Cosmological Constraints from Baryonic Acoustic Oscillations

Lev Kruglyak

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In this paper, we use measurements of baryon acoustic oscillation (BAO) measurements by the Dark Energy Spectroscopic Instrument (DESI) to obtain constraints on cosmological parameters in the ΛCDM model. This analysis has already been done in [2], here we attempt to recreate some of the constraints obtained in that paper.

The ΛCDM Model

We begin with a brief review of the ΛCDM model. The base geometry is the FLRW metric – we assume further that it is flat. Energy density in the model is thus split into five species: baryonic matter $\Omega_{\rm b}$, cold (i.e. non-relativistic) dark matter $\Omega_{\rm c}$, electromagnetic radiation Ω_γ, curvature $\Omega_{\rm K}$, neutrinos Ω_ν, and dark energy $\Omega_{\rm DE}$. Note that the curvature energy density $\Omega_K = 0$ is trivial, so we do not include it here. Baryonic and cold dark matter are grouped as $\Omega_{\rm bc} = \Omega_{\rm b} + \Omega_{\rm c}$, and non-relativistic matter including neutrinos is grouped as $\Omega_{\rm m} = \Omega_{\rm bc} + \Omega_{\rm m}$. Using standard equation of state parameters for $\Omega_{\rm bc}$, Ω_γ, and Ω_K, we can write the time-dependent Hubble parameter as:

$$\frac{H(z)}{H_0} = \left[\Omega_{\rm bc}(1+z)^3 + \Omega_\gamma(1+z)^4 + \Omega_\nu \frac{\rho_\nu(z)}{\rho_{\nu,0}} + \Omega_{\rm DE} \frac{\rho_{\rm DE}(z)}{\rho_{\rm DE,0}} \right]^{1/2}. \tag{1}$$

The energy density of neutrinos begins scaling as $(1+z)^4$ at high redshifts and transitions to $(1+z)^3$. Throughout, we assume the existence of a single species of massive neutrino with mass $m_\nu = 0.06\,{\rm eV}$ and $N_{\rm eff} = 3.044$. In the base ΛCDM model, we assume that dark energy is constant over time, namely that $\rho_{\rm DE}(z)/\rho_{\rm DE,0} = 1$. We remove this assumption later in the paper.

When performing Bayesian analysis on the ΛCDM model, we sample along the six independent parameters of the model: $(\omega_{\rm c}, \omega_{\rm b}, \theta_*, \ln(10^{10} A_s), n_s, \tau)$. The first of these two parameters describe the ratios of cold dark matter and baryonic matter in the universe, i.e. $\omega_{\rm c} = \Omega_{\rm c} h^2$ and $\omega_{\rm b} = \Omega_{\rm b} h^2$ where $h = H_0/100\,{\rm km\,s^{-1}\,Mpc^{-1}}$. The next parameter, θ_* is the angular size of the sound horizon at the surface of last scattering, i.e.

$$\theta_* = \frac{r_s(z_*)}{D_A(z_*)} \quad \text{where} \quad r_s(z) = \int_z^\infty \frac{c_s(z')}{H(z')}\,dz' \quad \text{and} \quad D_A(z) = \frac{1}{1+z} \int_0^z \frac{c}{H(z')}\,dz' \tag{2}$$

where $z_* \approx 1090$ and $c_s(z)$ is the speed of sound in the primordial photon-baryon fluid. To speed up computation, it is common to approximate θ_* by a parameter $\theta_{\rm MC}$ given by fitting formula in [4]. The remaining parameter $\ln(10^{10} A_s), n_s$, and τ are the amplitude and spectral index of the primordial scalar perturbations and the optical depth respectively.

To perform Bayesian analysis with the CMB and DESI datasets, we use `Cobaya` [5], a general purpose Python program for exploring posterior distributions using Monte Carlo samplers. We use `CAMB` [3] as the theoretical model backend for Cobaya analysis and use a Markov Chain Monte Carlo (MCMC) sampler with convergence criterion $R - 1 < 0.01$ in the Gelman-Rubin statistic.

