

Parameters definition

NP : Number of Parameters should be fitted
 β : Vector of independent variable appearing in the function
N_f : Number of parallel functions
F : Vector of functions which must be fitted to experimental data
 β_0 : First guess of parameters
N_file : number of file, must be same as Number of parallel functions
Name_files : Vector Names of experimental data files

datafit prints the number of iterations, the *Std Deviation of Residuals* (Goodness of Fit) statistic, the supplementary information required, and returns *f*.

```
restart;  
with(plottools) : with(LinearAlgebra) : with(plots) : with(StringTools) : with(stats) :  
  with(CurveFitting) : with(VectorCalculus) :  
with(linalg) : with(ArrayTools) :
```

```
nlf := proc(NP, Betaa, N_f, F, B0, N_file, Name_files)  
  description "Levenberg-Marquardt algorithm to calculate material parameters";  
  local i, ii, ij, ik, kk, kkk, c_;  
  local A, X_exp, Y_exp, Jac;  
  local Total_n_points, Ntot;  
  local LSP, Jac_total, Jac_total0;  
  local Res, Res0, Res1, Betaa1, Delta1, Betaa0;  
  local limit_iteration, Lambda_0, v;  
  local AA1, BB1, lc, LSP_;
```

```
  Betaa0 := B0;
```

```
  #=====
```

```
  # read experimental data
```

```
  #=====
```

```
  Total_n_points := 0 :
```

```
  for i from 1 to N_file do
```

```
    A := Matrix(readdata(Name_files[i], [float, float])) :  
    Ntot[i] := Size(A, 1) :
```

```
    for ii from 1 to Ntot[i] do
```

```
      X_exp[ii + Total_n_points] := A[ii, 1];
```

```
      Y_exp[ii + Total_n_points] := A[ii, 2];
```

```
    end do:
```

```
    Total_n_points := Total_n_points + Ntot[i] :
```

```

end do:
unassign('i','ii') :

#=====
# Caculate Jacobian Vector of functions
#=====
for ii from 1 to N_f do
    for ij from 1 to NP do
        Jac[ii, ij] := diff(F[ii](x, Betaa[i] $ i = 1 ..NP), Betaa[ij]);
    end do:
end do:
unassign('ii','ij') :

#=====
#Calculation of J and Res= y_exp-f(x,beta). x=At each data point.
# J is derived as a vector and can be calculated using numerical differentiation.
#=====

c_, LSP := 0, 0 :
Jac_total := Matrix( Total_n_points, NP);
Jac_total0 := Matrix( Total_n_points, NP);
Res := Matrix( Total_n_points, 1);

for ii from 1 to N_f do
    for ij from 1 to Ntot[ii] do
        c_ := c_ + 1;
        for ik from 1 to NP do
            Jac_total[c_, ik] := eval(Jac[ii, ik], x = X_exp[c_]);
        end do;
        Res[c_, 1] := Y_exp[c_] - F[ii](X_exp[c_], Betaa0[i] $ i = 1 ..NP) :
        LSP := LSP + Res[c_, 1]^2;
    end do;
end do;
unassign('ii','ij','ik','i') :

#=====
#Set initial Levenberg Marquardt Damping parameter
#=====
v := 2.10 :

Lambda_0 :=  $\frac{3}{v^6}$  :

limit_iteration := 50 :

Betaa1 := [0 $ i = 1 ..NP] :
Delta1 := Matrix(Np, 1) :
Res1 := Matrix( Total_n_points, 2);

```

```

#=====
# Begin iterations.
#=====
for kkk from 1 to limit_iteration do
    Jac_total0 := eval(Jac_total, Betaa = Betaa0 ) :

    for kk from 1 to 2 do
        AA1 := transpose(Jac_total0) . Jac_total0 + Lambda_0 / v^(kk-1)
        *IdentityMatrix(NP) :
        BB1 := transpose(Jac_total0) . Res:
        Delta1 := 1 / AA1 . BB1:
        for i from 1 to NP do
            Betaa1[kk][i] := Betaa0[i] + Delta1[i, 1];
        end do:
        unassign('i') :
        c_, LSP_[kk] := 0, 0 :
        for ii from 1 to N_f do
            for ij from 1 to Ntot[ii] do
                c_ := c_ + 1 :
                Res1[c_, kk] := Y_exp[c_] - F[ii](X_exp[c_], Betaa1[kk][i])
$ i = 1 ..NP) :
                LSP_[kk] := LSP_[kk] + Res1[c_, kk]^2 :
            end do:
        end do:
    end do:
    unassign('kk','ij','ik','c_','i') :

#=====
===

#Choosing between shortening and increasing the steps using lambda1 and
lambda2

#=====
===

if min(LSP_[1], LSP_[2]) < LSP then

    if LSP_[1] ≤ LSP_[2] then
        for i from 1 to NP do
            Betaa0[i] := Betaa1[1][i];
        end do:
        Res := convert(Res1[ .., 1], Matrix) :
        LSP := LSP_[1] :
    else
        for i from 1 to NP do
            Betaa0[i] := Betaa1[2][i];
        end do:
        Res := convert(Res1[ .., 2], Matrix) :

```

```

        LSP := LSP_[2]:
    end if:
    unassign('ii','ij','ik','c_','i'):

else
    if LSP_[1] < LSP_[2] then
        Lambda_0 := Lambda_0*v^2:
    else
        Lambda_0 := Lambda_0/v^2:
    end if:
end if:

end do:
RETURN(Betaa0);

end proc:

