# Lecture 7

# Morphology

# Morphology Lecture (with first Brief Segmentation-Continued-reviewed)

# Segmentation-review

- Image Segmentation
    1. Multi-level thresh-holding
    2. Smoothing and thresh-holding
    3. P-tile method
    4. Iterative thresh-holding
    5. Thresh-holding based on local properties
    6. Dynamic thresh-holding
    7. Watershed algorithm
    8. Object segmentation from motion
    9. Region growing
- Region and simple object representation
- Split and Merge

# Morphology Lecture

# MORPHOLOGY

- morphological operators - tools for extracting image components that are useful in the representation and description of region shape (examples: erosion, dilation, etc.)
- the language of mathematical morphology is **set theory**
- **erosion** - removes pixels from the periphery of a region (it also removes single pixels) shrinking
- **dilation** - adds a layer of pixels around a periphery of a region (it also fill small holes within regions) Expanding

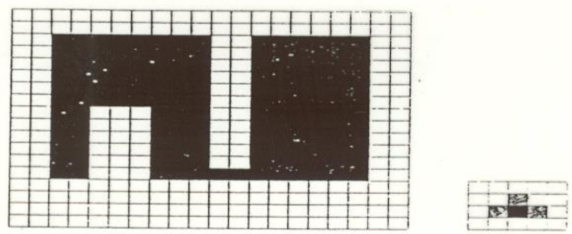


Figure 2.25: The original test image $A$ (left) and structuring element $B$ (right). Note that the origin of the structuring element is darker than the other pixels in $B$.

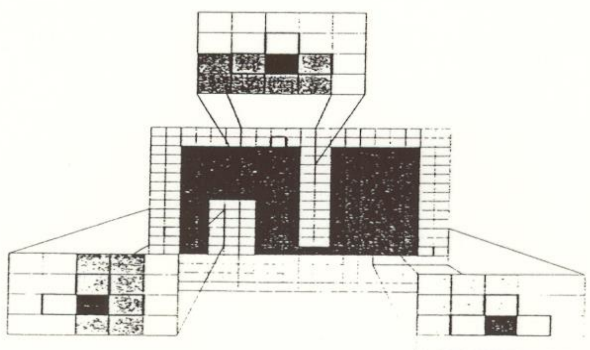
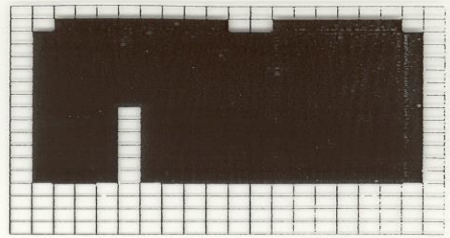


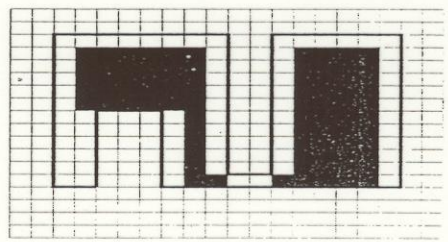
Figure 2.26: Translations of the structuring element $B$ to 1 pixels in $A$ where the entire structuring element is *not* contained within $A$. During a dilation operation, every pixel in the structuring element will be present in the final image. During an erosion operation, the pixel at the origin of the structuring element will be deleted.



$$A \oplus B = \bigcup_{b_i \in B} A_{b_i}$$

If $A_{b_i}$ are translations of the binary image $A$ by the 1 pixels of the binary image $B$, then the union of the translations of $A$ by the 1 pixels of $B$ is called the dilation of $A$ by $B$.

Figure 2.27: The dilation of $A$ by $B$. The boundary of the original figure $A$ is shown as a bold line.



The erosion of a binary image $A$ by a binary image $B$ is 1 at a pixel $p$ if and only if every 1 pixel in the translation of $B$ to $p$ is also in $A$.

$$A \ominus B = \{p | B_p \subseteq A\}$$

Figure 2.28: The erosion of $A$ by $B$. The boundary of the original figure $A$ is shown as a bold line.

