MATH 231BR: ADVANCED ALGEBRAIC TOPOLOGY
HOMEWORK 5
DUE: TUESDAY, MARCH 7 AT 12:00AM (MIDNIGHT) ON CANVAS

In the below, I use LAT to refer to Miller's *Lectures on Algebraic Topology*, available at: `https://math.mit.edu/~hrm/papers/lectures-905-906.pdf`.

1. Problem 1: Edge of homotopy LES (10 points)

Do Exercise 47.10 of LAT.

2. Problem 2: Fiber sequences and weak equivalences (15 points)

Given a map $f : X \to Y$ and a point $y \in Y$, let $F(f, y)$ denote the homotopy fiber of f above the point y. Given a commutative diagram

$$\begin{tikzcd} X_1 \arrow[r] \arrow[d, "f_1"] & X_2 \arrow[d, "f_2"] \\ Y_1 \arrow[r, "g"] & Y_2, \end{tikzcd}$$

prove that if $Y_1 \to Y_2$ is an n-equivalence and $F(f_1, y) \to F(f_2, g(y))$ is an n-equivalence for all $y \in Y_1$, then $X_1 \to X_2$ is an n-equivalence.

Extending the fiber sequences one step further, deduce that if $X_1 \to X_2$ is an n-equivalence and $Y_1 \to Y_2$ is an $(n+1)$-equivalence, then $F(f_1, y) \to F(f_2, g(y))$ is an n-equivalence for all $y \in Y_1$.

3. Problem 3: A factorization (10 points)

Prove that a map $X \to Y$ of path-connected spaces may be factored as $X \to Z_n \to Y$ with $X \to Z_n$ an isomorphism on π_i for $i \leq n$ and $Z_n \to Y$ an isomorhpism on π_i for $i > n$.

4. Problem 4: Connectivity of a product (15 points)

Suppose that X and Y are pointed CW complexes with X m-connected and Y n-connected. Prove that the inclusion $X \vee Y \to X \times Y$ is an $(m+n+1)$-equivalence and $X \wedge Y$ is $(m+n+1)$-connected.