Image
Spatial Domain Analysis

Lecture 5

Spring 2019

Prof. Megherbi

- Spatial Domain Processing
    1. masking and convolution
    2. Low and high pass spatial filters
    3. Derivative filters
- The relationship between spatial and frequency filters
- Miscellaneous operations
    1. simple intensity transform
    2. Histogram equalization
    3. Image subtraction
    4. Image averaging

Spatial domain processing

- Take notes

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

Figure 4.20 *A 3 × 3 mask with arbitrary coefficients (weights).*

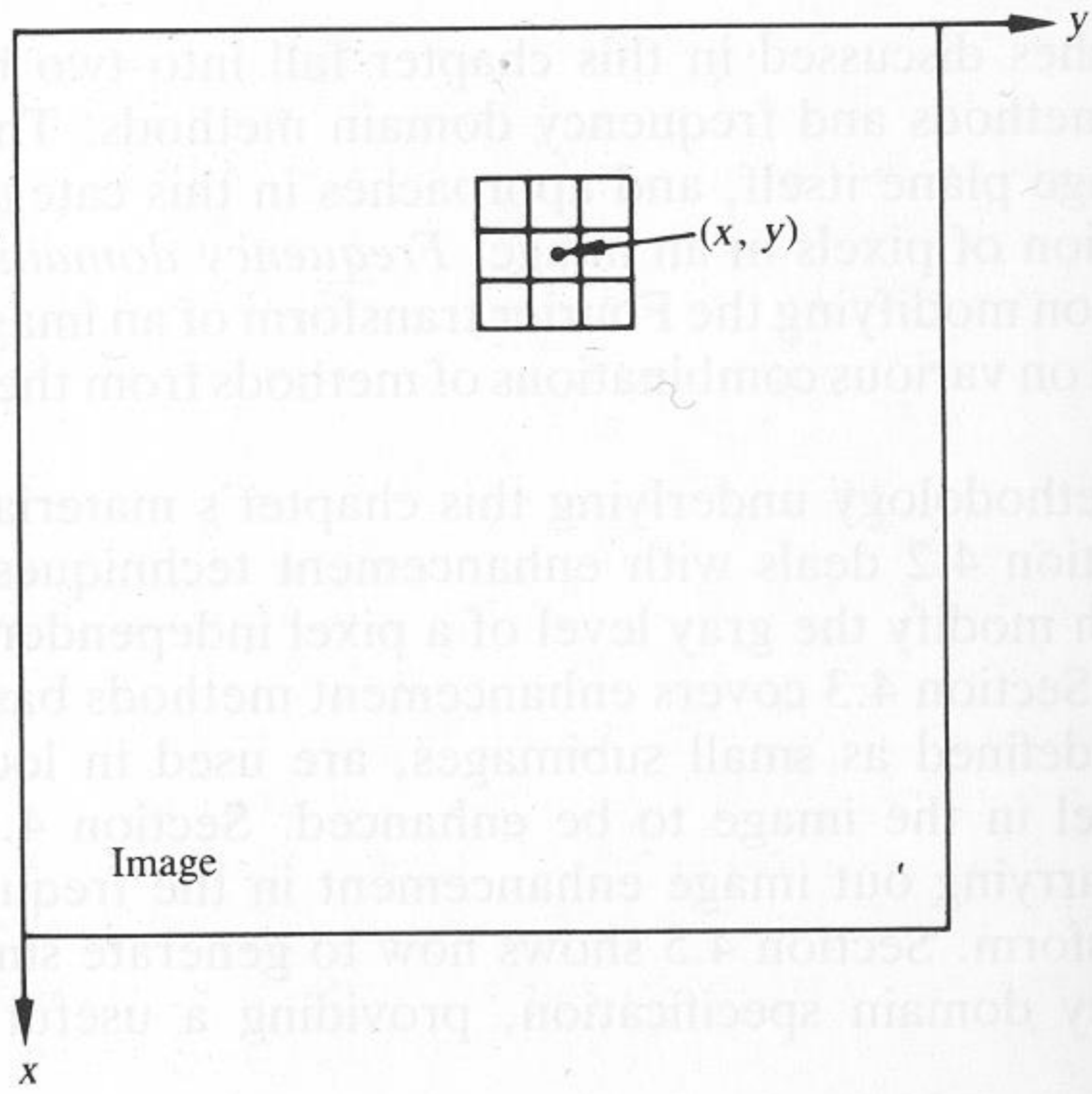

Figure 4.1 *A 3 × 3 neighborhood about a point (x, y) in an image.*

Examples of masks

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Low pass filters

- take notes

$$\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(a)

$$\frac{1}{25} \times \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

(b)

$$\frac{1}{49} \times \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

(c)

Figure 4.21 *Spatial lowpass filters of various sizes.*

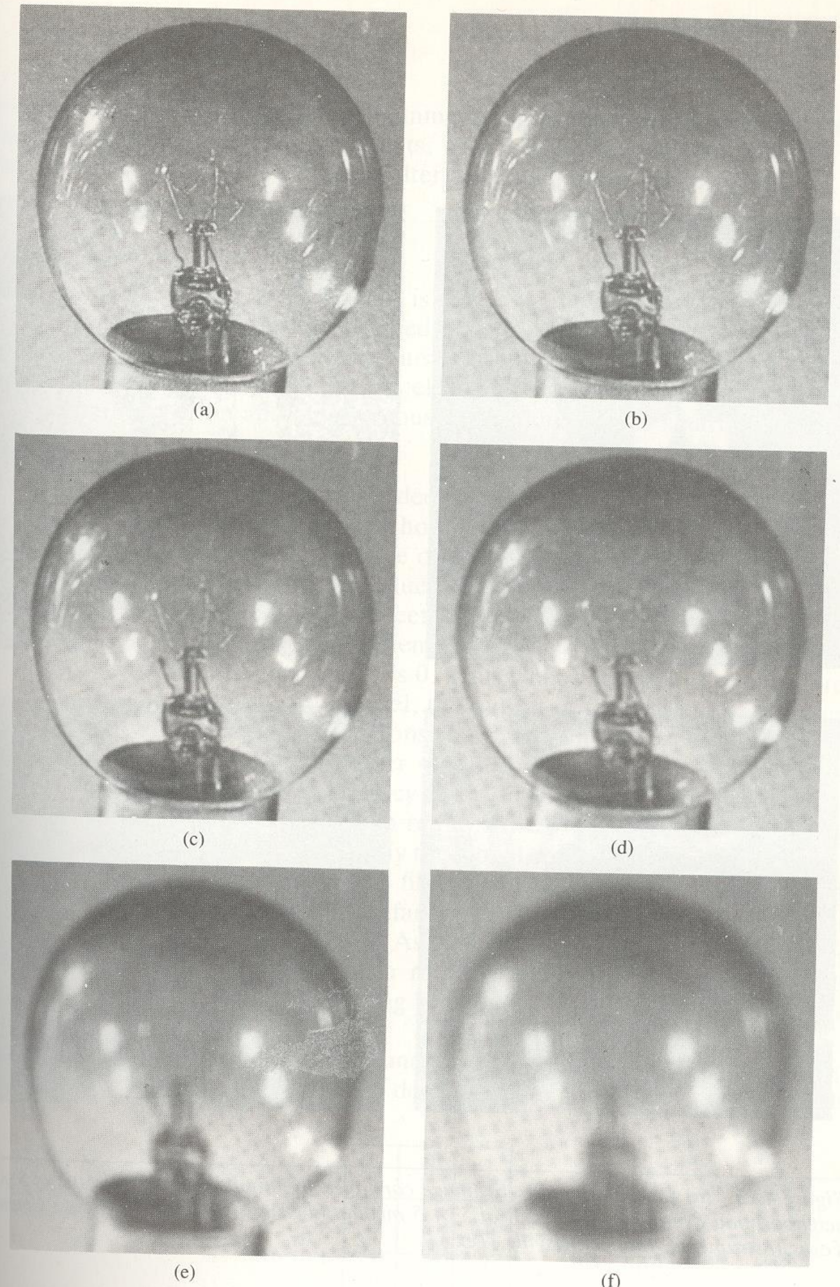
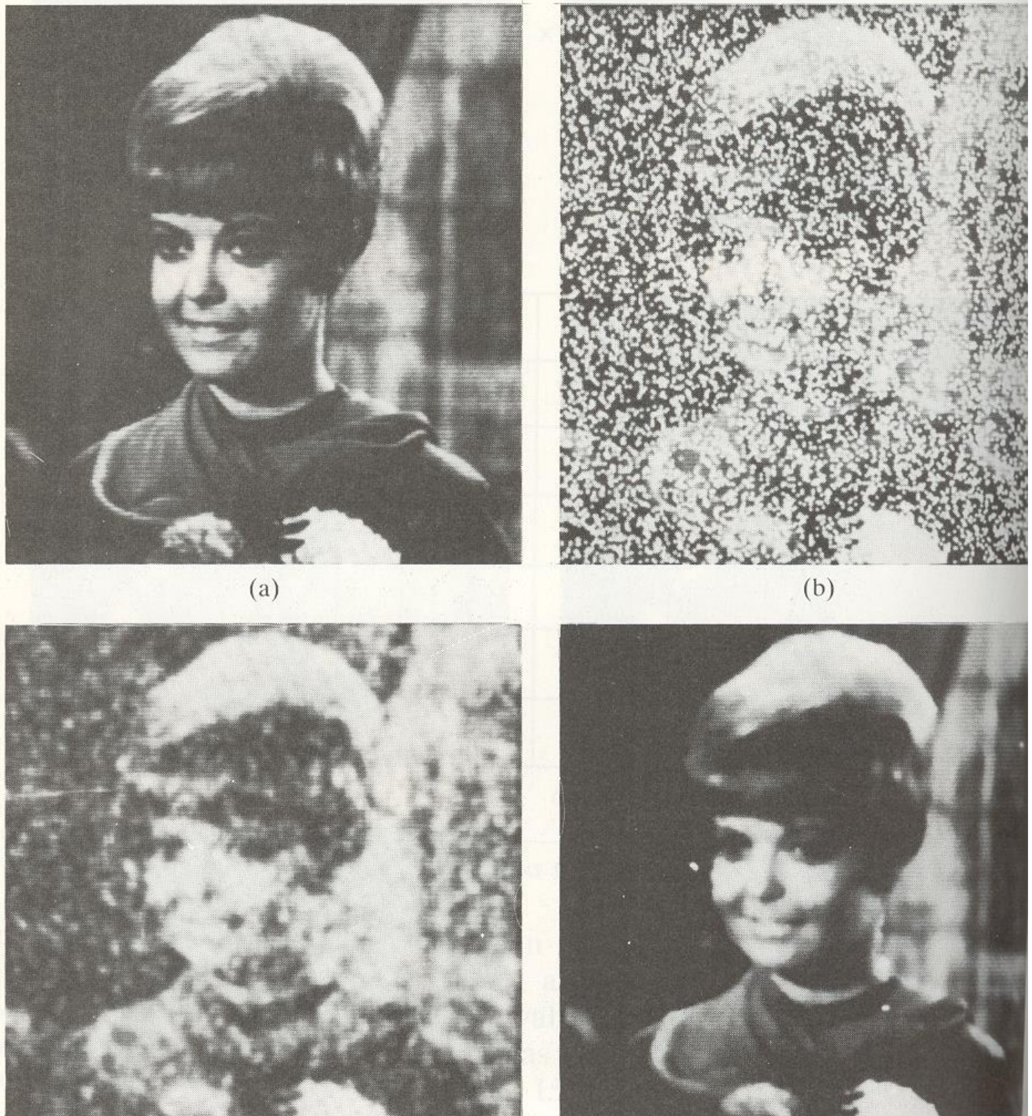


Figure 4.22 *(a) Original image; (b)–(f) results of spatial lowpass filtering with a mask of size $n \times n$, $n = 3, 5, 7, 15, 25$.*



(a) (b)

(c) (d)

Figure 4.23 *(a) Original image; (b) image corrupted by impulse noise; (c) result of 5 × 5 neighborhood averaging; (d) result of 5 × 5 median filtering. (Courtesy of Martin Connor, Texas Instruments, Inc., Lewisville, Tex.)*