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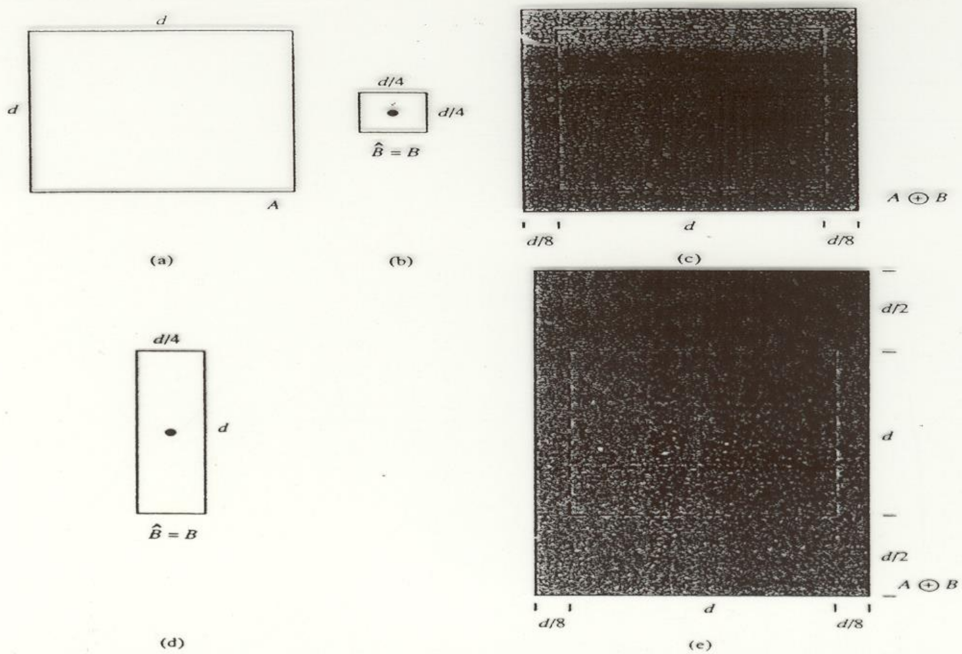


*Figure 8.26* (a) Original set $A$; (b) square structuring element and its reflection; (c) dilation of $A$ by $B$, shown shaded; (d) elongated structuring element; (e) dilation of $A$ using this element.

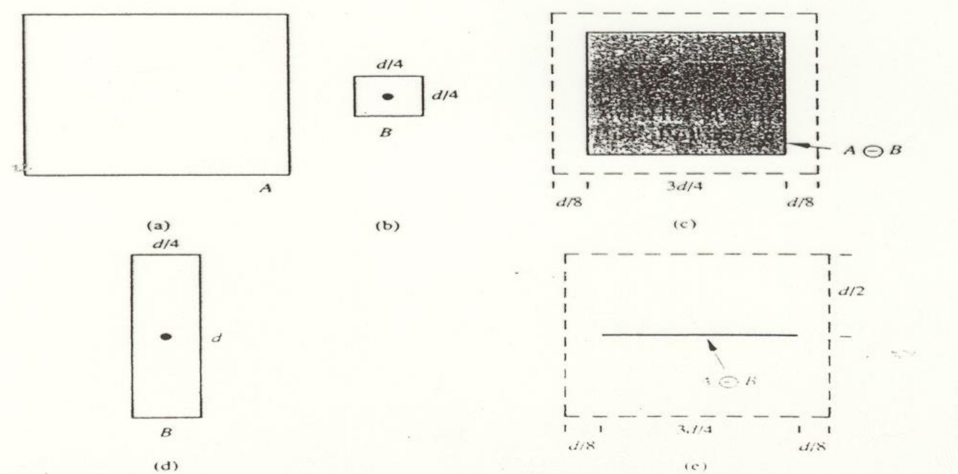


*Figure 8.27* (a) Original set $A$; (b) structuring element $B$; (c) erosion of $A$ by $B$, shown shaded; (d) elongated structuring element; (e) erosion of $A$ by this element.

