Physics 212 Problem Set 4

Lev Kruglyak

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Problem 1. Confirm the equation

$$\mathcal{S}_{(2)} = \frac{1}{2}\int d\eta\, d^3x \left[(v')^2 - (\partial_i v)^2 + \frac{v''}{z}v^2\right],$$

where $v = z\mathcal{R}$ and $z^2 = a^2\dot{\phi}^2/H^2$, starting from the equation

$$\mathcal{S}_{(2)} = \frac{1}{2}\int d^4x\, a^3 \frac{\dot{\phi}^2}{H^2}\left[\dot{\mathcal{R}}^2 - a^{-2}(\partial_i \mathcal{R})^2\right].$$

Problem 2.

(a). Assuming slow-roll inflation, express the tensor-to-scalar ratio in terms of slow-roll parameters.

(b). Write down the tilt of the scalar power spectrum (given by $n_s - 1 = d\ln\Delta^2_{\mathcal{R}}/d\ln k$) in terms of the slow-roll parameters (going up to first order only in slow-roll parameters).

(c). Express the tilt of the tensor power spectrum (given by $n_t = d\ln\Delta^2_t/d\ln k$) in terms of slow-roll parameters (going also up to first order only in slow-roll parameters).

(d). Using your answers in (a) and (b), write r in terms of n_t. This relation is known as the "single-field slow-roll consistency relation". Any violation of this condition found in the data would violate the assumption of slow-roll single-field inflation, and would shed light on the physics of inflation!

Problem 3. Consider an inflationary potential of the form $V(\phi) = \lambda\phi^4$.

(a). Setting $\Delta^2_{\mathcal{R}}(k) = 10^{-9}$ (consistent with observations), show what value this implies for λ. see hint