Homework 8

Problem 1. Two aliens have arrived on Earth, each claiming to possess a machine that can solve the SAT problem in polynomial time. However, only one of these machines is genuine, while the other is a counterfeit. Your task is to design a protocol for solving the SAT problem in polynomial time by asking questions to their machines.

Solution. Let the two aliens be A and B, and their machines be M_A and M_B respectively. Suppose M_A corresponds to a algorithm A which can decide whether $\varphi(x_1, x_2, \cdots, x_n)$ is satisfiable or not in polynomial time.

Consider algorithm S as follows:

1. For $i = 1, 2, \cdots, n$ do:

   (a) Use M_A to check whether $\varphi(a_1, \cdots, a_{i-1}, 0, x_{i+1}, \cdots, x_n)$ is satisfiable or not.

   (b) If so, set $a_i = 0$. Otherwise, set $a_i = 1$.

2. Check if $a = (a_1, a_2, \cdots, a_n)$ is a "satisfying" assignment for φ. Output a if it is.

Since one of the machines is a counterfeit, we couldn't guarantee that the output is a truly satisfying assignment. However, SAT $\in$ NP, so we can verify the output in polynomial time. If the output is a truly satisfying assignment, then M_A is the genuine machine. Otherwise, M_B is the genuine machine.

Now that we have decided which machine is genuine, we can use it to solve the SAT problem in polynomial time.