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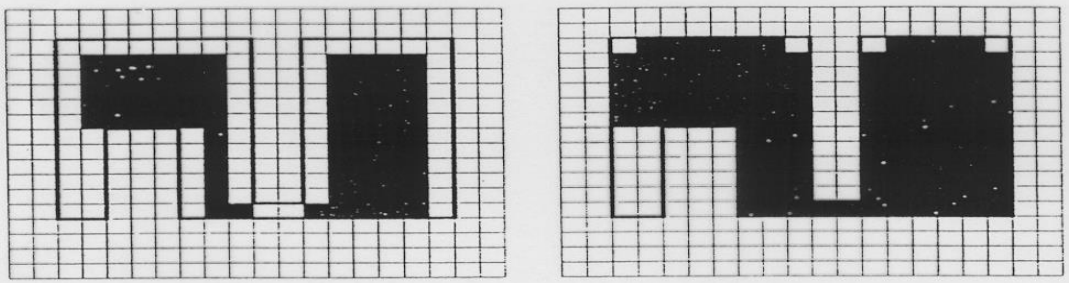
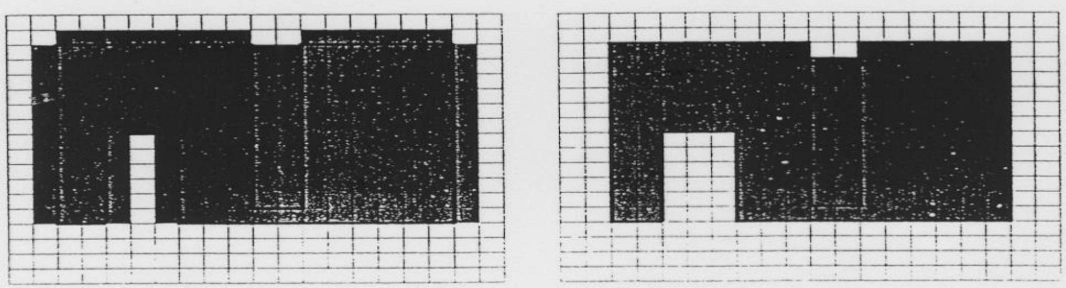
- erosion and dilation are often used in **filtering** the images - if the nature of noise is known, then a suitable structuring element can be used and a sequence of erosion and dilation operations can be applied for removing the noise
- **opening** - a combination of an erosion followed by a dilation (opening up the spaces between touching regions, removing pixels in regions which are too small to contain the structuring element)
- **closing** - a combination of an dilation followed by an erosion (fusing narrow brakes, eliminating small holes, filling gaps smaller than the structuring element)



erosion
+
dilation

Figure 2.32: Opening operation. *Left:* Initial erosion. *Right:* Succeeding dilation. The boundary of the original figure $A$ is shown as a bold line.



dilation
+
erosion

Figure 2.33: Closing operation. *Left:* Initial dilation. *Right:* Succeeding erosion. The boundary of the original figure $A$ is shown as a bold line.

- the structuring element (probe) does not have to be compact or regular - it can be any pattern of pixels
- morphological operations can be used for **boundary extraction**:

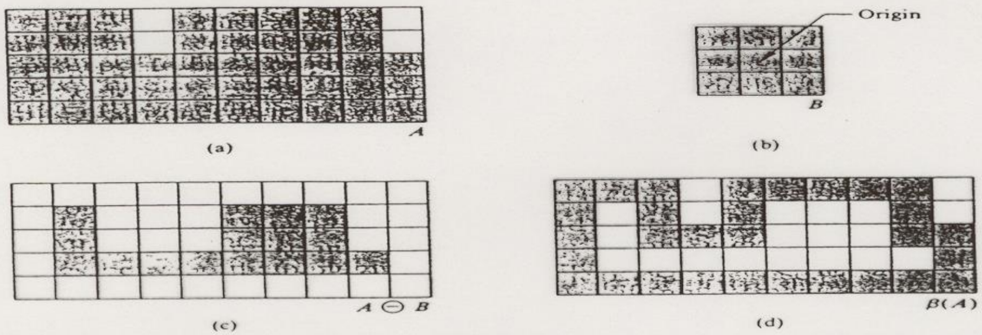
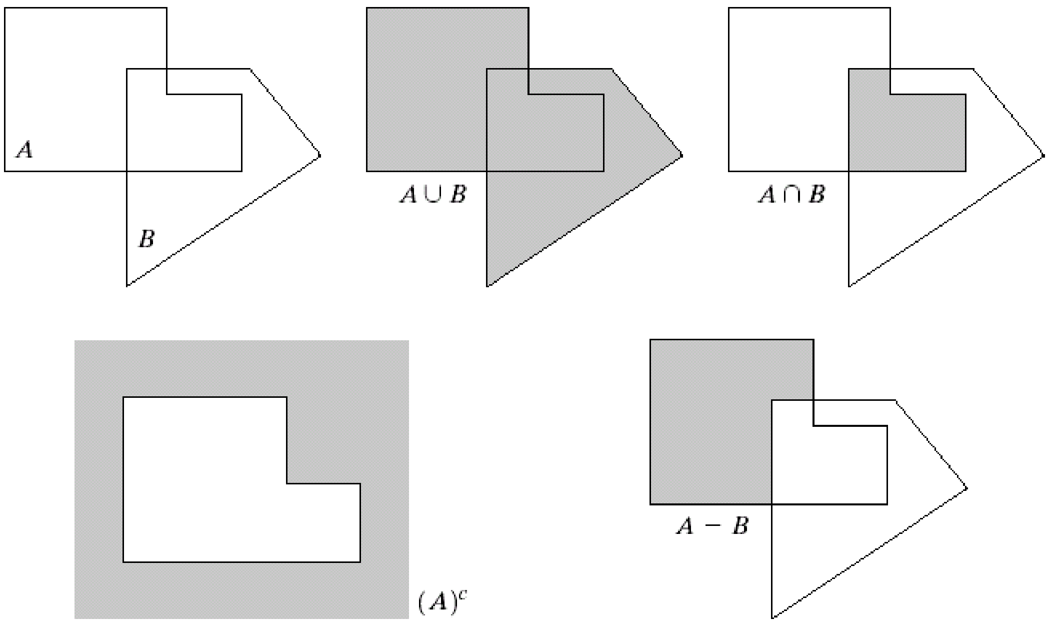