Exploring the CMB H_0, Ω_m Degeneracy

We begin by using the full CMB data from Planck 2018 to derive CMB priors which speed up later computations. We begin with simple uniform priors on the six cosmological parameters, taken from Table 2 of [1].We combine this with the full CMB likelihood from Planck 2018, including TT, EE modes for low ℓ and NPIPE TTTEEE modes for high ℓ as well as PR4 lensing.

parameter	ω_c	ω_b	$100\theta_\mathrm{MC}$	$\ln(10^{10}A_s)$	n_s	τ
default	$\mathcal{U}[0.001, 0.99]$	$\mathcal{U}[0.005, 0.1]$	$\mathcal{U}[0.5, 10]$	$\mathcal{U}[1.61, 3.91]$	$\mathcal{U}[0.8, 1.2]$	$\mathcal{U}[0.01, 0.8]$

Figure 1: Priors for $(\omega_\mathrm{c}, \omega_\mathrm{b}, 100\theta_\mathrm{MC}, \ln(10^{10}A_s), n_s, \tau)$.

To help calibrate later BAO data, we also add on a BBN consistent prior on ω_b, namely

$$\omega_\mathrm{b} = \Omega_m h^2 \sim \mathcal{N}(0.02218, 0.00055).$$

Running `Cobaya` with these priors, with Ω_m, H_0, and θ_* as derived parameters, we can graph the posterior distribution on H_0 and Ω_m. Graphing the direction of $\Omega_\mathrm{m}h^3$, we get a graph of the degeneracy direction of the CMB data. This is a well-known degeneracy, showing that despite the tight bounds which the CMB imposes on many parameters, varied data sources such as BAO are needed to impose constraints on important cosmological parameters.
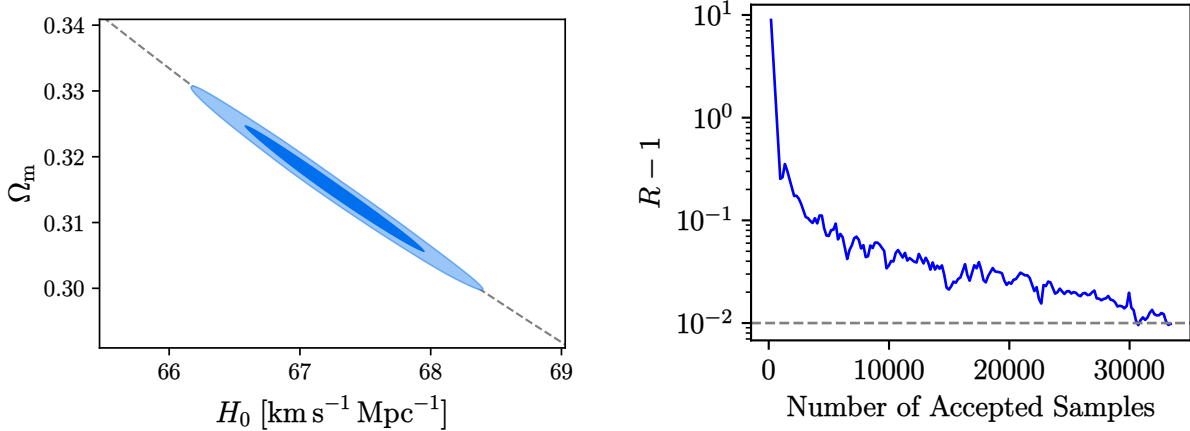

Figure 2: The H_0, Ω_m CMB degeneracy. (Planck 2018 + lensing)

Using the `GetDist` python package, we can use the full data of the CMB run to derive the covariance matrix for the parameters $\theta_*, \omega_\mathrm{b}$, and ω_c. This is useful when we want to approximately include a CMB likelihood without using the entire dataset. Of course, we must be careful to check with the full dataset to ensure the results are within approximation error.

