \leftarrow \text{Sim}_2(\texttt{NZIK.crs}, x, w)$
    4. return $\mathcal{A}_2(\pi, \texttt{state})$

We also have that a NZIK has ideal functionality $\mathcal{F}_{\texttt{NZIK}}^{Rel}$ which is defined as follows:

> **The $\mathcal{F}_{\texttt{NZIK}}^{Rel}$ functionality for zero-knowledge**:
> Upon receiving $(\texttt{prove}, sid, x, w)$ from P_i, with sid being used for the first time, if $(x, w) \in Rel$, broadcast $(\texttt{proof}, sid, i, \pi)$, otherwise broadcast $\perp$.

1.3 Data Independent Priority Queue

In this work, we will use data independent queues as studied in [Tof11, MZ14, MDPB23]. Data independent data structures are unique as their control flow and memory access do not depend on input data ([MZ14]).

Definition 1.6 (Word RAM model [MZ14]). In the word RAM model, the RAM has a constant number of public and secret registers and can perform arbitrary operations on a constant number of registers in constant time.

Definition 1.7 (Data Independent Data Structure [MZ14])**.** In the word RAM model, a data independent data structure is a collection of algorithms where all the algorithms uses RAM such that the RAM can only set its control flow based on registers that are public.

Data independent queues are especially useful as they allow for efficient computation within MPC and FHE as control flow is not dependent on underlying ciphertexts data. We use a data independent queue as outlined in [MDPB23] which allows for

- `PQ.Insert`: Inserts a tag and value, (p, x) into `PQ` according to the tag's priority.

- `PQ.ExtractFront`: Removes and returns the (p, y) with highest tag priority.

- `PQ.Front`: Returns the (p, y) with highest tag priority without removing the element.

Moreover, we note that the order is stable. I.e. the first inserted among equal tagged elements has a higher priority.

1.4 Resevoir Sampling

Resevoir sampling is an online algorithm which allows for randomly selecting k elements from a stream of n elements while using $\tilde{O}(k)$ space. Algorithm R ([Vit85]) is a simple algorithm which relies on a priority queue with interface:

- `Resevoir.Init`(k) initialize the resevoir sampling algorithm and data structure

- `Resevoir.Insert`$(\mu_i, e_i, \text{coin}_i)$ where μ_i is i-th item, e_i is independentally sampled randomness, and coin_i is a random coin with probability $1/m$ of equaling 1.

  - If $i \leq k$, insert the item into the queue along with e_i as its tag via `PQ.Insert`(e_i, μ_i).
  - If $i > k$ and $\text{coin}_i = 1$, replace the smallest labeled item in the queue with the new item if the coin is 1.
  - If $i > k$ and $\text{coin}_i = 0$, do nothing.

- $\mu_{a_1}, \mu_{a_2}, ..., \mu_{a_k} \leftarrow$ `Resevoir.Output`$()$ where $a_1, ..., a_k$ are a uniformly random ordered susbset of $[n]$

  - Call `PQ.ExtractFront` k times setting μ_{a_ℓ} to the ℓ-th call to `PQ.ExtractFront` where $\ell \in [k]$.

2 Ideal MSLE Functionality

We use a similar notion of ideal functionality for a multi-secret leader election from the ideal functionality of single secret leader election of [CFG22] except that we add a `register_elect` phase for each election.

3 Semi Honest Reservoir Sampling Based Protocol

We outline a semi-honest protocol in fig. 2 for the MSLE functionality.

Figure 1: Description of the Multi Secret Leader Election functionality

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3.1 Semi Honest Simulation Security

We will now show that the above protocol is semi-honest simulation secure.

Theorem 3.1 (Semi-honest simulation security). *Assuming the existence of a PRF, a* TFHE *scheme with semantic security (definition 1.3) and simulation security (definition 1.4), then the protocol outline in fig. 2 is semi-honest simulation secure.*

Proof. We will show that the simulator outlined in fig. 3 is a simulator for the view of corrupt parties $C \subset [n]$ and $|C| < t$ by showing that the simulator's view and the protocol's view are indistinguishable for all calls via the following three lemmas.