*Figure 8.33* (a) Set A; (b) structuring element B; (c) A eroded by B; (d) boundary extracted by taking the set difference between A and its erosion.

- morphological operations can also be used for **region filling** and for **extraction of connected components** (Gonzalez and Woods, section 8.4)
- morphological operations can be used for **optical character recognition**:
  - a. create a model for each character:
    - extract the character to be recognized
    - use expanding or closing to fill holes and cavities
    - shrink the character image to remove unwanted regions and to reduce the size so that it will fit inside an instance of the character
  - b. preprocess the character image (fill the holes, remove unwanted pixels)
  - c. use the character model as a structuring element and perform erosion
  - d. compute the connected components
  - e. apply the size filter to discard regions that are too small
  - f. compute the position of each region that passes through the size filter --> this provides the position of each recognized instance of the character model in the image

# Morphological Image Processing




a b c
d e

**FIGURE 9.1** (a) Two sets $A$ and $B$. (b) The union of $A$ and $B$. (c) The intersection of $A$ and $B$. (d) The complement of $A$. (e) The difference between $A$ and $B$.

# Morphological Image Processing

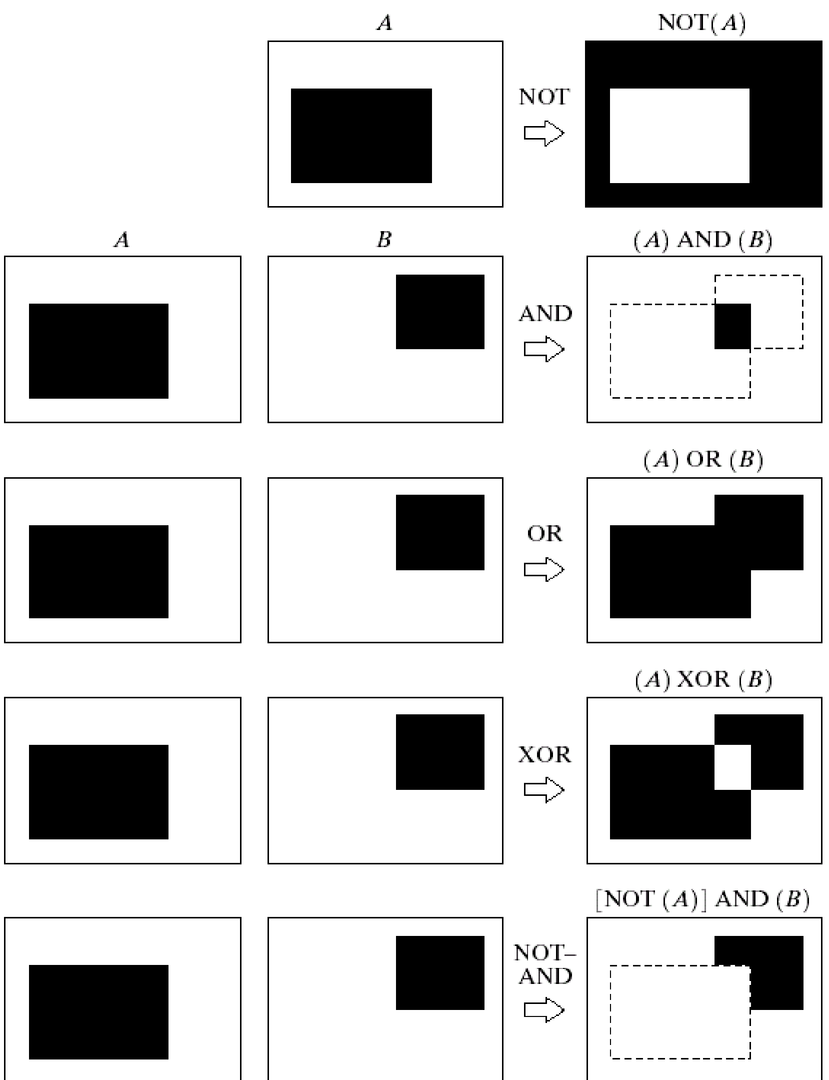



**FIGURE 9.3** Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.

# Morphological Image Processing

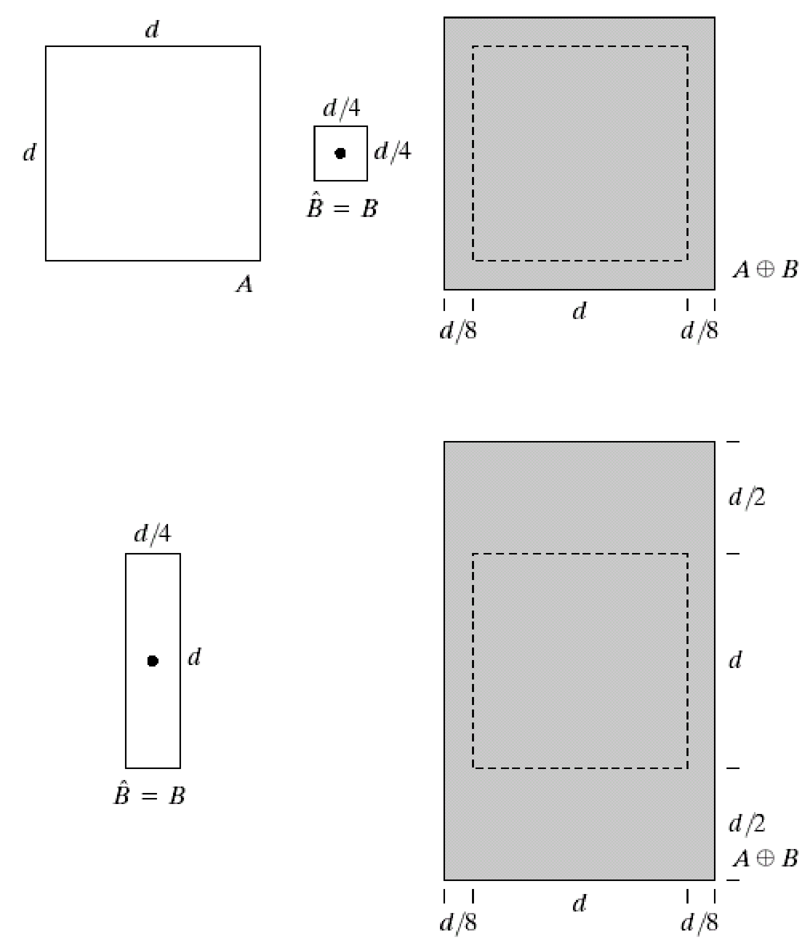
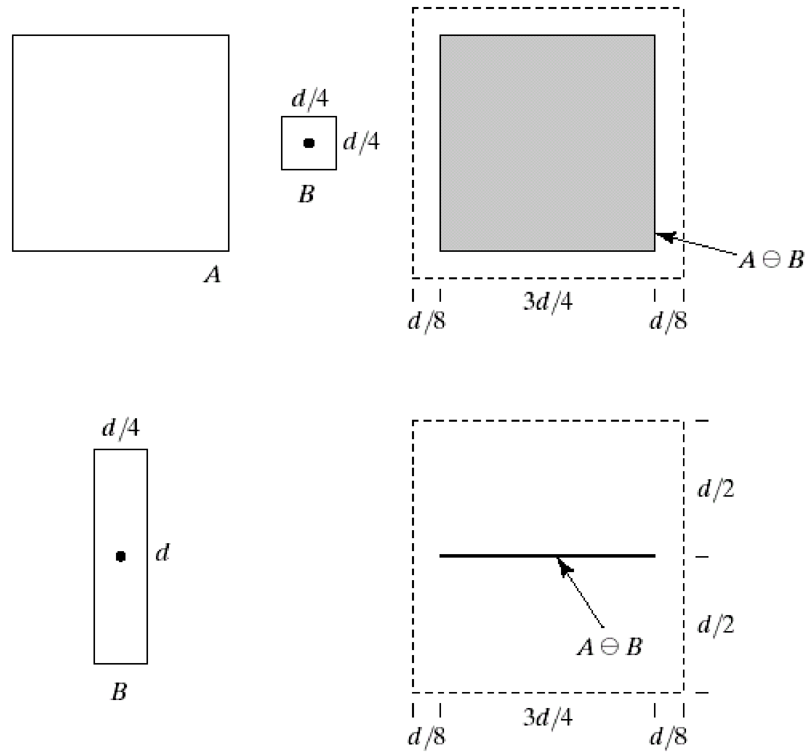