High pass filters

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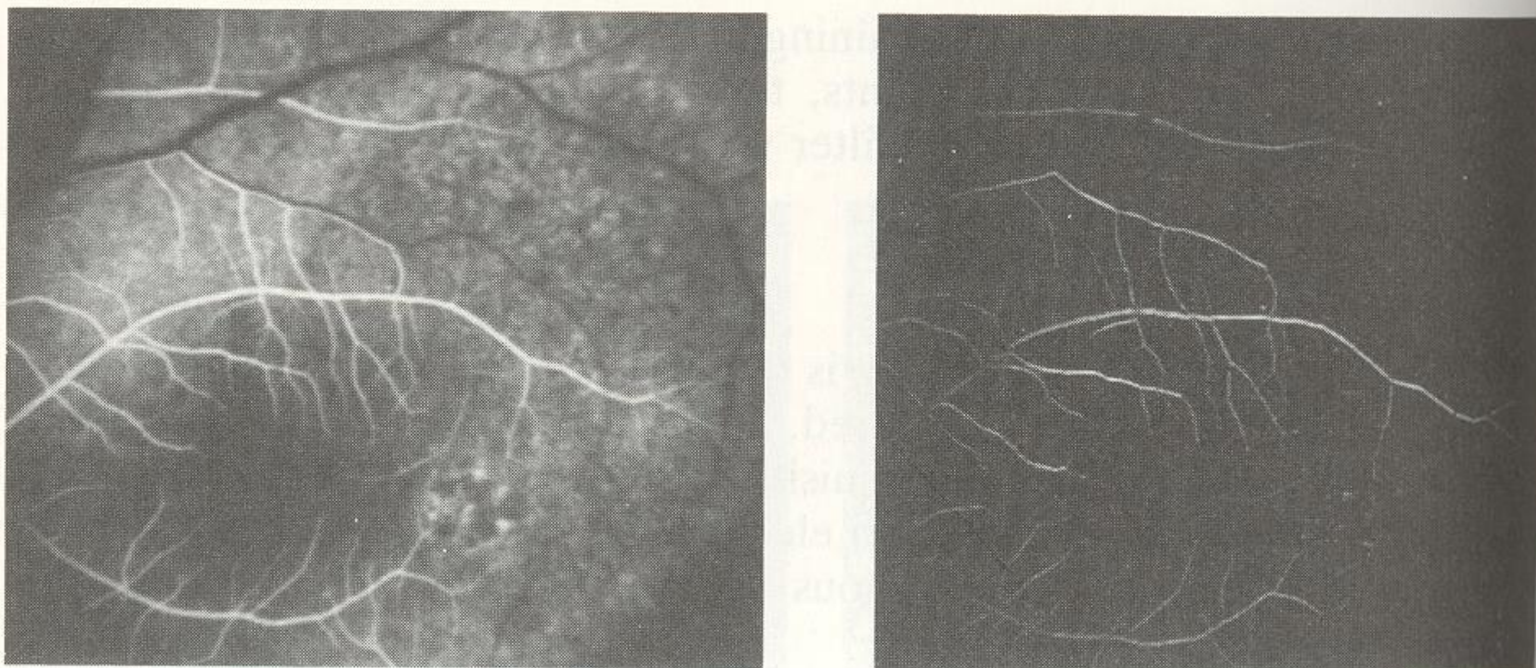
$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline -1 & -1 & -1 \\ \hline -1 & w & -1 \\ \hline -1 & -1 & -1 \\ \hline \end{array}$$

Figure 4.26 *Mask used for high-boost spatial filtering. The value of the center weight is* $w = 9A - 1$, *with* $A \geq 1$.

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline -1 & -1 & -1 \\ \hline -1 & 8 & -1 \\ \hline -1 & -1 & -1 \\ \hline \end{array}$$

Figure 4.24 *A basic highpass spatial filter.*

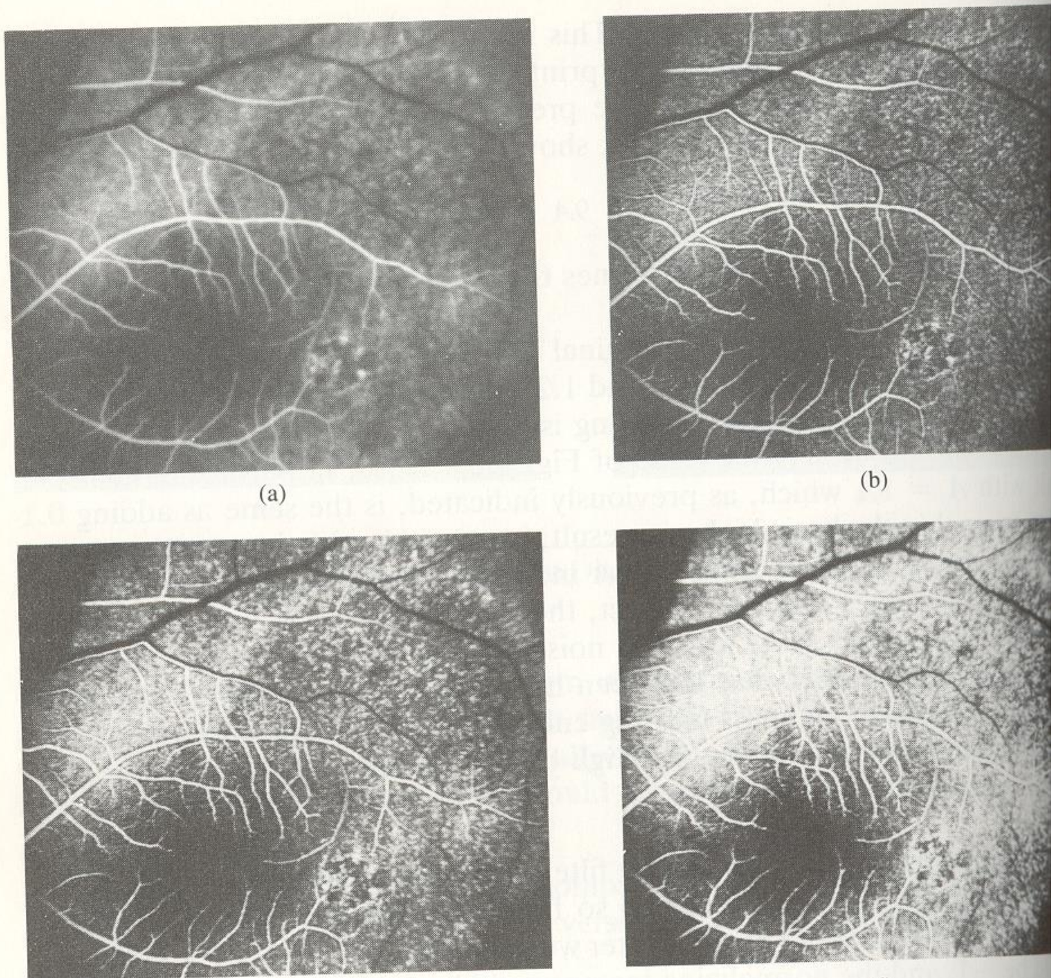


(a) (b)

Figure 4.25 *(a) Image of a human retina; (b) highpass filtered result using the mask in Fig. 4.24.*

High-boost filtering

- take notes



(a) (b) (c) (d)

Figure 4.27 *(a) Original image; (b)–(d) result of high-boost filtering using the mask in 4.26, with A = 1.1, 1.15, and 1.2, respectively. Compare these results with those show Fig. 4.25.*

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

(a)

1	0
0	−1

0	1
−1	0

(b) Roberts

−1	−1	−1
0	0	0
1	1	1

−1	0	1
−1	0	1
−1	0	1

(c) Prewitt

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

(d) Sobel

Figure 4.28 *A 3 × 3 region of an image (the z's are gray-level values) and various ma used to compute the derivative at point labeled z_5. Note that all mask coefficients sum t indicating a response of 0 in constant areas, as expected of a derivative operator.*

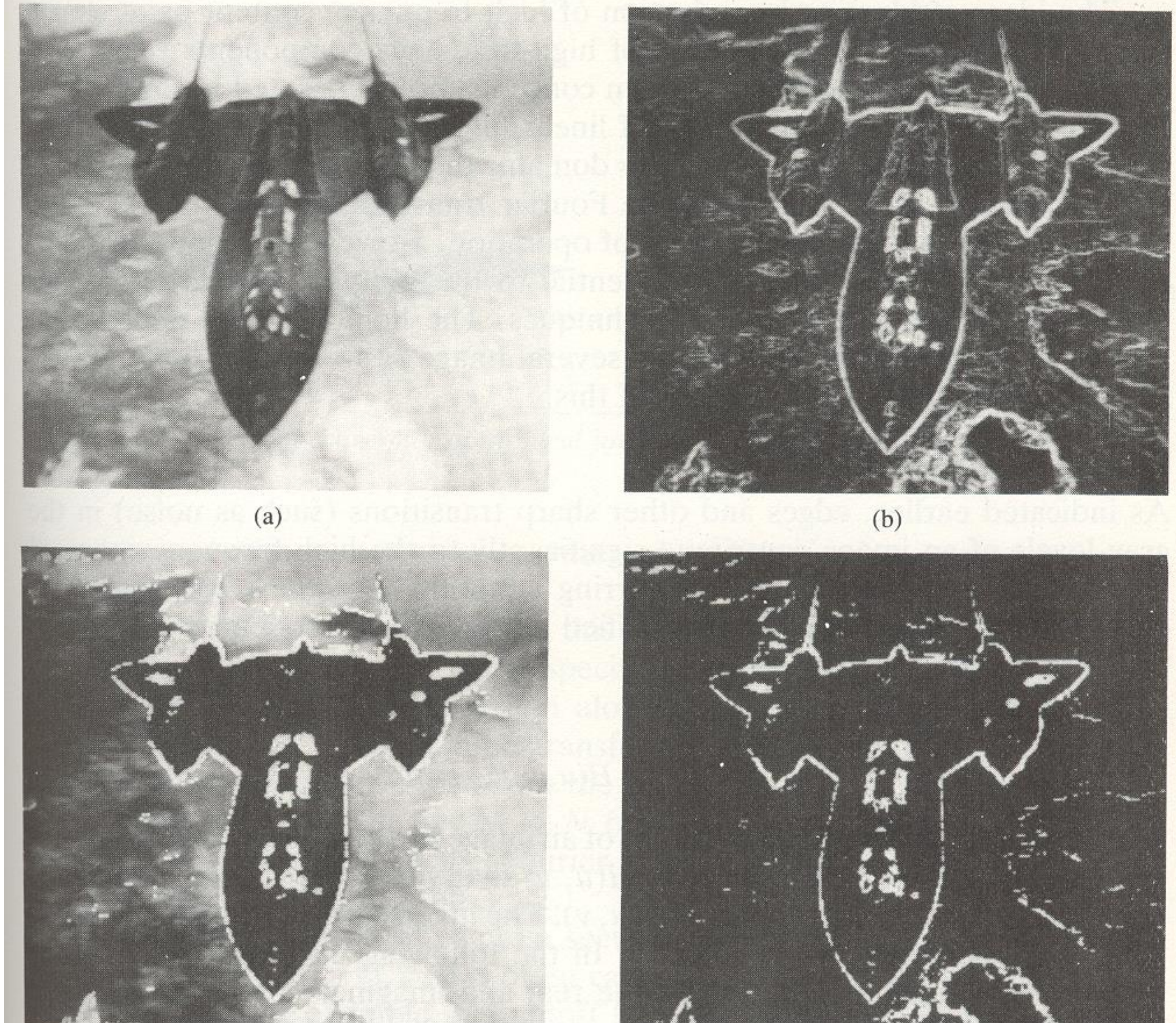

(a) (b) (c) (d)

Figure 4.29 Edge enhancement by gradient techniques (see text).