Lemma 3.2 (`register_elect`, `register_elect_add` are simulation secure). *Proof.* The simulator firsts simulates the view of the protocol for all calls to `register_elect` from P_i, $i \in C$. Note that the simulator uses the same inputs, s_i, r_i as in the honest protocol and thus the view is identical. Moreover, the view of $(\texttt{register\_elect\_add}, eid, \texttt{ct}_i')$ is identical to the real protocol as $\texttt{ct}_i' = \texttt{ct}_i$. For $j \notin C$, note that $c_j' \overset{\text{c}}{\equiv} c_j$ by the hiding property of commitments and thus $\texttt{ct}_j' \overset{\text{c}}{\equiv} \texttt{ct}_j$. Then, assuming that prior calls to `register_elect` by P_j are simulated by the simulator, the view of $(\texttt{register\_elect\_add}, eid, \texttt{ct}_j')$ is indistinguishable to the real protocol as `register_elect_add` is completely determined by prior calls to `register_elect_add` and $\texttt{ct}_j'$.

We then note that, after α, for $\alpha \in [n]$, calls to `register_elect_add`, the simulator's $\mathcal{R}_{eid,\texttt{Sim}_C} \overset{\text{c}}{\equiv} \mathcal{R}_{eid}$ of the protocol as $\mathcal{R}$ is completly determined by the calls to `register_elect_add`. $\square$

Lemma 3.3 (`elect`, `elect_done` are simulation secure). *Proof.* Now note that, when `elect` is called by P_i for $i \in C$, $\texttt{ct}_\ell' \overset{\text{c}}{\equiv} \texttt{ct}_\ell$ for all $\ell \in [k]$ as $\mathcal{R}_{eid}$ in the simulator is indistinguishable from that of the real protocol. We also have that, by the ideal functionality of `elect`, $\{a_\ell'\}_{\ell \in [k]}$ is a random subset of $[n]$ with random order with size k and that by the correctness of `Resevoir.Output`, $\{a_\ell\}_{\ell \in [k]}$ is a randomly chosen ordered subset of $[n]$. So then, $\{a_\ell'\}_{\ell \in [k]} \overset{\text{c}}{\equiv} \{a_\ell\}_{\ell \in [k]}$. Remembering that $c_\beta' \overset{\text{c}}{\equiv} c_\beta$ for all $\beta \in [n]$, we have that $c_{a_1}', ..., c_{a_k}' \overset{\text{c}}{\equiv} c_{a_1}, ..., c_{a_k}$. So then, we have that the decryption shares, p_i', for all $i \in [n]$ are simulated by the simulator such that $p_i' \overset{\text{c}}{\equiv} p_i$ in the real protocol and thus

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The MSLE Protocol, Party P_i realizing $\mathcal{F}_{\text{MSLE}}$ in the semi-honest setting:

Each party has as input a TFHE secret key share sk_i of secret key sk with associated public key pk. Initialize a set of finished elections $\mathcal{E}$.

- setup

    - Set $k_{\text{TFHE}} \xleftarrow{\$} U_q$.
    - Publish CRS, $CRS = \text{Enc}_{\text{TFHE}}(k_{\text{TFHE}})$.

- $(\texttt{register\_elect}, eid)$

    - Sample $s_i, r_i \xleftarrow{\$} \mathbb{F}_q$, $\text{ct}_i \leftarrow \text{Enc}_{\text{TFHE}}(c_i)$
    - Broadcast $(\texttt{register\_elect\_add}, eid, \text{ct}_i)$ to all parties and run $(\texttt{register\_elect\_add}, eid, \text{ct}_i)$

- $(\texttt{register\_elect\_add}, eid, \text{ct}_j)$ from party P_j for $j \in [n]$

    - If b_{eid} is not stored, set $b_{eid} = 0$
    - $b_{eid} \leftarrow b_{eid} + 1$
    - If $\mathcal{R}_{eid}$ is not stored, $\mathcal{R}_{eid} \leftarrow \texttt{Reservoir.Init}(k)$
    - Set $\text{Enc}_{\text{TFHE}}(e_i), \text{Enc}_{\text{TFHE}}(\text{coin}_i) \leftarrow \texttt{TFHE.Eval}(C_{PRF}, CRS, \text{ct}_j)$ where C is the PRF circuit with polynomial stretch and CRS is the key to the PRF. TODO: defn PRF with stretch
    - Call $\mathcal{R}_{eid} \leftarrow \texttt{TFHE.Eval}(C, \mathcal{R}_{eid}, \text{ct}_j, \text{Enc}_{\text{TFHE}}(e_i), \text{Enc}_{\text{TFHE}}(\text{coin}_i))$ where C is the reservoir sampling circuit for $\texttt{Reservoir.Insert}$. $\texttt{Reservoir}^{eid}.\texttt{Insert}(\text{Enc}_{\text{TFHE}}(c_j), \text{Enc}_{\text{TFHE}}(e_i), \text{Enc}_{\text{TFHE}}(\text{coin}_i))$.
    - Store $\mathcal{R}_{eid}$.

