Homework 8

Problem 1. Two aliens have arrived on Earth, each claiming to possess a machine that can solve the SAT problem in polynomial time. However, only one of these machines is genuine, while the other is a counterfeit. Your task is to design a protocol for solving the SAT problem in polynomial time by asking questions to their machines.

Solution. Let the two aliens be A and B, and their machines be M_A and M_B respectively. Given a formula φ as input, we ask M_A and M_B whether φ is satisfiable or not.

1. If both machines output "yes", then the answer is "yes".
2. If both machines output "no", then the answer is "no".
3. If one machine outputs "yes" and the other outputs "no", then we proceed to the next step. Without loss of generality, assume that M_A outputs "yes" and M_B outputs "no". Consider algorithm S as follows:
   (a) For $i = 1, 2, \cdots, n$ do:
      i. Use M_A to check whether $\varphi(a_1, \cdots, a_{i-1}, 0, x_{i+1}, \cdots, x_n)$ is satisfiable or not.
      ii. If so, set $a_i = 0$. Otherwise, set $a_i = 1$.
   (b) Check if $a = (a_1, a_2, \cdots, a_n)$ is a satisfying assignment for φ. If so, then the answer is "yes". Otherwise, the answer is "no".

Therefore, the protocol above can solve the SAT problem in polynomial time.