Derivative filters

- Take notes

−1	−1	−1
−1	8	−1
−1	−1	−1

Figure 7.2 *A mask used for detecting isolated points different from a constant background.*

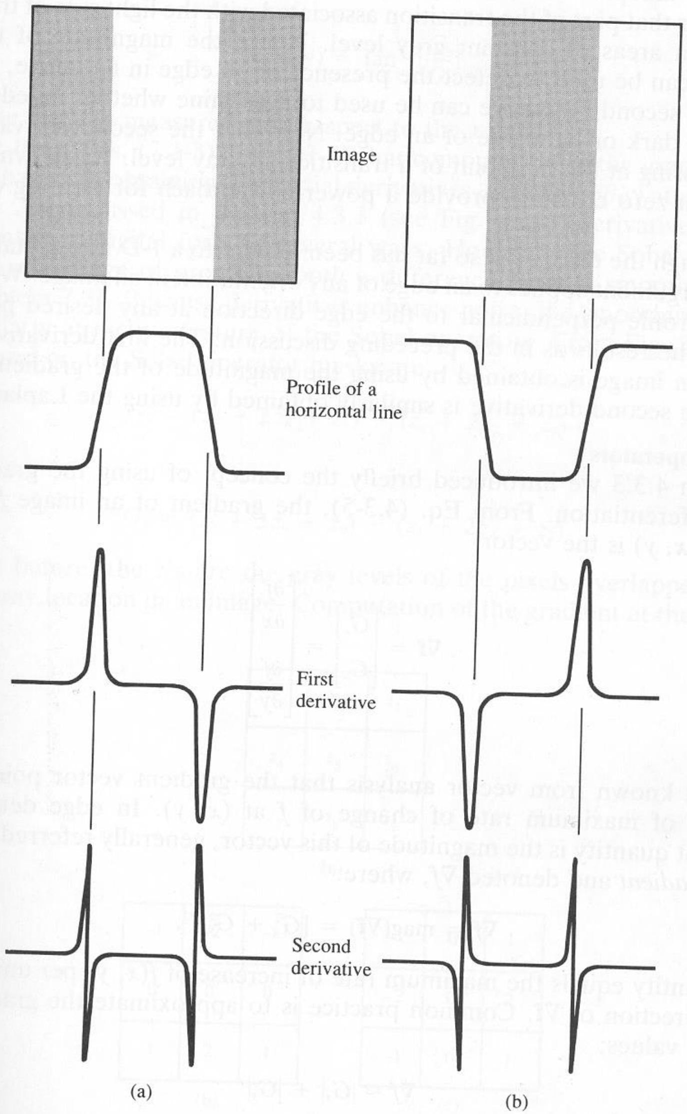


Figure 7.4 *Edge detection by derivative operators: (a) light stripe on a dark background; (b) dark stripe on a light background. Note that the second derivative has a zero crossing at the location of each edge.*

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

(a)

−1	−2	−1
0	0	0
1	2	1

(b)

−1	0	1
−2	0	2
−1	0	1

(c)

Figure 7.5 *(a) 3 × 3 image region; (b) mask used to compute G_x at center point of the 3 × 3 region; (c) mask used to compute G_y at that point. These masks are often referred to as the Sobel operators.*

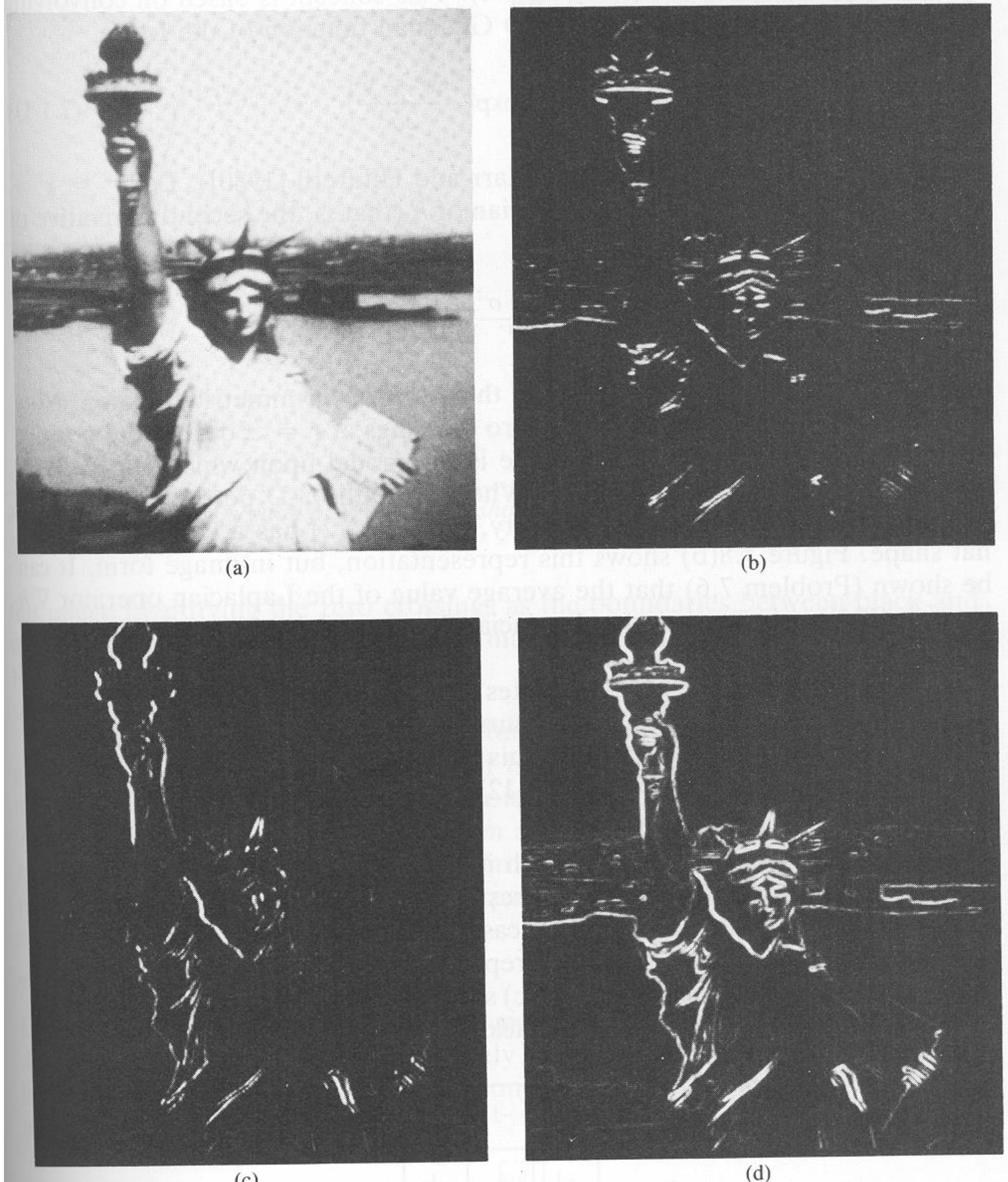


Figure 7.6 *(a) Original image; (b) result of applying the mask in Fig. 7.5(b) to obtain G_x; (c) result of using the mask in Fig. 7.5(c) to obtain G_y; (d) complete gradient image obtained by using Eq. (7.1-5).*

0	−1	0
−1	4	−1
0	−1	0

Figure 7.7 *Mask used to compute the Laplacian.*

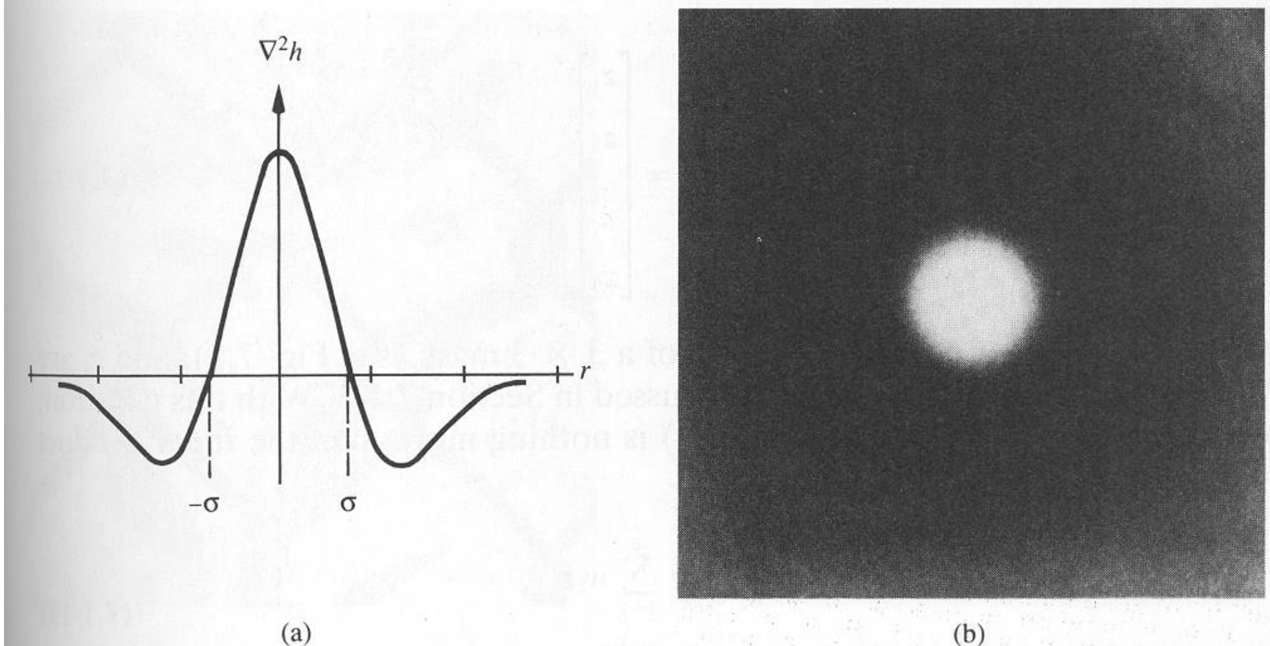


Figure 7.8 (a) Cross section of $\nabla^2 h$; (b) $\nabla^2 h$ shown as an intensity function (image). (From Marr [1982].)

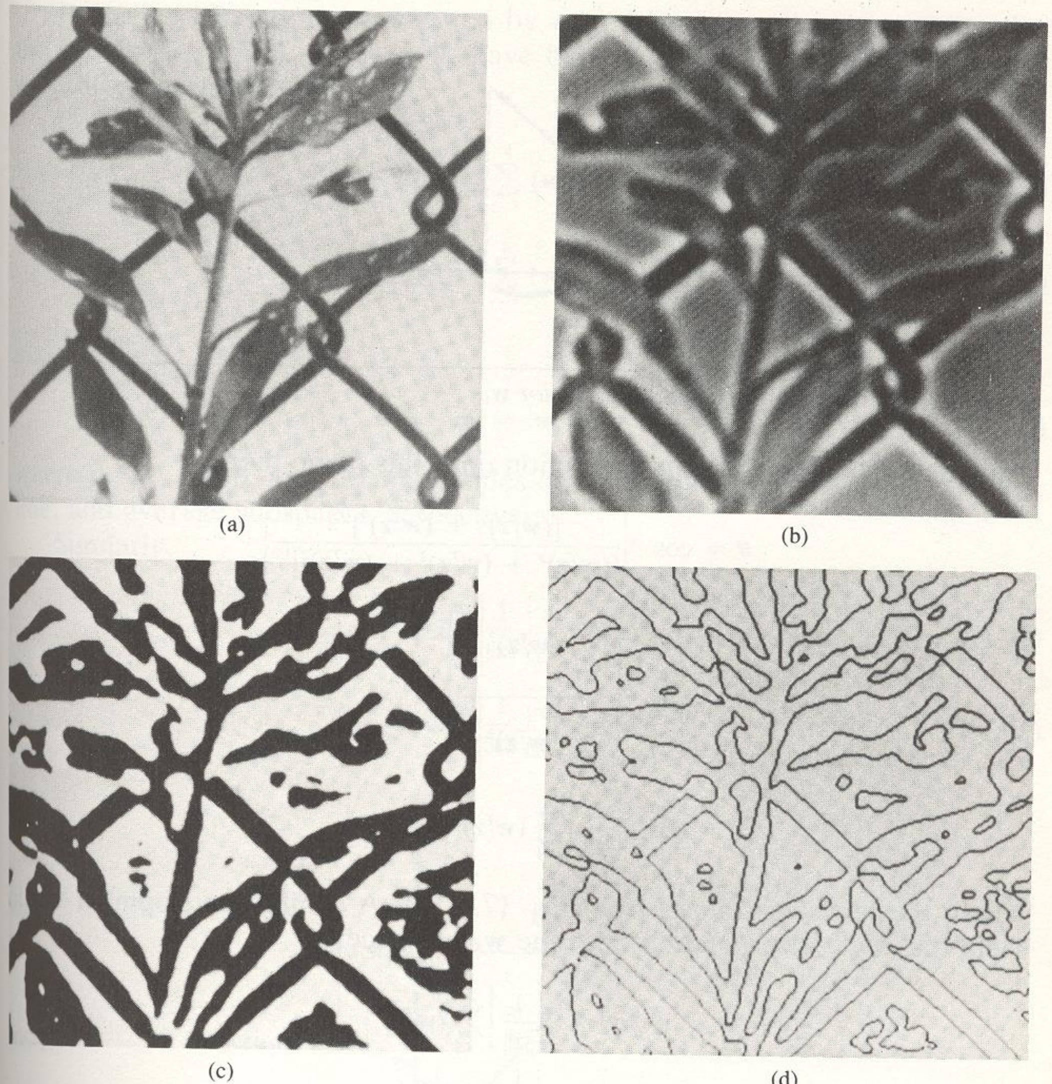


Figure 7.9 (a) Original image; (b) result of convolving (a) with $\nabla^2 h$; (c) result of making (b) binary to simplify detection of zero crossings; (d) zero crossings. (From Marr [1982].)

