MATH 231BR: ADVANCED ALGEBRAIC TOPOLOGY
HOMEWORK 4
DUE: TUESDAY, FEBRUARY 28 AT 12:00AM (MIDNIGHT) ON CANVAS

In the below, I use LAT to refer to Miller's *Lectures on Algebraic Topology*, available at: `https://math.mit.edu/~hrm/papers/lectures-905-906.pdf`.

1. Problem 1: Homotopy groups of infinite projective spaces (10 points)

Let $\mathbb{RP}^\infty = \varinjlim_n \mathbb{RP}^n$ and $\mathbb{CP}^\infty = \varinjlim_n \mathbb{CP}^n$ along the usual inclusions $\mathbb{RP}^n \hookrightarrow \mathbb{RP}^{n+1}$ and $\mathbb{CP}^n \hookrightarrow \mathbb{CP}^{n+1}$. Use the fibrations

$$S^0 \to S^\infty \to \mathbb{RP}^\infty$$

and

$$S^1 \to S^\infty \to \mathbb{CP}^\infty$$

to compute the homotopy groups of $\mathbb{RP}^\infty$ and $\mathbb{CP}^\infty$.

2. Problem 2: Homotopy groups of Hopf fibrations (10 points)

Use the Hopf fibrations to prove that

$$\pi_n S^2 \cong \pi_n S^3 \oplus \pi_{n-1} S^1,$$
$$\pi_n S^4 \cong \pi_n S^7 \oplus \pi_{n-1} S^3,$$

and

$$\pi_n S^8 \cong \pi_n S^{15} \oplus \pi_{n-1} S^7.$$

3. Problem 3: Fibrations and fiber homotopy equivalence (15 points)

Do Exercise 43.11 of LAT.

4. Problem 4: H-spaces are simple (15 points)

Given an H-space $(X, *)$, prove that the action of $\pi_1(X, *)$ on $\pi_n(X, *)$ is trivial for all $n \geq 1$. (You already proved this last week for $n = 1$, since the action in this case is just conjugation!) Conclude that a connected H-space is simple.