r/6thForm

> CompChallenge 2019

Challenge A | Quad

A quadratic equation is expressed in the form $ax^2 + bx + c = 0$, where a, b, c are integers.

Write a function, Quad, that takes any integers **a**, **b** and **c** as arguments.

The function should return a string in the form "ax^2 + bx + c = 0". However, note that:

- If a, b or c are negative, the ' $+$ ' should be replaced with a ' – ' for that term.
- If b or c are 0, that term is completely removed from the equation. a will not be 0.
- If a or b is -1 or 1, the 1 is excluded from the string.

Test your solution with

- Quad(1, 2, 3) which should return "x^2 + 2x + 3 = 0"
- Quad(-3, 4, -10) which should return "- 3x^2 + 4x – 10 = 0"
- Quad(-1, 0, 1) which should return "- x^2 + 1 = 0"

[15 Points]

Challenge B | QuadRoot

We say the solutions (or roots) of a quadratic equation are α and β. If we set

$$(x-\alpha)(x-\beta)=0$$

and expand out, we'll be left with an equation where only α and β are the solutions.

Write a second function, QuadRoot, that takes 2 integer arguments, **alpha** and **beta**.

The function should return a string in the same form as Quad, but **alpha** and **beta** should be solutions of the quadratic equation. You may use your answer to Challenge A to assist you.

Test your solution with

- QuadRoot(6, 9) which should return "x^2 - 15x + 54 = 0"
- QuadRoot(-80, 0) which should return "x^2 + 80x = 0"
- QuadRoot(7, -7) which should return "x^2 - 49 = 0"

[5 Points]

Challenge C | Poly

📰 For quadratic equations, we say the highest power is in x^2. However, a polynomial equation can have any integer for its maximum power (which we'll call n). Anything expressed in the form

$$c_n x^n + \cdots + c_2 x^2 + c_1 x + c_0 = 0$$

is a perfectly valid equation – and don't worry about all the c_i symbols, these are just integers like a, b, and c from before. For example, we could have $[x^7 - 5x^4 + x^3 - x^2 + 9 = 0]$ or $[-x - 100 = 0]$.

💡 Write a third function, Poly, that takes an array of integers as its argument.

The returned string for this function follows similar rules to Quad, noting that:

- We begin with a non-zero x^n term (where n is the highest power of the polynomial).
- Other non-zero terms should be sorted in order of descending power.
- The starting power of x will always be at least 1. ($42 = 0$ obviously doesn't work)

You may use your answer from Challenge A to assist you.

⚙ Test your solution with

- Poly([-3, 4]) which should return "- 3x + 4 = 0"
- Poly([8, -6, -1, 1, -5, 2]) which should return "8x^5 – 6x^4 – x^3 + x^2 – 5x + 2 = 0"
- Poly([1, 0, 0, 0, -1, 0, 0, 0, 0, 0, 8, 3]) which should return "x^11 - x^7 + 8x + 3 = 0"

[10 Points]

Challenge D | PolyRoot

You might have seen this coming, but now we're going to discuss the integer solutions of *any* polynomial. We'll call them $\alpha_1, \alpha_2, \dots, \alpha_n$. In a similar way to the last part, if we set

$$(x - \alpha_1)(x - \alpha_2)(\dots)(x - \alpha_n) = 0$$

and expand out, we'll have a polynomial equation where $\alpha_1, \alpha_2, \dots, \alpha_n$ are the solutions.

Some things to consider:

Below are the expansions of a polynomial equation with 3 solutions, and 4 solutions.