$$\begin{array}{|c|c|c|}\hline 1 & \sqrt{2} & 1 \\ \hline 0 & 0 & 0 \\ \hline -1 & -\sqrt{2} & -1 \\ \hline \end{array} \qquad \begin{array}{|c|c|c|}\hline 1 & 0 & -1 \\ \hline \sqrt{2} & 0 & -\sqrt{2} \\ \hline 1 & 0 & -1 \\ \hline \end{array}$$

$\mathbf{w}_1$ $\quad$ $\mathbf{w}_2$

$$\begin{array}{|c|c|c|}\hline 0 & -1 & \sqrt{2} \\ \hline 1 & 0 & -1 \\ \hline -\sqrt{2} & 1 & 0 \\ \hline \end{array} \qquad \begin{array}{|c|c|c|}\hline \sqrt{2} & -1 & 0 \\ \hline -1 & 0 & 1 \\ \hline 0 & 1 & -\sqrt{2} \\ \hline \end{array}$$

$\mathbf{w}_3$ $\quad$ $\mathbf{w}_4$

Basis of edge subspace ($\mathbf{w}_1$–$\mathbf{w}_4$)

$$\begin{array}{|c|c|c|}\hline 0 & 1 & 0 \\ \hline -1 & 0 & -1 \\ \hline 0 & 1 & 0 \\ \hline \end{array} \qquad \begin{array}{|c|c|c|}\hline -1 & 0 & 1 \\ \hline 0 & 0 & 0 \\ \hline 1 & 0 & -1 \\ \hline \end{array}$$

$\mathbf{w}_5$ $\quad$ $\mathbf{w}_6$

$$\begin{array}{|c|c|c|}\hline 1 & -2 & 1 \\ \hline -2 & 4 & -2 \\ \hline 1 & -2 & 1 \\ \hline \end{array} \qquad \begin{array}{|c|c|c|}\hline -2 & 1 & -2 \\ \hline 1 & 4 & 1 \\ \hline -2 & 1 & -2 \\ \hline \end{array}$$

$\mathbf{w}_7$ $\quad$ $\mathbf{w}_8$

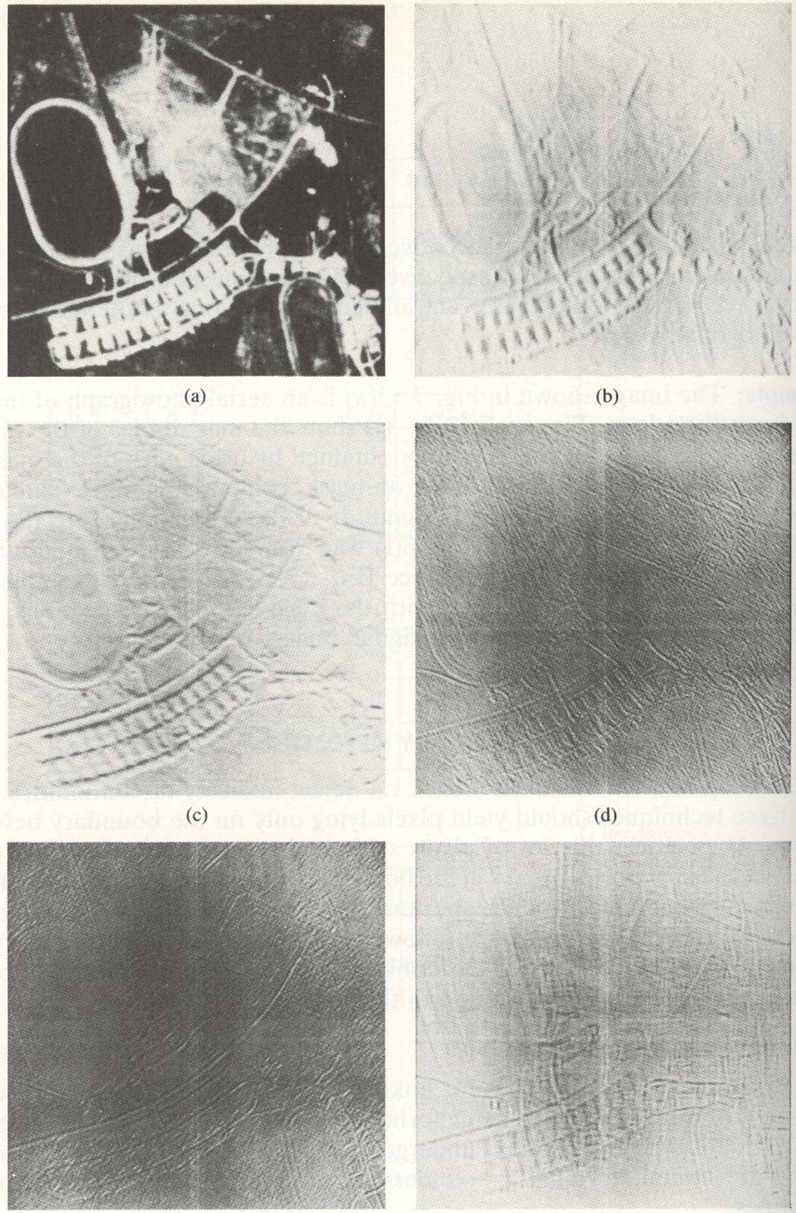
Basis of line subspace ($\mathbf{w}_5$–$\mathbf{w}_8$)

$$\begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

$\mathbf{w}_9$

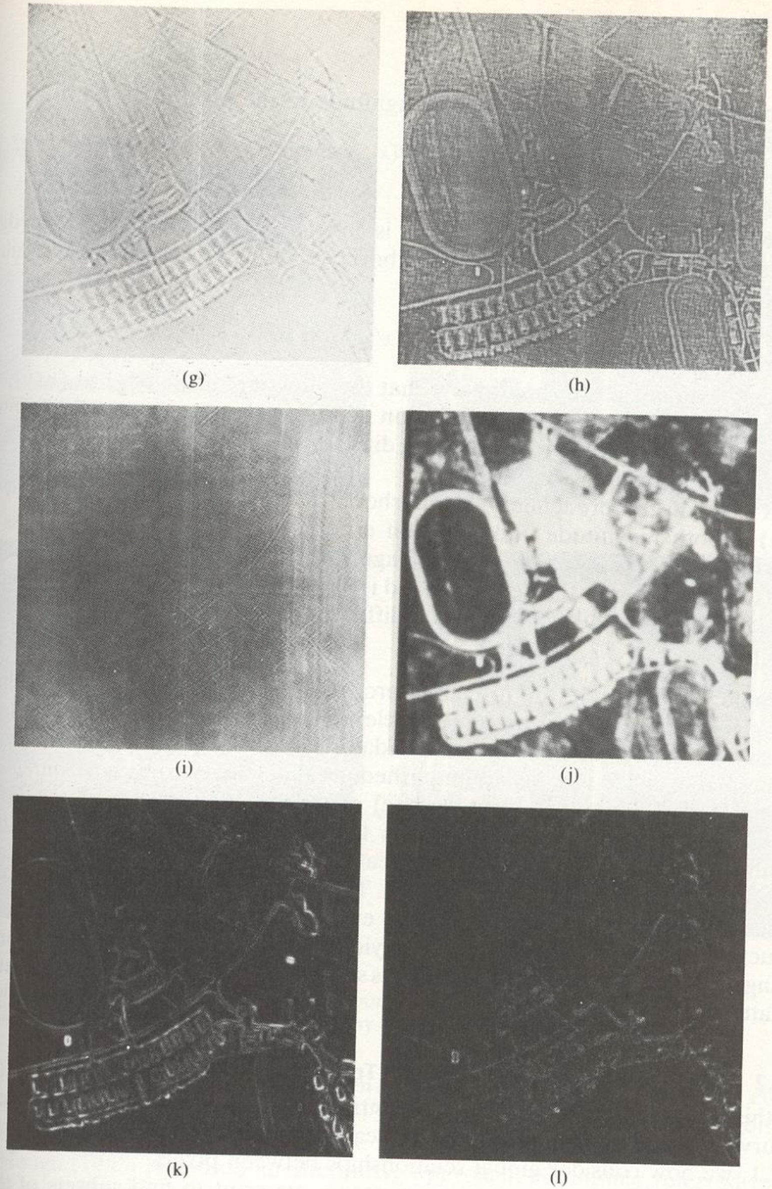
"Average" subspace

Figure 7.12 *Orthogonal masks (the* **w**'s *as shown are not normalized). (From Frei and Chen [1977].)*



(a) (b) (c) (d) (e) (f)

Figure 7.13 *(a) Original image; (b)–(f) projections onto* $\mathbf{w}_1$, $\mathbf{w}_2$, $\mathbf{w}_3$, $\mathbf{w}_4$, *and* $\mathbf{w}_5$ *subspaces, respectively. (From Hall and Frei [1976].)*



(g) (h) (i) (j) (k) (l)

Figure 7.13 (Continued) (g)–(j) projections onto $\mathbf{w}_6$, $\mathbf{w}_7$, $\mathbf{w}_8$, and $\mathbf{w}_9$ subspaces; (k) magnitude of projection onto edge subspace; (l) magnitude of projection onto line subspace. (From Hall and Frei [1976].)

Relation between spatial and frequency filters

- Take notes

$$R = w_1 z_1 + w_2 z_2 + \cdots + w_9 z_9$$
$$= \sum_{i=1}^{9} w_i z_i$$

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

Figure 7.1 *A general 3 × 3 mask.*

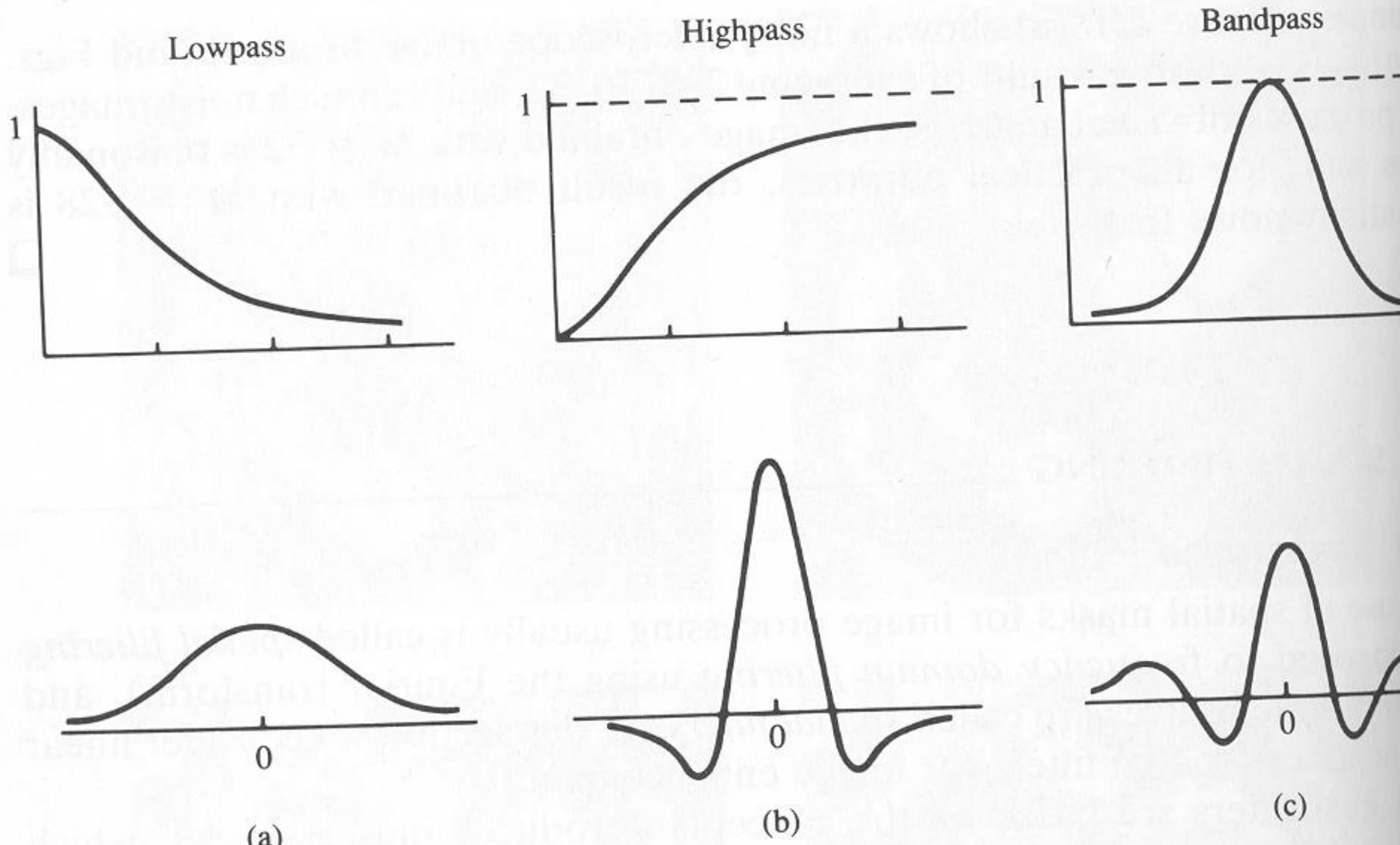
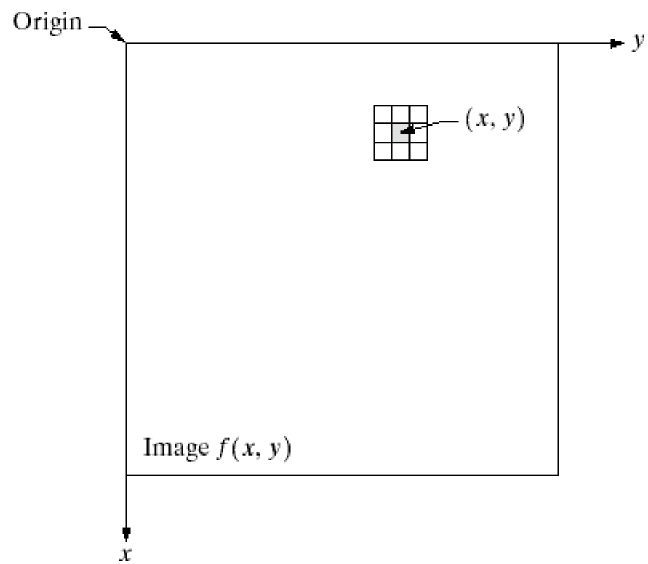