- $(\texttt{elect}, eid)$ called by user P_i when $b_{eid} = n$

    - For $\ell \in [k]$, set $\text{ct}_\ell \leftarrow \texttt{TFHE.Eval}(C_{\texttt{Resevoir.Output}}, \mathcal{R}_{eid})$
    - Set $p_i \leftarrow \texttt{TFHE.PartDec}(\text{ct}_1, ..., \text{ct}_k)$
    - Broadcast $(\texttt{elect\_done}, eid, p_i)$ and call $(\texttt{elect\_done}, eid, p_i)$

- $(\texttt{elect\_done}, eid, p_j)$ from party P_j

    - Retrive $\mathcal{P}^{eid}$ if it is stored, otherwise set $\mathcal{P}^{eid} \leftarrow \emptyset$.
    - Set $\mathcal{P}^{eid} \leftarrow \mathcal{P}^{eid} \bigcup \{p_j\}$
    - If $|\mathcal{P}^{eid}| = n$, set $c_{a_1}, ..., c_{a_\ell} \leftarrow \texttt{TFHE.FinDec}(\mathcal{P}^{eid})$
    - Store $\mathcal{P}^{eid}$.

- $(\texttt{reveal}, eid, \ell)$

    - Broadcast $(\texttt{result}, eid, \ell, i)$

Figure 2: Description of the MSLE protocol

Simulator for Threshold MSLE Sim_C where $C \subseteq [n]$ and $|C| < t$:
Initializie a set of finished elections $\mathcal{E}$ and set a random tape for the simulator.

- $(\texttt{register\_elect}, eid)$ for party P_j

    - If $j \in C$ Follow the protocol's $\texttt{register\_elect}$ using use $s_i, r_i, c_i, \texttt{ct}_j$ from the real protocol and call $(\texttt{register\_elect\_add}, eid, \texttt{ct}_j)$. Set $c'_j \leftarrow c_j$ and store c'_j.

    - If $j \notin C$, $s'_j, r'_j \stackrel{\$}{\leftarrow} \mathbb{F}_q, c'_j \leftarrow \texttt{comm}(s'_j, r'_j), \texttt{ct}'_j \leftarrow \texttt{Enc}_{\text{TFHE}}(c'_j)$ Store c'_j and follow the protocol by calling $(\texttt{register\_elect\_add}, eid, \texttt{ct}'_j)$.

- $(\texttt{elect}, eid, \texttt{out})$ from party P_j where $\texttt{out}$ is $\mathcal{F}_{\text{MSLE}}.\texttt{elect}$ output $(\texttt{outcome}, eid, q)_i$ for all $i \in [n]$ where $q \in \{1, ..., k, \perp\}$.

    - For $\ell \in [k]$, set $\texttt{ct}'_\ell \leftarrow \texttt{TFHE.Eval}(C_{\texttt{Resevoir.Output}}, \mathcal{R}_{eid})$
    - Let $a'_1, ..., a'_k \in [n]$ such that if $q_i = \ell$ then $a'_\ell = i$.
    - Call $p'_1, ..., p'_n \leftarrow \texttt{Sim}_{\text{TFHE}}(C_{\texttt{Ident}}, \{\texttt{ct}'_\ell\}, c'_{a'_1}, ..., c'_{a'_\ell}, [n], \texttt{st})$
    - Call $(\texttt{elect\_done}, eid, p_j)$ from the protocol.

- $(\texttt{reveal}, eid, \ell)$ for P_j and $\mathcal{F}_{\text{MSLE}}.\texttt{reveal}$ output $(\texttt{result}, eid, \ell, j)$ or $(\texttt{reject}, eid, \ell, j)$.

    - Broadcast $(\texttt{result}, eid, \ell, j)$

Figure 3: Description of the MSLE protocol

all calls to $\texttt{elect\_done}$ are indistinguishable from the real protocol as $\texttt{elect\_done}$ is completely determined by $\{p_i\}_{i \in [n]}$.

$\square$

Lemma 3.4 ($\texttt{reveal}$ is simulation secure). *Proof.* Finally, note that $\texttt{reveal}$ in the simulator is identical to that of the real protocol as in the semi-honest setting, only parties which have won an election will call $\texttt{reveal}$. $\square$

We thus have that the view of the simulator is identical to that of the real protocol by the above three lemmas. $\square$

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