Election Anomaly Detection

on the results of

2018 Russian presidential election

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Introduction

- The **2018 Russian presidential election** was held on **18 March 2018**
- The election conducts in one or two tours depending on results
- Voters across the country chose from **8 candidates** on the same day

- Current Government strived for a **70-70 principle**:
  - **70% voter turnout**
  - **70% votes share** for the incumbent president

Presentation objectives

This work tends to:

- Observe official data and detect anomalies
- Detect a fraud according only to official numbers
- Challenge an achievement of 70-70 principle

Candidates

Pavel Nikolaevich Grudinin

Communist party member, deputy of Moscow Duma

Sergey Nikolaevich Baburin

Deputy of the State Duma

Vladimir Volfovich Zhirinovsky

Perennial candidate, took part in 2000 and 2008 presidential elections

Vladimir Vladimirovich Putin

President of Russian Federation in 2000-2008 and 2012-present

Ksenia Anatolyevna Sobchak

Civic Initiative, opposition activist

Maxim Alexandrovich Suraikin

Chairman of the Central Committee of the Communists of Russia

Boris Yurievich Titov

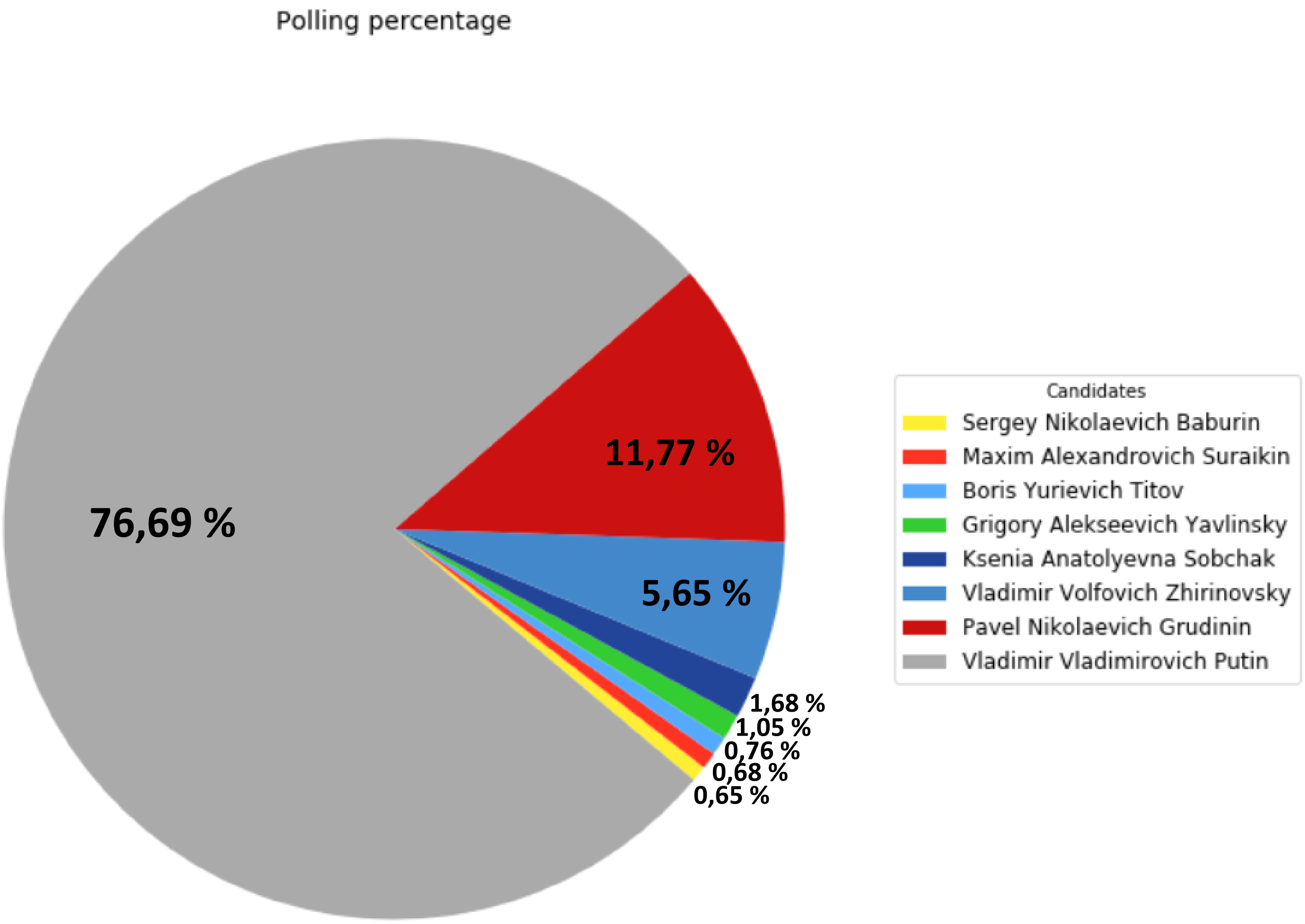
Leader of the democratic pro-Kremlin party