Figure 4.19 *Top: cross sections of basic shapes for circularly symmetric frequency dom*
filters. Bottom: cross sections of corresponding spatial domain filters.

Miscellaneous Operations in the spatial domain: Image Enhancement

- Simple intensity transform

FIGURE 3.1 A 3×3 neighborhood about a point (x, y) in an image.



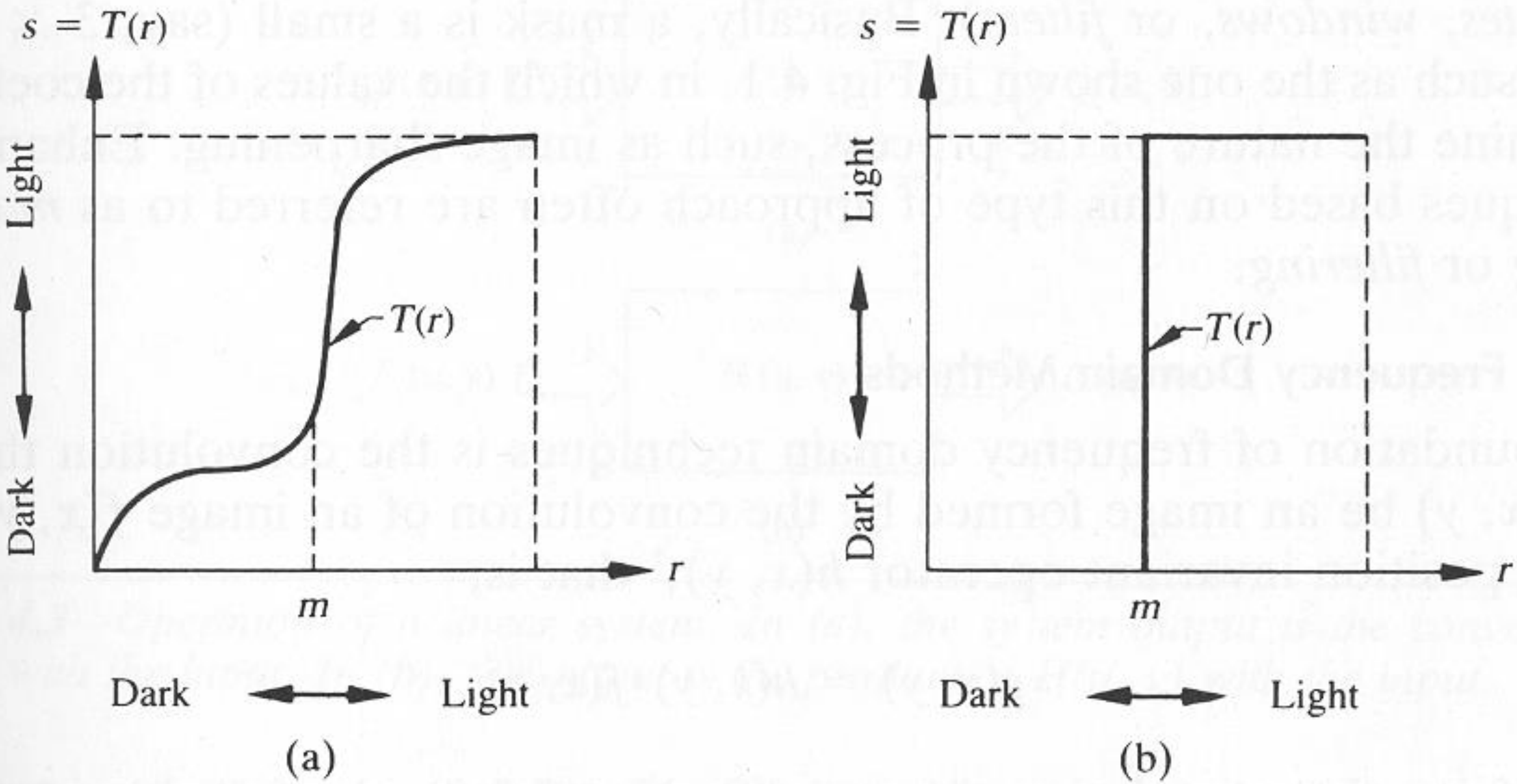


Figure 4.2 Gray-level transformation functions for contrast enhancement.

Simple intensity transforms

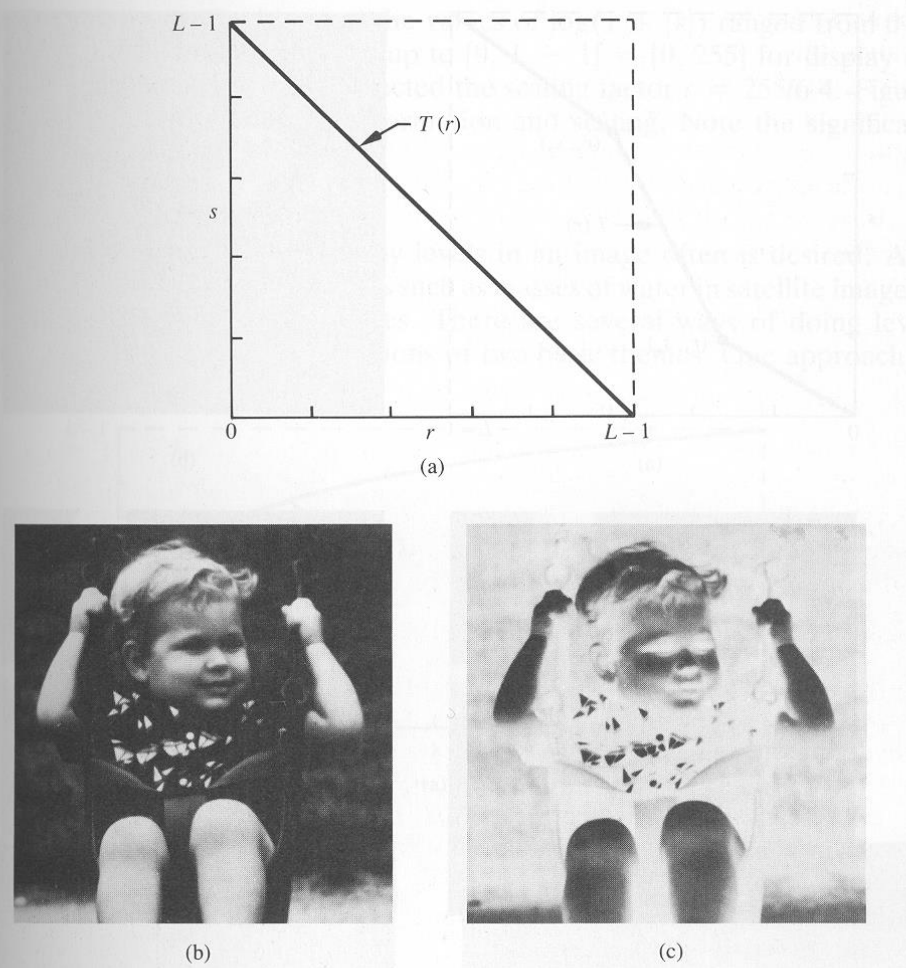
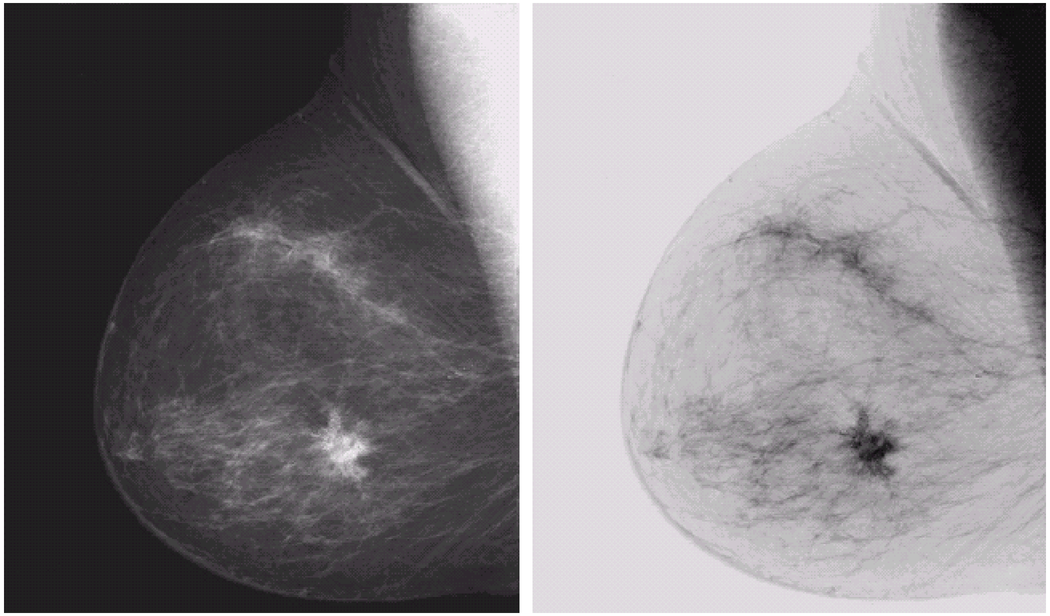


Figure 4.4 Obtaining the negative of an image: (a) gray-level transformation function; (b) an image; and (c) its negative. In (a), r and s denote the input and output gray levels, respectively.



a b

FIGURE 3.4 (a) Original digital mammogram. (b) Negative image obtained using the negative transformation in Eq. (3.2-1). (Courtesy of G.E. Medical Systems.)