$$(\theta_*, \omega_\mathrm{b}, \omega_\mathrm{c}) \sim$$
$$\mathcal{N}\left(\begin{bmatrix} 0.01041009 \\ 0.02219065 \\ 0.11969417 \end{bmatrix}, \begin{bmatrix} 5.10125839 \times 10^{-12} & 6.40183333 \times 10^{-11} & -6.09287501 \times 10^{-10} \\ 6.40183333 \times 10^{-11} & 1.62733275 \times 10^{-8} & -6.32279490 \times 10^{-8} \\ -6.09287501 \times 10^{-10} & -6.32279490 \times 10^{-8} & 1.08987992 \times 10^{-6} \end{bmatrix}\right)$$

Baryonic Acoustic Oscillations

Baryonic Acoustic Oscillations (BAO) provide a tool to measure the expansion history of the universe by providing a standard ruler given by the characteristic scale which is imprinted on matter clusters in the universe by pre-recombination pressure waves in the primordial baryon-photon fluid.

We now use the DESI DR2 data release, the BBN prior on ω_b, and a CMB derived prior on θ_*

$$100\theta_* \sim \mathcal{N}(1.04110, 0.00053)$$

to see what constraints they impose on the H_0, Ω_m degeneracy.
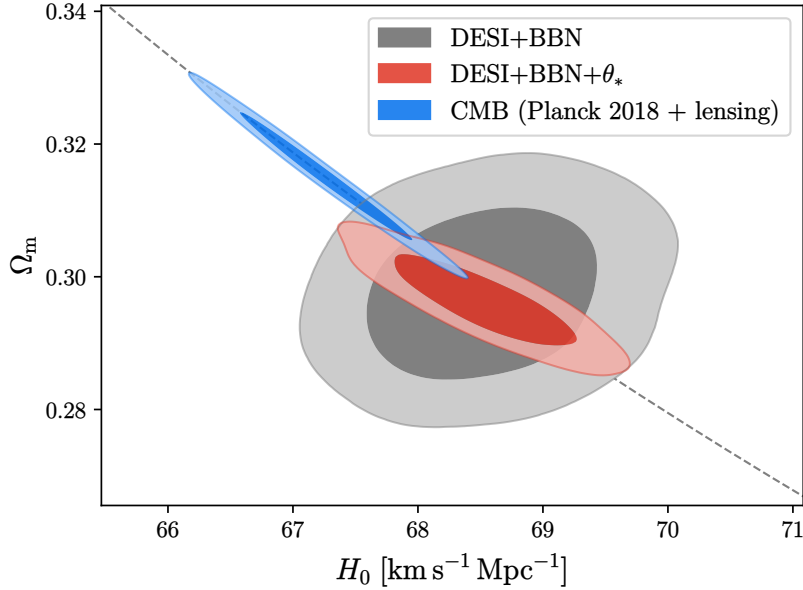
There is a mild tension ($\approx 2\sigma$) between the DESI+BBN data and the CMB data, but they both generally agree with the H_0, Ω_m degeneracy of the CMB. Using `GetDist`, we calculate 68% limits for H_0 and Ω_m, and summarize the bounds in the following table:

	H_0	Ω_m
DESI+BBN	68.53 ± 0.62	0.2975 ± 0.0085
DESI+BBN+θ_*	68.52 ± 0.49	$0.2963^{+0.0042}_{-0.0048}$
CMB	67.26 ± 0.46	0.3151 ± 0.0064
DESI+BBN+CMB	68.39 ± 0.28	0.2996 ± 0.0035
DESI+BBN+CMB (prior)	$68.32^{+0.33}_{-0.29}$	$0.3003^{+0.0037}_{-0.0043}$

Figure 3: 68% limits on H_0 and Ω_m from BAO and CMB datasets.

