1.

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Discrete Mathematical structures

(1)

Solution :-

enumerator for partition of n with odd-number of parts

$$G_1(x) = (1-x)^{-1}(1-x^3)^{-1}(1-x^5)^{-1} \cdots\cdots$$

enumerator for partioning of distinct parts of n

$$G_2(x) = (1+x)(1+x^2)(1+x^3)(1+x^4) \cdots\cdots$$

Consider $G_2(x)$

$$= \frac{(1+x)(1-x)}{(1-x)} \times \frac{(1+x^2)(1-x^2)}{(1-x^2)} \times \frac{(1+x^3)(1-x^3)}{(1-x^3)}$$

$$= \frac{(\cancel{1-x^2})}{(1-x)} \times \frac{\cancel{(1-x^4)}}{\cancel{(1-x^2)}} \times \frac{\cancel{(1-x^6)}}{(1-x^3)} \cdots$$

$$= (1-x)^{-1}(1-x^3)^{-1}(1-x^5)^{-1} \cdots\cdots$$

$$= G_1(x)$$

$\therefore$ hence proved.

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(2)

Solution :

Dearrangement problem :-

$$D_n = n!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} \cdots\cdots (-1)^n \frac{1}{n!}\right)$$

$n = 5$

$$D_5 = 5!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right)$$

$$= 120\left(\frac{60 - 20 + 5 - 1}{120}\right) = \frac{44}{120} \times 120 =$$

$$= 44$$

∴ There are 44 ways to return the 5 hats so that no guest receives their own hat.

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(3) Solution

$\{3, 6, 9, 0\}$ under addition modulo 12.
↳ Subgroup

$Z_{12} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

$\therefore$ left cosets

$0 :-$ $\{0+0, 0+3, 0+6, 0+9\} = \{0, 3, 6, 9\}$
$1 :-$ $\{1+0, \cancel{0}1+3, 1+6, 1+9\} = \{1, 4, 7, 10\}$
$2 :-$ $\{2+0, 2+3, 2+6, 2+9\}$ $\{2, 5, 8, 11\}$
$3 :-$ $\{3+0, 3+3, 3+6, 3+9\} = \{0, 3, 6, 9\}$
$\vdots$

~~reap~~ repeating

$\therefore$ distinct left cosets :-

$\{0, 3, 6, 9\}, \{1, 4, 7, 10\}, \{2, 5, 8, 11\}$.

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(4) Using the theorem :-

No of partitions of n with atmost k parts = no of partitions of n with no part greater than k.

$\therefore$ we need to find no of partitions of 7 with no part greater than 3.

$\therefore$ generating function
$$= (1-x)^{-1}(1-x^2)^{-1}(1-x^3)^{-1} \longrightarrow (1)$$

now, find coeff of x^7 in (1)

$$(1-x)^{-1}(1+x^2+x^4)(1+x^3+x^6)$$

$$(1+x^3+x^6+x^2+x^5+x^8+x^4+x^7+x^{10}+x^6+x^9+x^{12})(1-x)^{-1}$$

$$\Rightarrow (1+x^2+x^3+x^4+x^5+2x^6+x^7) \rightarrow$$

[$\because$ rest ignored as they are not contributing to x^7]

Ans :- ~~$\sum$ coeff of x^n~~

Sum of coefficients of $x^n + 2x^6 + 1$ where $n \in [2, 3, 4, 5, 7]$ in $(1-x)^{-1}$

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$= {}^{1+7-1}C_7 + {}^{1+5-1}C_5 + {}^{1+4-1}C_4 + {}^{1+3-1}C_3 + {}^{1+2-1}C_2 + {}^{1+1-1}C_1 + 1$

$= 1+1+1+1+1+2+1$

$= \underline{\underline{8}}$

$\boxed{\therefore 8 \text{ partitions}}$

(5) Solution:- coeff of x^{10} in $(1-x)^{-1}(1-x^2)^{-1}(1-x^5)^{-1}$

$(1-x)^{-1}(1+x^2+x^4+x^6+x^8+x^{10})(1+x^5+x^{10})$

$(1+x^5+x^{10}+x^2+x^7+x^{12}+x^4+x^9+x^{14}+x^6+x^{11}+x^{16}+x^8+x^{13}+x^{18}+x^{10}+x^{15}+x^{20})(1-x)^{-1}$

powers > 10 ignored :-

$(1+x^5+x^2+x^7+x^4+x^9+x^6+x^8+2x^{10})(1-x)^{-1}$

$\therefore$ coeff of x^{10} :-

${}^{1+10-1}C_{10} + {}^{1+8-1}C_8 + {}^{1+6-1}C_6 + {}^{1+5-1}C_5 + {}^{1+4-1}C_4 + {}^{1+3-1}C_3 + {}^{1+2-1}C_2 + {}^{1+1-1}C_1 + 2.$

$= 8 + 2 = 10$

$\boxed{\text{coefficient of } x^{10} = 10}$

END