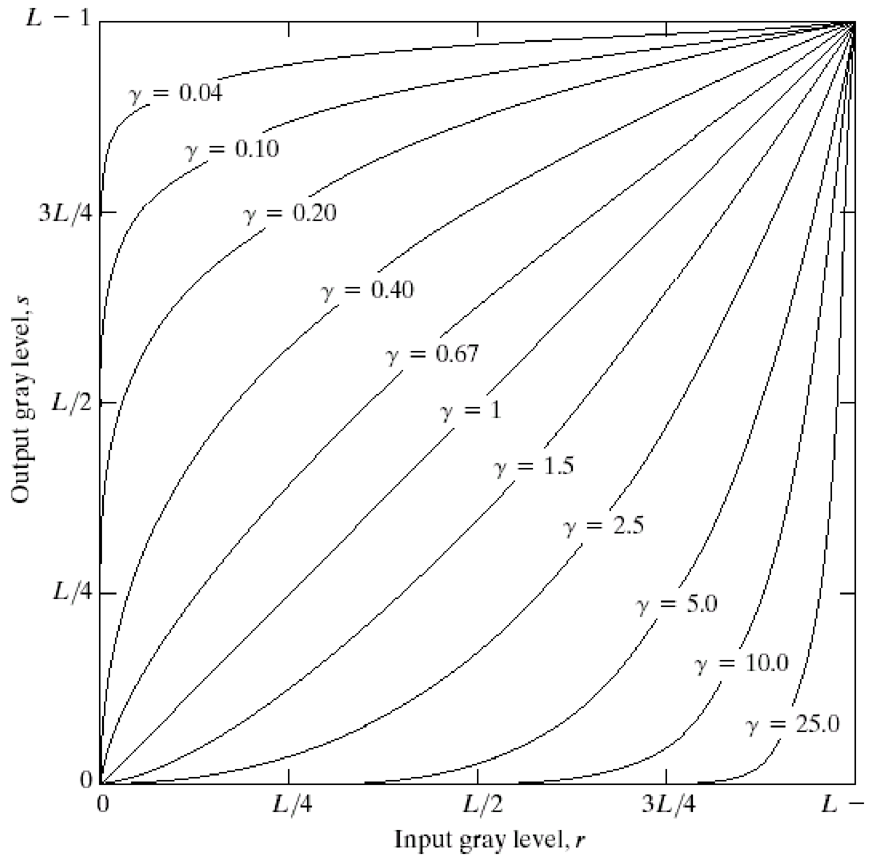


FIGURE 3.6 Plots of the equation $s = cr^{\gamma}$ for various values of γ ($c = 1$ in all cases).

a b
c d

FIGURE 3.7
(a) Linear-wedge gray-scale image.
(b) Response of monitor to linear wedge.
(c) Gamma-corrected wedge.
(d) Output of monitor.

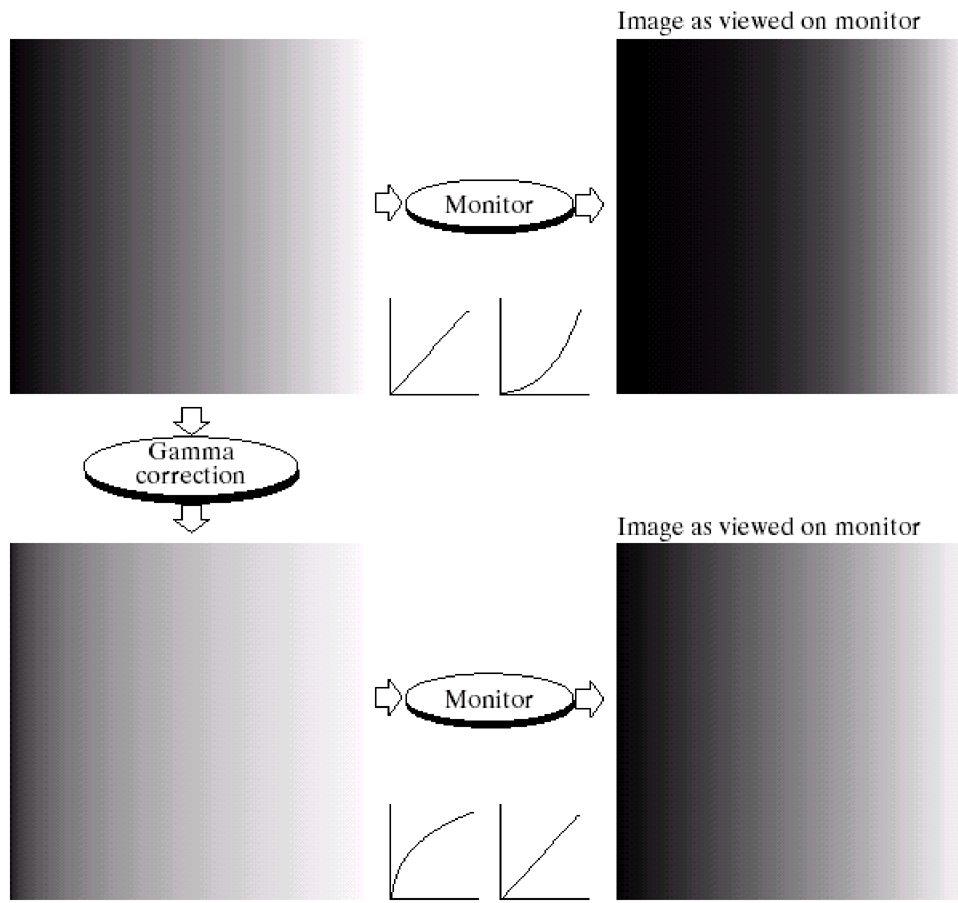
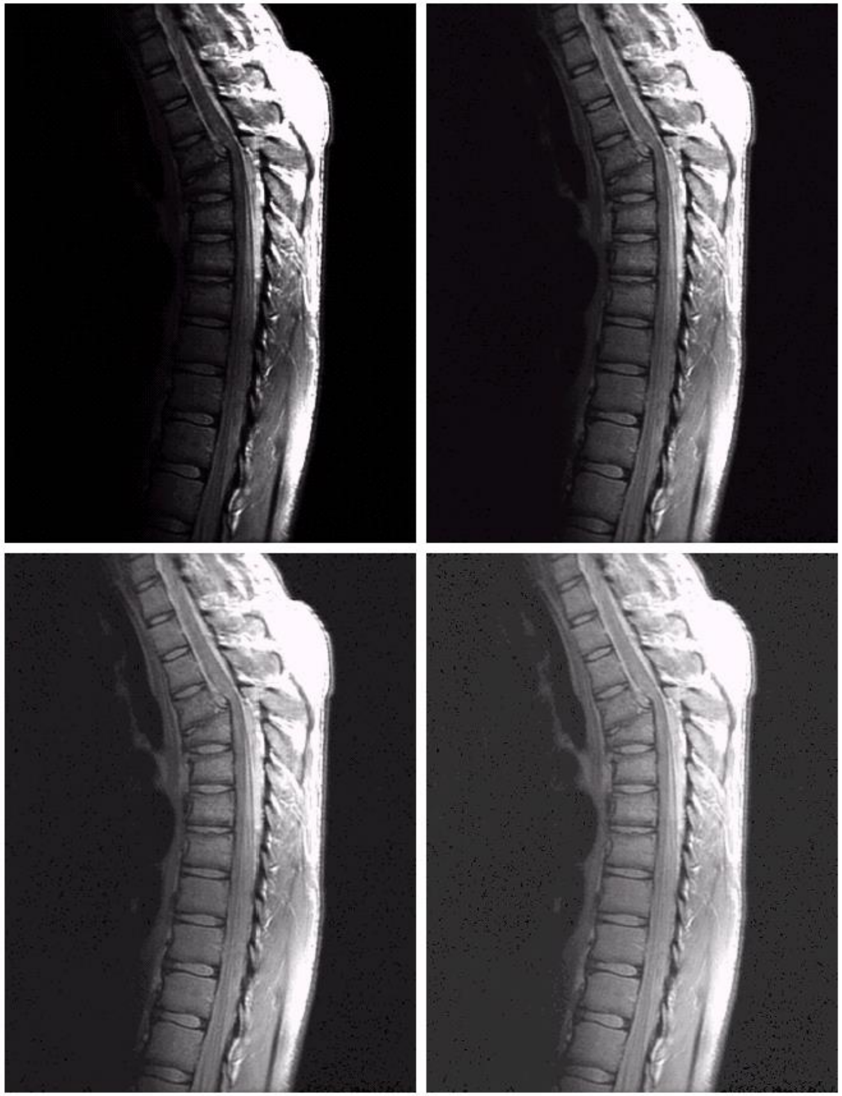


Image Enhancement in the Spatial Domain



a b
c d

FIGURE 3.8 (a) Magnetic resonance (MR) image of a fractured human spine. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 0.6, 0.4,$ and 0.3, respectively. (Original image for this example courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

a	b
c	d

FIGURE 3.9 (a) Aerial image. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 3.0, 4.0,$ and 5.0, respectively. (Original image for this example courtesy of NASA.)

