

Astron 140 Homework 2

Due Sept 23, 11:59pm

1. Show that the scalar product of two Lorentz 4-vectors is a Lorentz scalar. (10 points)
2. * \mathcal{O}' -frame moves with speed v in the x -direction relative to \mathcal{O} -frame. In the \mathcal{O} -frame there is a photon with frequency f that moves at an angle θ with respect to the x -axis of \mathcal{O} -frame. Show that its frequency f' measured in \mathcal{O}' -frame is

$$\frac{f'}{f} = \frac{1 - v \cos \theta}{\sqrt{1 - v^2}} .$$

Show that even when the motion of the photon is perpendicular to the x -axis of the \mathcal{O} -frame (i.e. $\theta = \pi/2$), there is a frequency shift (called transverse Doppler shift). At what angle θ does the photon have to move so that there is no Doppler shift between \mathcal{O} and \mathcal{O}' ? (10 points)

3. Prove that conservation of four-momentum forbids a reaction in which an electron and positron annihilate and produce a single photon. Prove that the production of two photons is allowed. (5 points)
4. Calculate the energy required to accelerate a particle of rest mass $m_0 \neq 0$ from speed v to speed $v + \delta v$ ($\delta v \ll v$), to first order in δv . Show that it would take an infinite amount of energy to accelerate the particle to the speed of light. (5 points)
5. Write the equation of motion for Newton's theory of gravitation in terms of the gravitational potential $\Phi(\vec{x})$. What is the distinctive feature of this equation of motion (as opposed to that for other forces)? (5 points)
6. Give the simplest experimental evidence for the ratio between the inertial and gravitational mass being a universal constant (i.e. independent of the material composition of the object). (5 points)
7. State Einstein's Equivalence Principle. Use this equivalence principle to explain the observation that a helium balloon leans forward in a forwardly accelerating vehicle. Give an example of your own of the equivalence principle. (10 points)