**FIGURE 9.4** (a) Set $A$. (b) Square structuring element (dot is the center). (c) Dilation of $A$ by $B$, shown shaded. (d) Elongated structuring element. (e) Dilation of $A$ using this element.

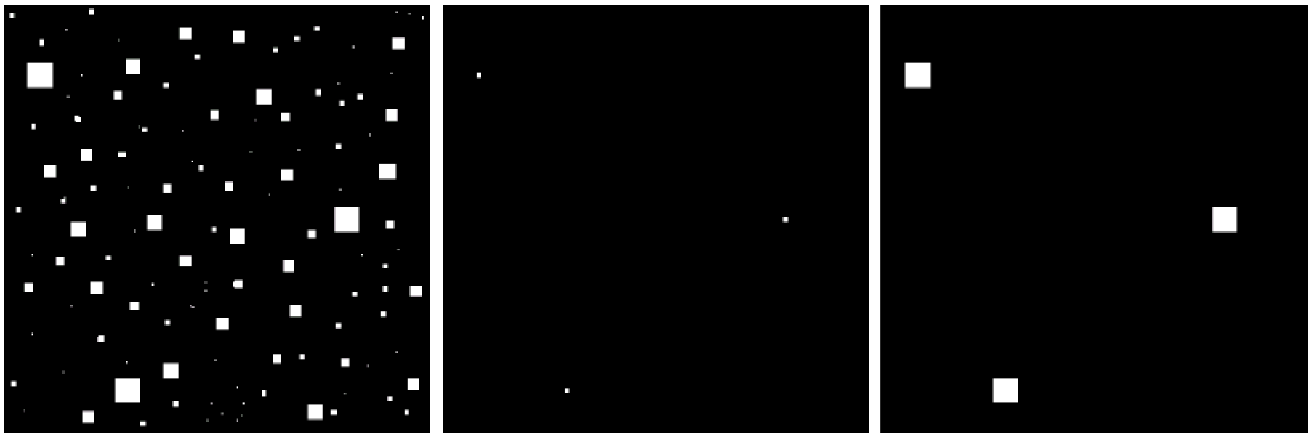
# Morphological Image Processing




a b c
d e

**FIGURE 9.6** (a) Set A. (b) Square structuring element. (c) Erosion of $A$ by $B$, shown shaded. (d) Elongated structuring element. (e) Erosion of $A$ using this element.