Contrast stretching

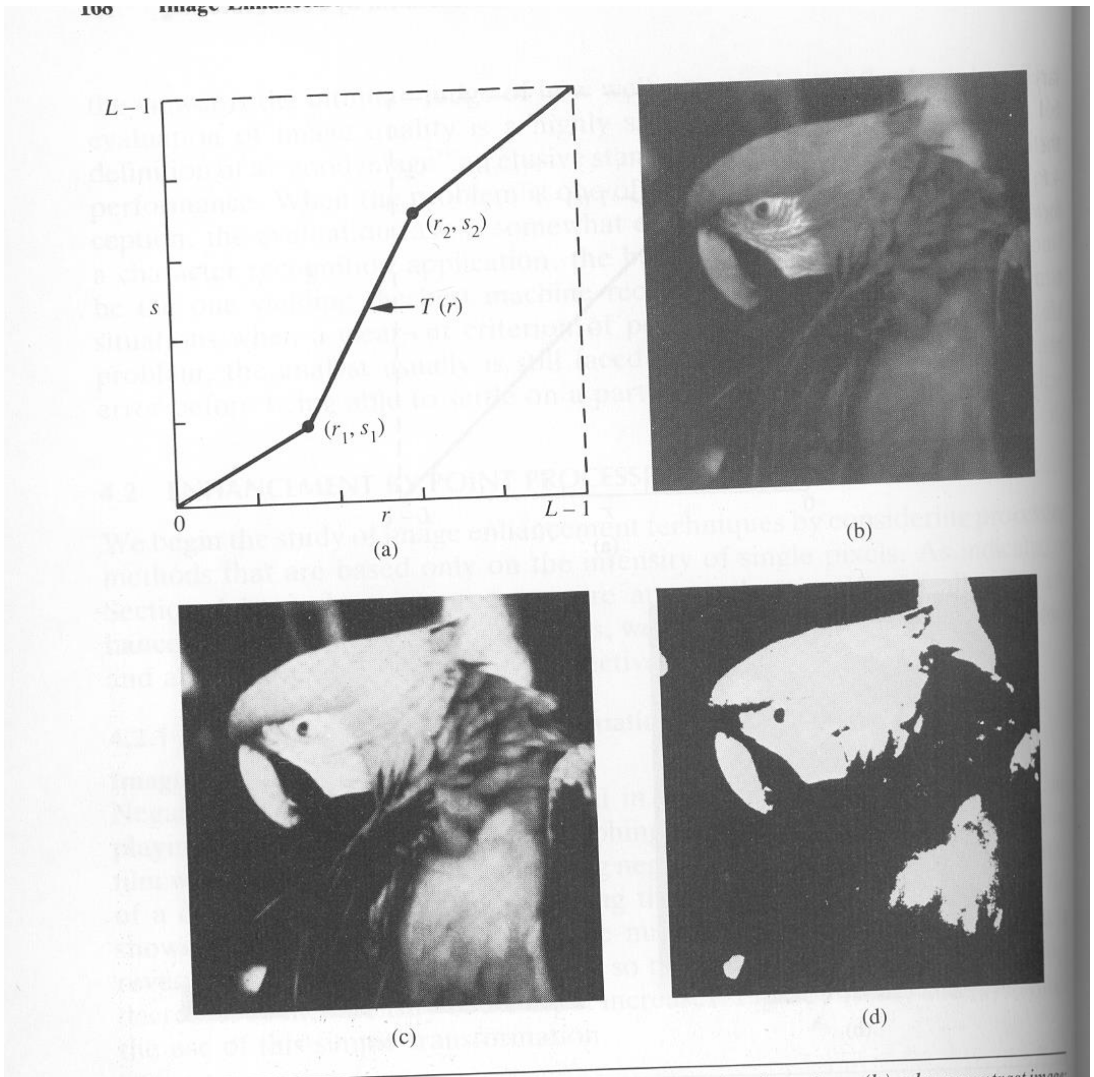
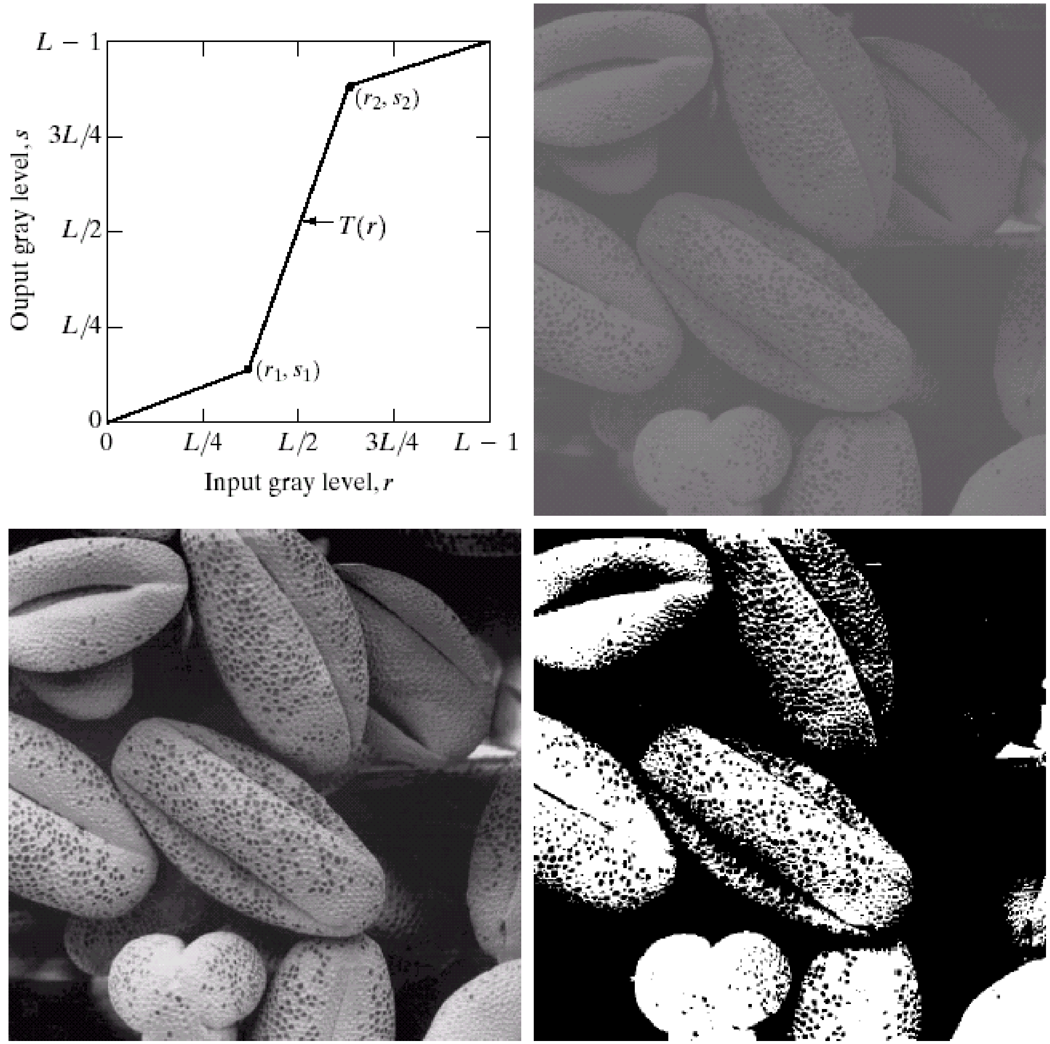
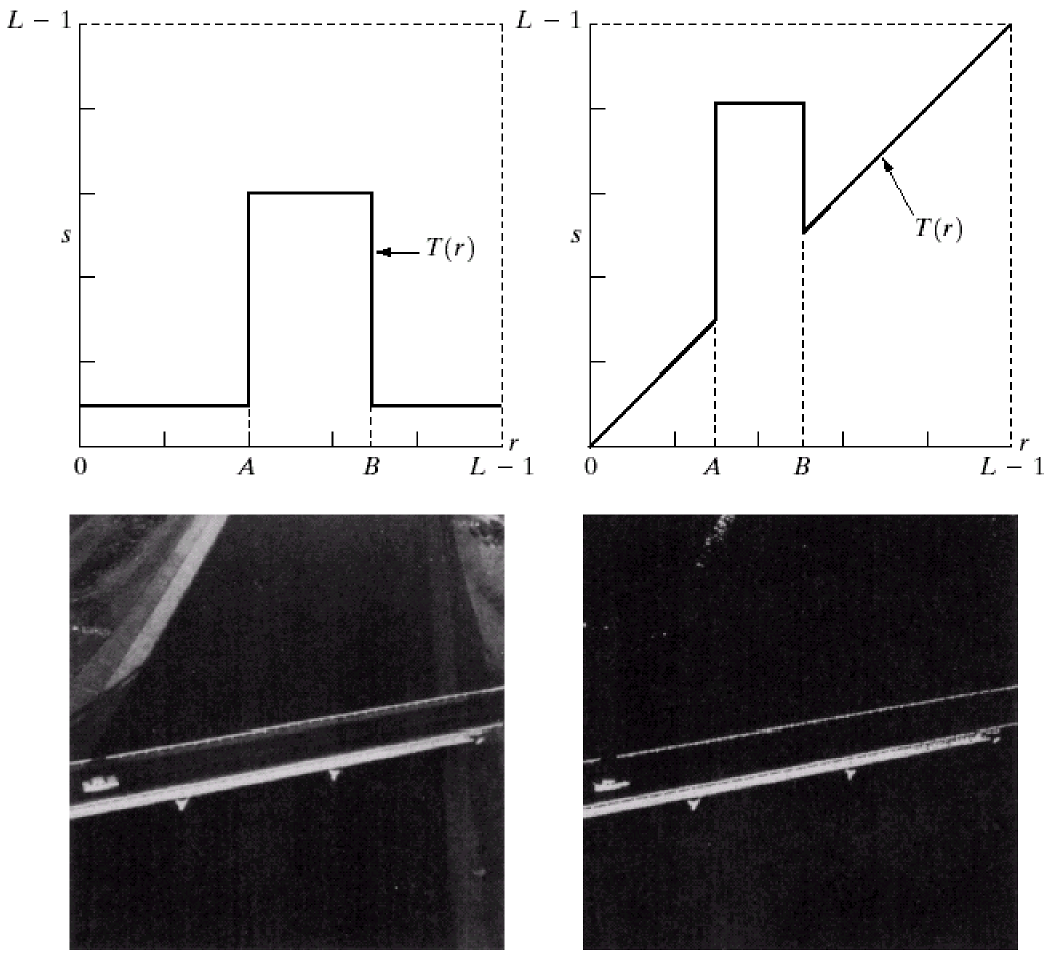


Figure 4.5 *Contrast stretching: (a) form of transformation function; (b) a low-contrast image; (c) result of contrast stretching; (d) result of thresholding.*



a b
c d

FIGURE 3.10 Contrast stretching. (a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)



a	b
c	d

FIGURE 3.11 (a) This transformation highlights range $[A, B]$ of gray levels and reduces all others to a constant level. (b) This transformation highlights range $[A, B]$ but preserves all other levels. (c) An image. (d) Result of using the transformation in (a).

Histogram equalization

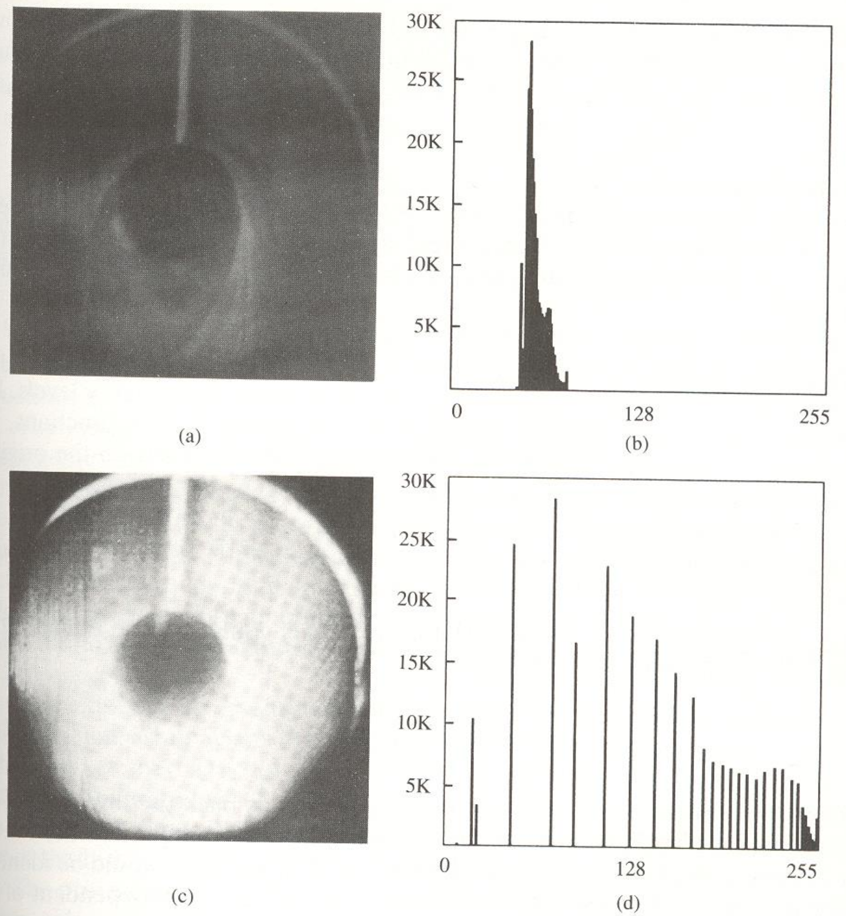
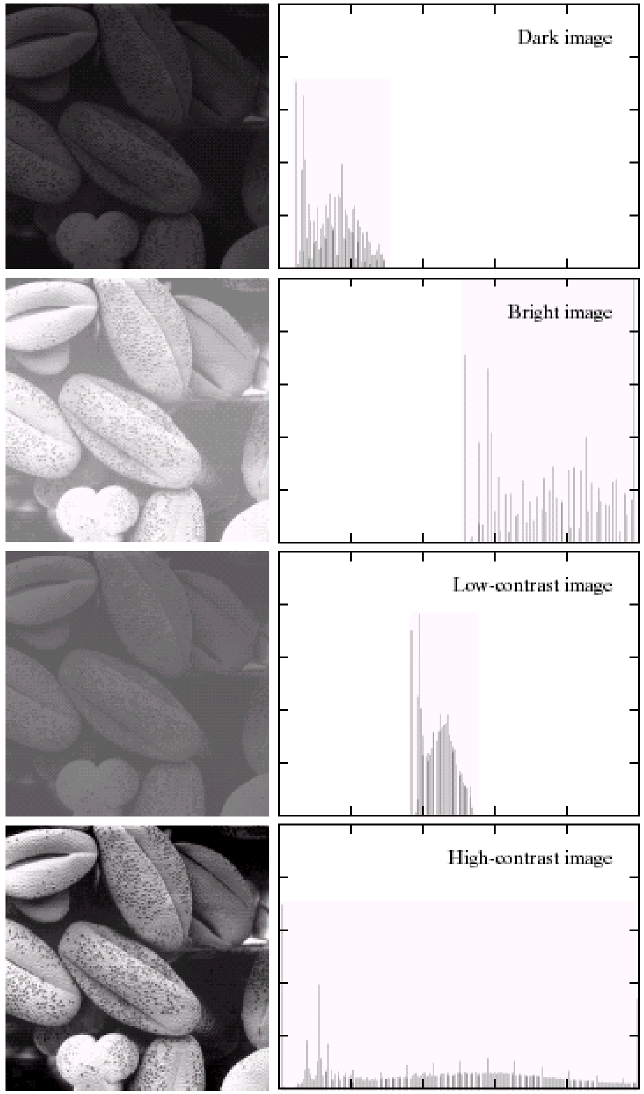
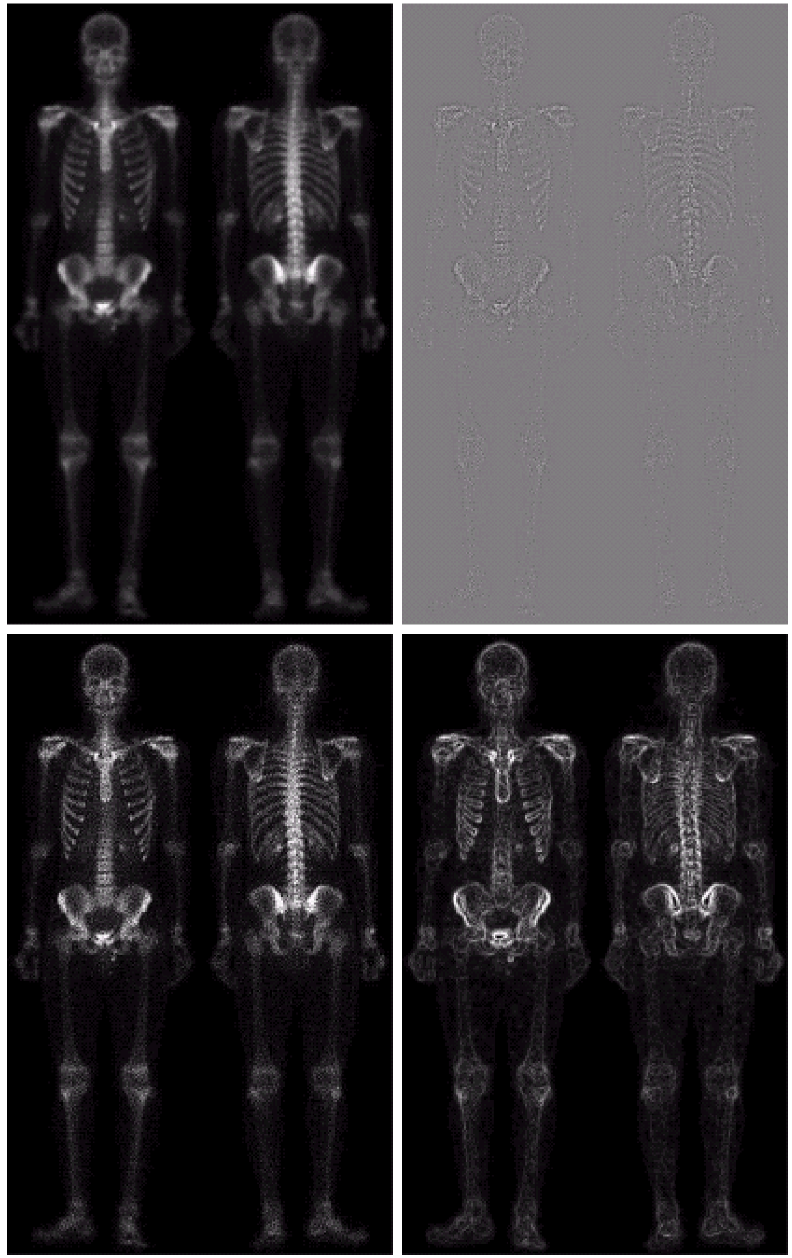


Figure 4.13 (a) Original image and (b) its histogram; (c) image subjected to histogram equalization and (d) its histogram.