We can also combine the BAO data from DESI and the BBN priors directly with the derived likelihood on $(\theta_*, \omega_\mathrm{b}, \omega_\mathrm{c})$ obtained from the CMB data. For completeness, we also include a run with the full CMB data to ensure accuracy.
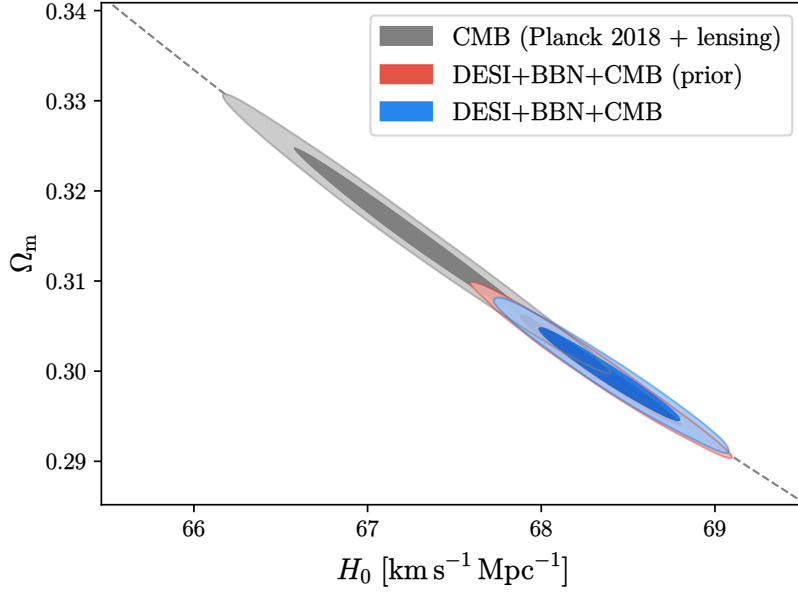
This puts the slight tension between the DESI and CMB datasets and the CMB dataset alone into view. Notice that using the derived CMB prior instead of the full CMB dataset does not affect the posterior distribution all that much. We also note that modifications of ΛCDM can alleviate this BAO tension, see [2] for more details.

The Behavior of Dark Energy

Finally, we discuss some failed attempts to recreate bounds on the behavior of dark energy using the DESI BAO data. In the base ΛCDM model, dark energy is assumed to be constant, i.e. $\rho_{\text{DE}}(z)/\rho_{\text{DE},0} = 1$. However, it is common to extend the theory by allowing dark energy to have a varying equation of state parameter. The usual parametrization is:

$$w(z) = w_0 + w_a(1 - a) \quad \implies \quad \frac{\rho_{\text{DE}}(z)}{\rho_{\text{DE},0}} = a^{3(1+w_0+w_a)}e^{-3w_a(1-a)}. \tag{3}$$

This model is known as the w_0w_aCDM model, and can help alleviate some of the tension described earlier in the paper. Note that the ordinary ΛCDM model has $w_0 = -1$ and $w_a = 0$. A non-zero w_a is extremely interesting since it implies that dark energy varies over time. This comes with a host of theoretical problems, and could lead to exciting new physics outside of the ΛCDM model.

Perhaps due to a lack of time, some theoretical misunderstanding, or bug in my code, all of my runs involving the w_0w_aCDM model gave me results agreeing with the ΛCDM model. I used the DESI DR2 dataset alongside the CMB dataset, including using just the CMB prior to speedup iteration time.

References

[1] DESI Collaboration et al. *DESI 2024 VII: Cosmological Constraints from the Full-Shape Modeling of Clustering Measurements*. 2024.

[2] DESI Collaboration et al. *DESI DR2 Results II: Measurements of Baryon Acoustic Oscillations and Cosmological Constraints*. 2025.

[3] Antony Lewis, Anthony Challinor, and Anthony Lasenby. "Efficient computation of CMB anisotropies in closed FRW models". In: *Astrophys. J.* 538 (2000), pp. 473–476.

[4] Antony Lewis, Anthony Challinor, and Anthony Lasenby. "Efficient Computation of Cosmic Microwave Background Anisotropies in Closed Friedmann-Robertson-Walker Models". In: *The Astrophysical Journal* 538.2 (2000), pp. 473–476.

[5] Jesús Torrado and Antony Lewis. "Cobaya: code for Bayesian analysis of hierarchical physical models". In: *Journal of Cosmology and Astroparticle Physics* 2021.05 (2021), p. 057.