$(x-\alpha_1)(x-\alpha_2)(x-\alpha_3)=0$		$(x-\alpha_1)(x-\alpha_2)(x-\alpha_3)(x-\alpha_4)=0$	
Power of x	Number in front of term	Power of x	Number in front of term
3	1	4	1
2	$-(\alpha_1+\alpha_2+\alpha_3)$	3	$-(\alpha_1+\alpha_2+\alpha_3+\alpha_4)$
1	$\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3$	2	$\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_1\alpha_4+\alpha_2\alpha_3+\alpha_2\alpha_4+\alpha_3\alpha_4$
0	$-(\alpha_1\alpha_2\alpha_3)$	1	$-(\alpha_1\alpha_2\alpha_3+\alpha_1\alpha_2\alpha_4+\alpha_1\alpha_3\alpha_4+\alpha_2\alpha_3\alpha_4)$
		0	$\alpha_1\alpha_2\alpha_3\alpha_4$

Is there a pattern you can spot? What would it look like if there were 5 solutions?

Write a fourth function, PolyRoot, that takes an array/list of integer arguments.

The function should return a string in the same form as Poly, but every integer passed into the list argument must be a solution of the polynomial. You may use your answers from the previous challenges to assist you.

Test your solution with

- PolyRoot([-3]) which should return "x + 3 = 0"
- PolyRoot([8, -6, -1, 1, -5, 2]) which should return "x^6 + x^5 - 65x^4 - 125x^3 + 544x^2 + 124x – 480 = 0"
- PolyRoot([1, 0, 0, 0, -1, 0, 0, 0, 0, 0, 8, 3]) which should return "x^12 - 11x^11 + 23x^10 + 11x^9 - 24x^8 = 0"

Keep in mind that you can get a significant number of marks for partial solutions in this task.

[20 Points]

Challenge E | PolyRationalRoot

Rational numbers can be expressed in the form a/b, where a and b are integers, and b is greater than 0.

In a relation to quadratics, a/b and c/d are the two rational solutions of the equation $(bx - a)(dx - c) = 0$.

Rational numbers can also be written as decimals. For example,

$$\frac{1}{2} = 0.5, \qquad \frac{11}{8} = 1.375, \qquad -\frac{2}{3} = 0.\dot{6} = 0.6666 \ldots, \qquad \frac{58}{55} = 1.0\dot{5}\dot{4} = 1.0545454 \ldots$$

Some fractions have infinitely repeating (recurring) decimal numbers, making them a little harder to convert.

Some things to consider:

If the highest term in a polynomial was x^3, and each solution was rational, what numbers would be in front of each term? What about x^4 and x^5? Can you deduce a general solution from this?

How would we convert a decimal to a fraction? Are there any rules or steps that can be used in an algorithm?

What would it mean for a polynomial to be in its 'lowest form'?

Write a final (for real this time) function, PolyRationalRoot, that takes an array/list of string arguments.

List elements can be *Integers, fractions or decimals*. Any recurring parts of decimals will be highlighted with brackets in the string. **Examples:** "3", "-54", "-3/2", "8/5", "12/6", "-0.8", "4.1527", "0.(3)", "-1.0(54)", "4.(9)"

The return value should be in the same form as PolyRoot, but every value passed into the list argument must be a solution of the polynomial. You may use your answers from the previous challenges to assist you.

The returned polynomial must be in the lowest form possible. For example, although $2x^2 + 6x + 14 = 0$ is valid, it isn't in its lowest form - we can divide everything by 2 to get $x^2 + 3x + 7 = 0$.

Test your solution with

- PolyRationalRoot(["-3/2", "0"]) which should return "2x^2 + 3x = 0"
- PolyRationalRoot(["1.0(54)", "3/1"]) which should return "55x^2 - 223x + 174 = 0"
- PolyRationalRoot(["0.(8)", "-0.75", "-3/6", "-5", "4/2"]) which should return "72x^5 + 242x^4 - 695x^3 - 443x^2 + 458x + 240 = 0"
- PolyRationalRoot(["-5.(9)", "114/19", "4.0", "-131/95", "0", "25/200", "-391/23"]) which should return "760x^7 + 10833x^6 - 66782x^5 - 456495x^4 + 1428100x^3 + 2394252x^2 - 320688x = 0"

Keep in mind that you can get a significant number of marks for partial solutions in this task.

[20 Points]