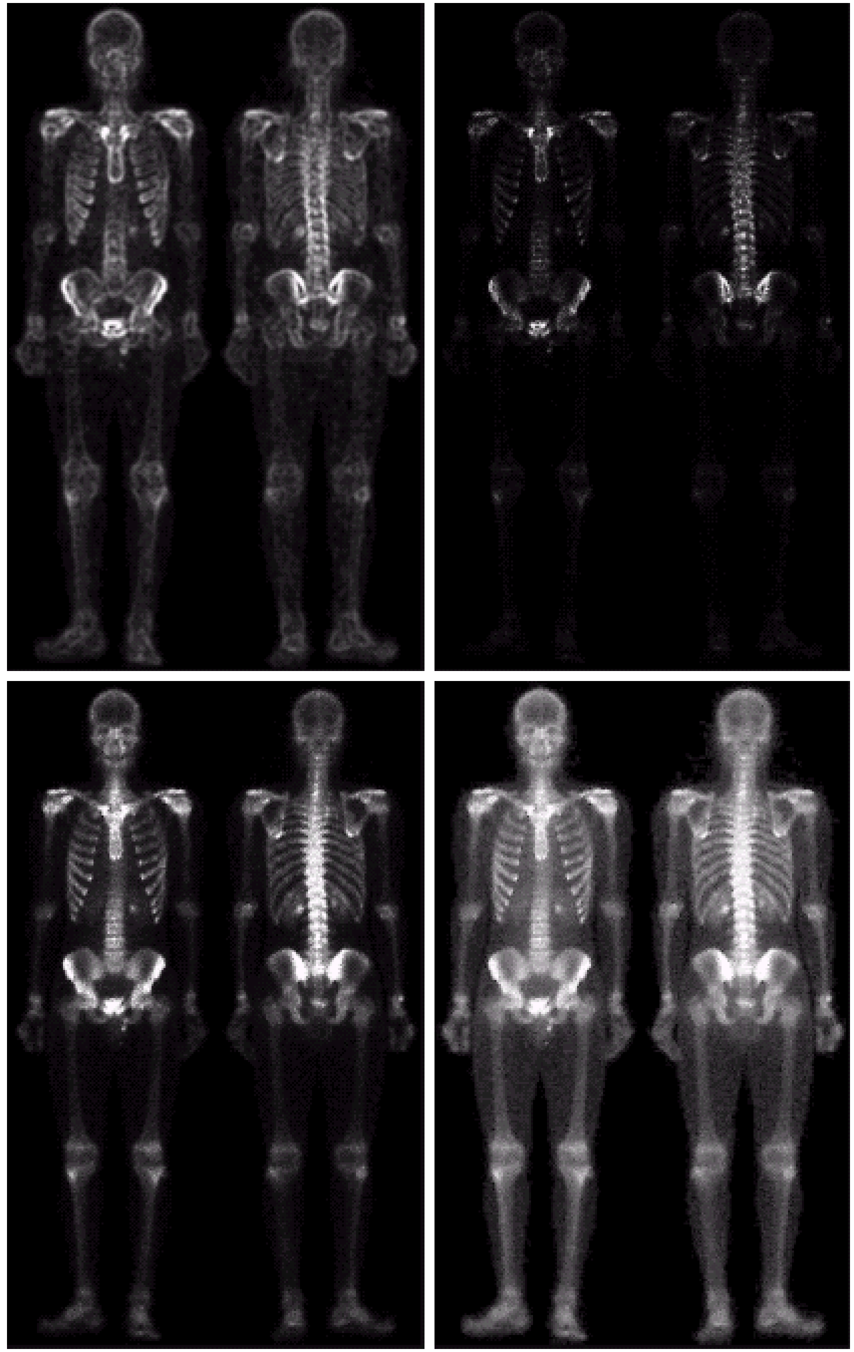
a b

FIGURE 3.15 Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)



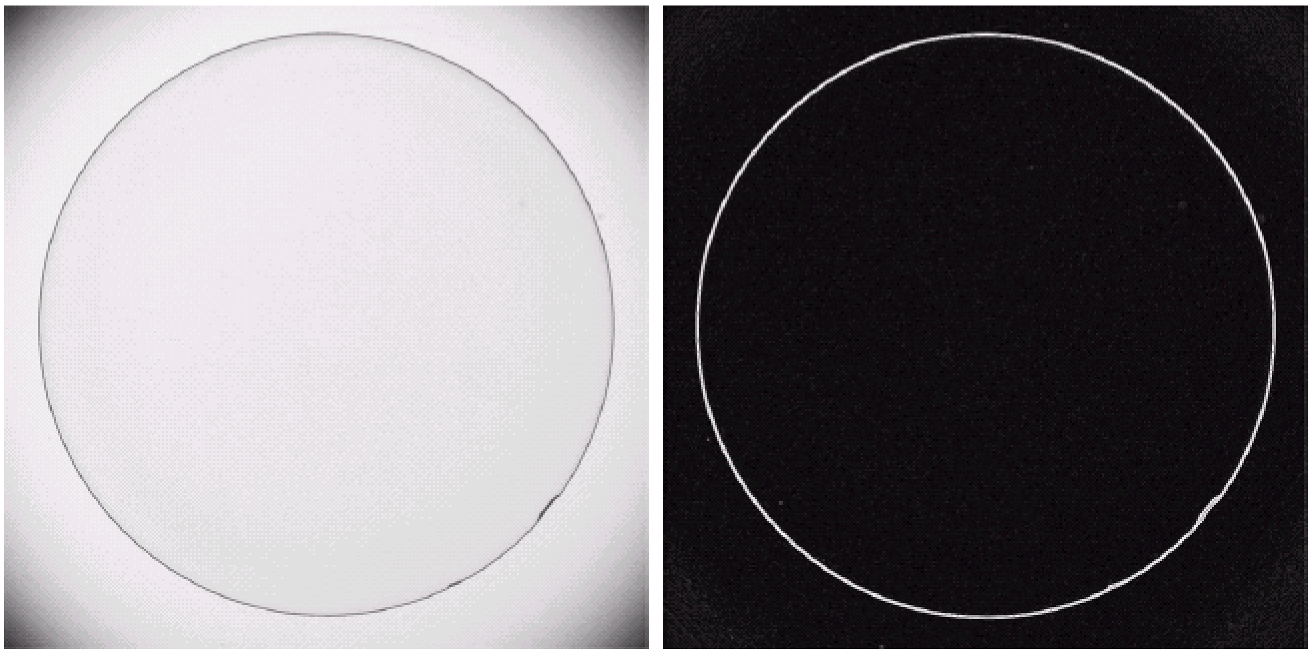
a b
c d

FIGURE 3.46
(a) Image of whole body bone scan. (b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b). (d) Sobel of (a).



e f
g h

FIGURE 3.46
(Continued)
(e) Sobel image smoothed with a 5×5 averaging filter. (f) Mask image formed by the product of (c) and (e). (g) Sharpened image obtained by the sum of (a) and (f). (h) Final result obtained by applying a power-law transformation to (g). Compare (g) and (h) with (a). (Original image courtesy of G.E. Medical Systems.)



a b

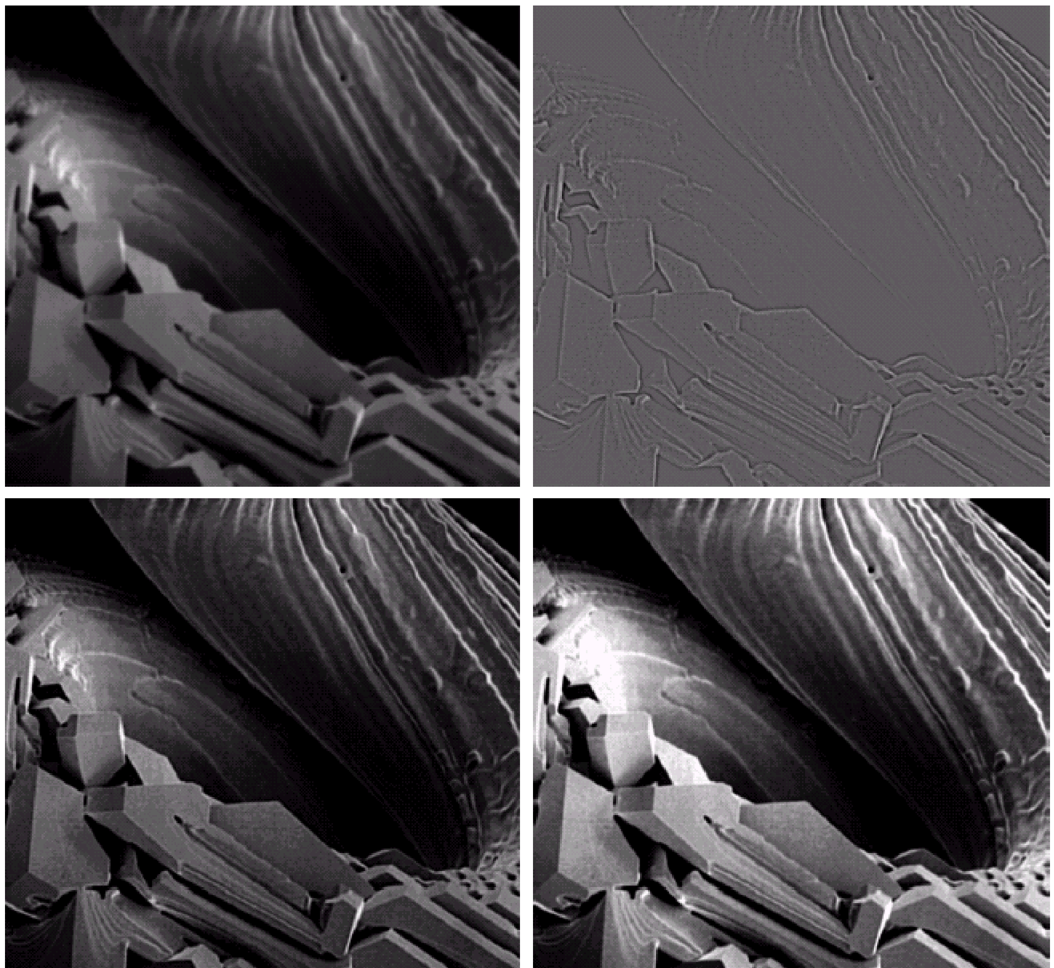
FIGURE 3.45 Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock). (b) Sobel gradient. (Original image courtesy of Mr. Pete Sites, Perceptics Corporation.)

0	−1	0
−1	$A + 4$	−1
0	−1	0

−1	−1	−1
−1	$A + 8$	−1
−1	−1	−1

a b

FIGURE 3.42 The high-boost filtering technique can be implemented with either one of these masks, with $A \geq 1$.



a b
c d

FIGURE 3.43 (a) Same as Fig. 3.41(c), but darker. (a) Laplacian of (a) computed with the mask in Fig. 3.42(b) using $A = 0$. (c) Laplacian enhanced image using the mask in Fig. 3.42(b) with $A = 1$. (d) Same as (c), but using $A = 1.7$.