Grigory Alekseevich Yavlinsky

Former chairman of social-liberal party

Brief election results

Vladimir Putin gained 76,69 % of votes with a voter turnout of 67,54 % and won the election preventing a second tour



Brief election results

Vladimir Putin gained the majority of votes in **every region** of the country

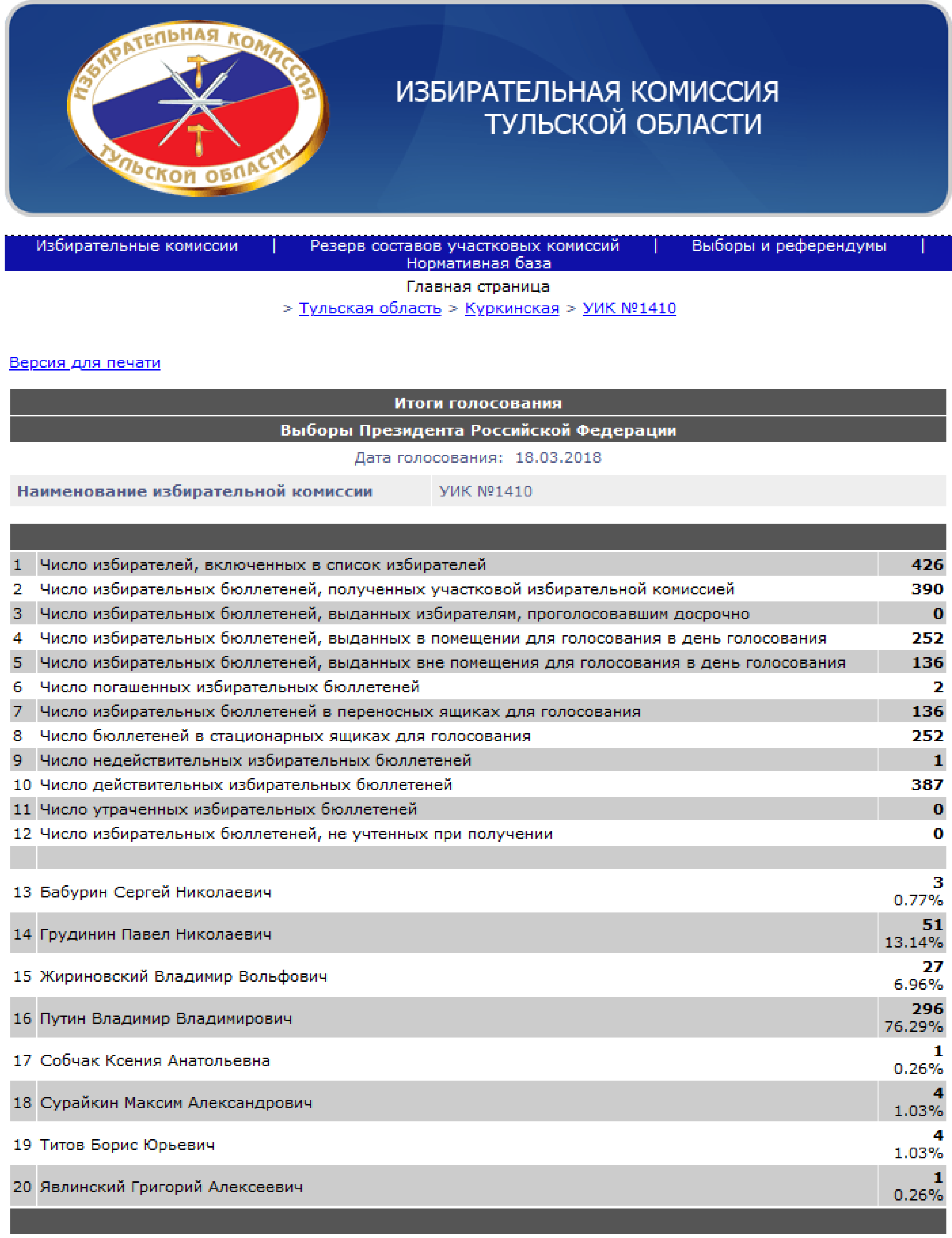
Dataset

Following the tendency of open-data, every polling station across the country was obliged to publish the polling data on Internet after accomplishment of votes counting procedure

The election data has never been centralized or published in a single data file by authorities. Instead, every polling station created a separate Web-page containing the data relevant to specific election

The full dataset of 97 893 polling stations has been compiled by running the crawler script[1] produced by Alexander Danilov

[1] GitLab program repository: https://gitlab.com/modos189/cikinfo



ИЗБИРАТЕЛЬНАЯ КОМИССИЯ ТУЛЬСКОЙ ОБЛАСТИ

Избирательные комиссии | Резерв составов участковых комиссий | Выборы и референдумы | Нормативная база

Главная страница

> Тульская область > Куркинская > УИК №1410

Версия для печати

Итоги голосования

Выборы Президента Российской Федерации

Дата голосования: 18.03.2018