# Morphological Image Processing



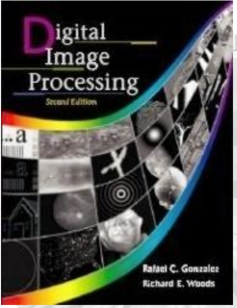
a b c

**FIGURE 9.7** (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

# Morphological Image Processing

**TABLE 9.2** Summary of morphological operations and their properties.

| Operation | Equation | Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26). |
|---|---|---|
| Translation | $(A)_z = \{w \mid w = a + z, \quad \text{for } a \in A\}$ | Translates the origin of $A$ to point $z$. |
| Reflection | $\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$ | Reflects all elements of $B$ about the origin of this set. |
| Complement | $A^c = \{w \mid w \notin A\}$ | Set of points not in $A$. |
| Difference | $A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$ | Set of points that belong to $A$ but not to $B$. |
| Dilation | $A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$ | "Expands" the boundary of $A$. (I) |
| Erosion | $A \ominus B = \{z \mid (B)_z \subseteq A\}$ | "Contracts" the boundary of $A$. (I) |
| Opening | $A \circ B = (A \ominus B) \oplus B$ | Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I) |
| Closing | $A \bullet B = (A \oplus B) \ominus B$ | Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I) |




# Morphological Image Processing

| | | |
|---|---|---|
| Hit-or-miss transform | $A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$ $= (A \ominus B_1) - (A \oplus \hat{B}_2)$ | The set of points (coordinates) at which, simultaneously, $B_1$ found a match ("hit") in $A$ and $B_2$ found a match in $A^c$. |
| Boundary extraction | $\beta(A) = A - (A \ominus B)$ | Set of points on the boundary of set $A$. (I) |
| Region filling | $X_k = (X_{k-1} \oplus B) \cap A^c$; $X_0 = p$ and $k = 1, 2, 3, \ldots$ | Fills a region in $A$, given a point $p$ in the region. (II) |
| Connected components | $X_k = (X_{k-1} \oplus B) \cap A$; $X_0 = p$ and $k = 1, 2, 3, \ldots$ | Finds a connected component $Y$ in $A$, given a point $p$ in $Y$. (I) |
| Convex hull | $X_k^i = (X_{k-1}^i \circledast B^i) \cup A$; $i = 1, 2, 3, 4$; $k = 1, 2, 3, \ldots$; $X_0^i = A$; and $D^i = X_{\text{conv}}^i$. | Finds the convex hull $C(A)$ of set $A$, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. (III) |

**TABLE 9.2** Summary of morphological results and their properties. *(continued)*

# Morphological Image Processing

**TABLE 9.2** Summary of morphological results and their properties. *(continued)*

| Operation | Equation | Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26). |
|---|---|---|
| Thinning | $A \otimes B = A - (A \circledast B)$ <br> $= A \cap (A \circledast B)^c$ <br><br> $A \otimes \{B\} =$ <br> $((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$ <br> $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$ | Thins set $A$. The first two equations give the basic definition of thinning. The last two equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV) |
| Thickening | $A \odot B = A \cup (A \circledast B)$ <br><br> $A \odot \{B\} =$ <br> $((\ldots(A \odot B^1) \odot B^2 \ldots) \odot B^n)$ | Thickens set $A$. (See preceding comments on sequences of structuring elements.) Uses IV with 0's and 1's reversed. |

# Morphological Image Processing

| | | |
|---|---|---|
| Skeletons | $S(A) = \bigcup_{k=0} S_k(A)$<br>$S_k(A) = \bigcup_{k=0}^{K} \{(A \ominus kB) - [(A \ominus kB) \circ B]\}$<br>Reconstruction of $A$:<br>$A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$ | Finds the skeleton $S(A)$ of set $A$. The last equation indicates that $A$ can be reconstructed from its skeleton subsets $S_k(A)$. In all three equations, $K$ is the value of the iterative step after which the set $A$ erodes to the empty set. The notation $(A \ominus kB)$ denotes the $k$th iteration of successive erosion of $A$ by $B$. (I) |
| Pruning | $X_1 = A \otimes \{B\}$<br>$X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$<br>$X_3 = (X_2 \oplus H) \cap A$<br>$X_4 = X_1 \cup X_3$ | $X_4$ is the result of pruning set $A$. The number of times that the first equation is applied to obtain $X_1$ must be specified. Structuring elements V are used for the first two equations. In the third equation $H$ denotes structuring element I. |

**TABLE 9.2** Summary of morphological results and their properties. *(continued)*