MATH 231BR: ADVANCED ALGEBRAIC TOPOLOGY
HOMEWORK 6
DUE: FRIDAY, MARCH 24 AT 12:00AM (MIDNIGHT) ON CANVAS

In the below, I use LAT to refer to Miller's *Lectures on Algebraic Topology*, available at: `https://math.mit.edu/~hrm/papers/lectures-905-906.pdf`.

1. Problem 1: Fiber sequences of Eilenberg–Maclane spaces (10 points)

Do Exercise 51.7 of LAT, replacing "fibration with fiber weakly equivalent to" with "map with homotopy fiber weakly equivalent to."

2. Problem 2: Minimal cell structures (10 points)

Do Exercise 51.8 of LAT.

3. Problem 3: Simply-connected 3-manifolds (5 points)

Let M denote a simply-connected compact 3-manifold. Prove that $M \simeq S^3$.

4. Problem 4: (Co)homological characterization of $\mathbb{CP}^n$ (15 points)

(1) Let X denote a simple space with homology groups $H_*(X;\mathbb{Z}) \cong \mathbb{Z}[0] \oplus \mathbb{Z}[2] \oplus \cdots \oplus \mathbb{Z}[2n]$ and cohomology ring $H^*(X;\mathbb{Z}) \cong \mathbb{Z}[x]/(x^{n+1})$, where $|x| = 2$. Prove that $X \simeq \mathbb{CP}^n$.

(2) Prove that $[\mathbb{CP}^n, \mathbb{CP}^n] \cong \mathbb{Z}$ via the map sending a map $\mathbb{CP}^n \to \mathbb{CP}^n$ to the induced homomorphism on H_2.

5. Problem 5: A consequence of obstruction theory (10 points)

Do Exercise 53.8 of LAT.