Наименование избирательной комиссии: УИК №1410

1	Число избирателей, включенных в список избирателей	426
2	Число избирательных бюллетеней, полученных участковой избирательной комиссией	390
3	Число избирательных бюллетеней, выданных избирателям, проголосовавшим досрочно	0
4	Число избирательных бюллетеней, выданных в помещении для голосования в день голосования	252
5	Число избирательных бюллетеней, выданных вне помещения для голосования в день голосования	136
6	Число погашенных избирательных бюллетеней	2
7	Число избирательных бюллетеней в переносных ящиках для голосования	136
8	Число бюллетеней в стационарных ящиках для голосования	252
9	Число недействительных избирательных бюллетеней	1
10	Число действительных избирательных бюллетеней	387
11	Число утраченных избирательных бюллетеней	0
12	Число избирательных бюллетеней, не учтенных при получении	0
13	Бабурин Сергей Николаевич	3 0.77%
14	Грудинин Павел Николаевич	51 13.14%
15	Жириновский Владимир Вольфович	27 6.96%
16	Путин Владимир Владимирович	296 76.29%
17	Собчак Ксения Анатольевна	1 0.26%
18	Сурайкин Максим Александрович	4 1.03%
19	Титов Борис Юрьевич	4 1.03%
20	Явлинский Григорий Алексеевич	1 0.26%

Example of Web-page of the polling station №1410 of Tula containing the 2018 presidential election data

