Math 132 Makeup For Problem 5 on Homework 5

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Due: ??

Problem 1. Show that for each $a > 0$, the solid hyperboloid

$$x^2 + y^2 - z^2 \leq a$$

is a manifold with boundary.

Let $f : \mathbb{R}^3 \to \mathbb{R}$ be given by $f(x, y, z) = z^2 + y^2 - z^2$. By basic calculus, this has derivative $df_{x,y,z} = [2x, 2y, -2z]$. This is a surjective map as long as $(x, y, z) \neq 0$, so for any $a > 0$, it follows that a is a regular value of f. Therefore, the desired space, $f^{-1}((-\infty, a])$ is a smooth manifold with boundary $f^{-1}(a)$, since $(-\infty, a]$ is a smooth manifold with boundary $\{a\}$.