

Math 132
Problem Set 5
Spring, 2023

This problem set is due on Friday, March 3rd Please make your answers as complete and clear as possible. You are allowed to discuss these problems with others in the class, but your writing should be your own.

1. This problem is about the Morse Lemma in dimension 1.
 - (a) Suppose $f : U \rightarrow \mathbb{R}$ is a smooth function defined on an open neighborhood U of $0 \in \mathbb{R}$, that $f'(0) = 0$ and that $f''(0) \neq 0$. Show that there is a new coordinate x defined near $0 \in U$ in which f is given by $f(x) = f(0) \pm x^2$. (HINT: First show that f is given by $f(t) = f(0) \pm t^2 g(t)$ where g is a smooth function with $g(0) > 0$.)
 - (b) Suppose that $f : X \rightarrow \mathbb{R}$ is a smooth function on a 1-manifold X and $p \in X$ is a non-degenerate critical point. Show that there is a local coordinate system $\Phi : U \rightarrow \mathbb{R}$ with $\Phi(p) = 0$, in which $f \circ \Phi^{-1}(x) = a \pm x^2$ (where $a = f(p)$). This is the Morse Lemma in dimension 1.
2. Work Problem 2.2 of §2 of Chapter III in the Lecture Notes.
3. Work Problem 2.3 of §2 of Chapter III in the Lecture Notes.
4. Work Problem 5.3 of §5 of Chapter IV in the Lecture Notes.
5. Work Problem 5.6 of §5 of Chapter IV in the Lecture Notes.