```

Example

```

> NP := 3: Betaa := [a[i] $ i = 1..NP];
  F[1] := unapply(sum(Betaa[i]*x^(i-1), i = 1..NP), x, Betaa[i] $ i = 1
    ..NP);
  F[1] := unapply(Betaa[1]*x + Betaa[2]*x^2 + Betaa[3]*sin(x), x, Betaa[i] $ i
    = 1..NP);
  N_f := 1;
  #Betaa is the array that stores the values of the material parameters
  #F[1] is an example function for three material parameters
  Betaa0 := [0 $ i = 1..NP];
  N_file := 1;
  Name_files := ["exp-1st-unload-diani.txt"];
  Aa := nlf(NP, Betaa, N_f, F, Betaa0, N_file, Name_files);
  A := Matrix(readdata(Name_files[1], [float, float])):
  pl[1] := plot(A, color = BLACK, caption = "E fitted vs experiment", title
    = "Stress-Strain Curve for NpPEG", labels
    = ["Strain (Length/Original Length)", "Stress (MPa)"]):
  pl[2] := plot(F[1](x, Aa[i] $ i = 1..NP), x = 1..2, color = BLUE, caption
    = "fitted"):
  display(pl[1], pl[2]);

```

#the blue line is the curve generated by the algorithm, and the black line is the curve from the experimental data

$$\text{Betaa} := [a_1, a_2, a_3]$$

$$F_1 := (x, a_1, a_2, a_3) \rightarrow a_3 x^2 + a_2 x + a_1$$

$$F_1 := (x, a_1, a_2, a_3) \rightarrow a_2 x^2 + a_1 x + a_3 \sin(x)$$

$$N_f := 1$$

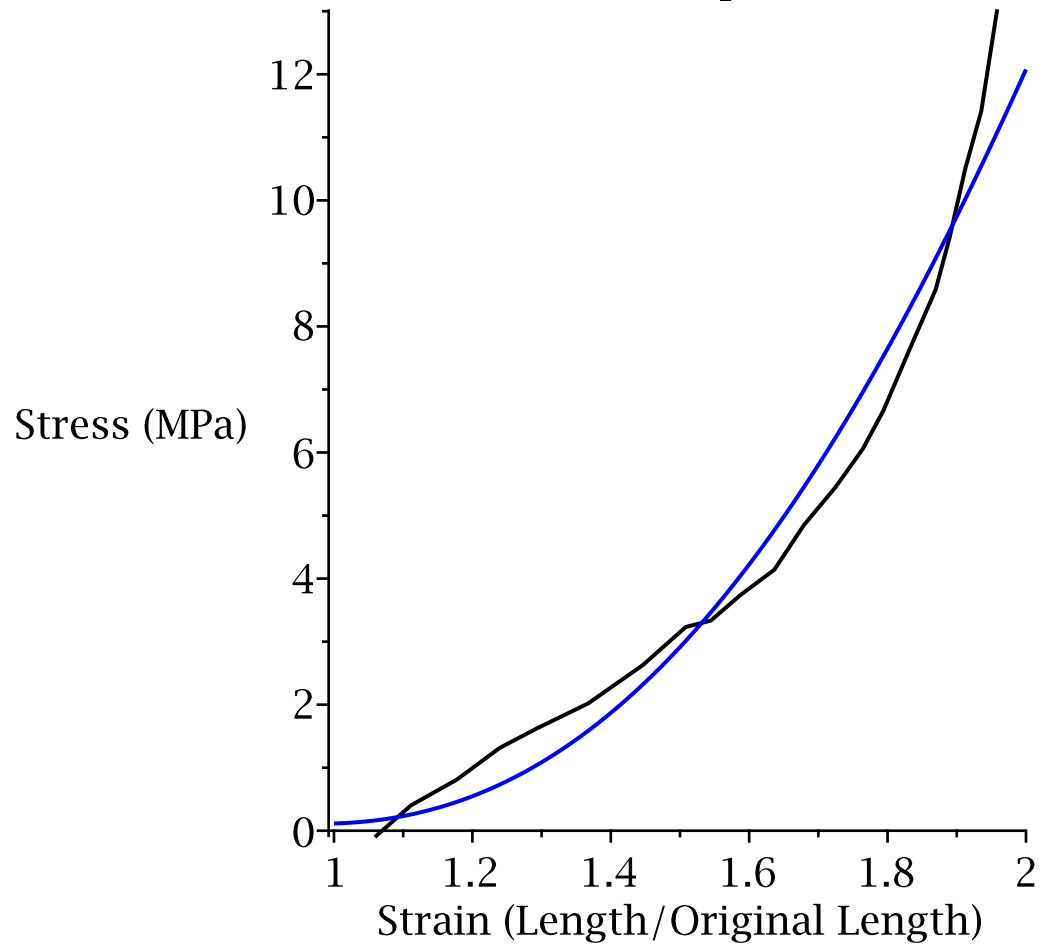
$Betaa0 := [0, 0, 0]$

$N_file := 1$

$Name_files := ["exp-1st-unload-diani.txt"]$

$Aa := [76.4863003235468, -19.9937431231326, -66.9985457439815]$

Stress-Strain Curve for NpPEG



E fitted vs experiment