Dataset

The collected dataset contains 97 893 rows corresponding to parsed polling stations, and 25 columns

	link	uik	kom1	kom2	kom3	1	2	...	14	15	16	17	18	19	20
83510	http://www.rostov.vybory.izbirkom.ru/region /ro...	1694	Ростовская область	Ростов-на-Дону, Ворошиловская	УИК №1694	2330.0	2250.0	...	120.0	42.0	1165.0	10.0	3.0	9.0	7.0
57051	http://www.kaluga.vybory.izbirkom.ru/region /ka...	322	Калужская область	Боровская	УИК №322	431.0	400.0	...	26.0	14.0	221.0	1.0	1.0	0.0	0.0
14226	http://www.bashkortostan.vybory.izbirkom.ru /re...	1717	Республика Башкортостан	Давлекановская	УИК №1717	1285.0	1050.0	...	111.0	45.0	522.0	6.0	11.0	4.0	8.0
60872	http://www.kirov.vybory.izbirkom.ru/region /kir...	660	Кировская область	Малмыжская	УИК №660	234.0	268.0	...	35.0	6.0	154.0	0.0	1.0	1.0	0.0
62243	http://www.kurgan.vybory.izbirkom.ru/region /ku...	125	Курганская область	Курган, Восточная	УИК №125	1661.0	1700.0	...	145.0	62.0	650.0	20.0	14.0	17.0	17.0
74327	http://www.omsk.vybory.izbirkom.ru/region /omsk...	995	Омская область	Марьяновская	УИК №995	110.0	116.0	...	12.0	3.0	65.0	2.0	0.0	0.0	0.0
13943	http://www.bashkortostan.vybory.izbirkom.ru /re...	1457	Республика Башкортостан	Благоварская	УИК №1457	413.0	426.0	...	41.0	15.0	332.0	1.0	4.0	1.0	1.0
37015	http://www.krasnoyarsk.vybory.izbirkom.ru /regi...	773	Красноярский край	Абанская	УИК №773	234.0	235.0	...	22.0	13.0	113.0	0.0	2.0	0.0	0.0
88487	http://www.saratov.vybory.izbirkom.ru /region/s...	1611	Саратовская область	Саратовская	УИК №1611	1318.0	1300.0	...	12.0	17.0	1234.0	3.0	0.0	3.0	0.0
78032	http://www.penza.vybory.izbirkom.ru/region /pen...	142	Пензенская область	Пенза, Ленинская	УИК №142	1927.0	1700.0	...	153.0	80.0	871.0	35.0	19.0	19.0	14.0
12416	http://www.khantu-mansy.vybory.izbirkom.ru /reg...	690	Ханты-Мансийский автономный округ - Югра	Сургутская городская	УИК №690	1554.0	1450.0	...	121.0	51.0	850.0	28.0	3.0	8.0	14.0
31227	http://www.altai_terr.vybory.izbirkom.ru /regio...	583	Алтайский край	Алейская	УИК №583	63.0	70.0	...	10.0	6.0	43.0	0.0	0.0	0.0	0.0
74628	http://www.omsk.vybory.izbirkom.ru/region /omsk...	1136	Омская область	Называевская	УИК №1136	679.0	660.0	...	41.0	44.0	328.0	5.0	3.0	0.0	0.0

Random sample from the dataset

Columns

link: link to polling station Web-page
uik: local ID of polling station
kom1: region of polling station
kom2: district of polling station
kom3: name of polling station
1: number of attached voters
2: number of bulletins assigned for voting
3: number of bulletins for preschedule voting
4: number of bulletins given inside on the voting day
5: number of bulletins given outside on the voting day
6: number of annulated bulletins
7: number of bulletins inside of transportable voting urns
8: number of bulletins inside of stationary voting urns
9: number of invalid bulletins
10: number of valid bulletins
11: number of lost bulletins
12: number of not calculated bulletins
13: votes for Sergey Nikolaevich Baburin
14: votes for Pavel Nikolaevich Grudinin
15: votes for Vladimir Volfovich Zhirinovsky
16: votes for Vladimir Vladimirovich Putin
17: votes for Ksenia Anatolyevna Sobchak
18: votes for Maxim Alexandrovich Suraikin
19: votes for Boris Yurievich Titov
20: votes for Grigory Alekseevich Yavlinsky

link	uik	kom1	kom2	kom3	1	2	...	14	15	16	17	18	19	20
http://www.altai_terr.vybory.izbirkom.ru/regio...	1240	Алтайский край	Павловская	УИК №1240	1367.0	1300.0	...	185.0	76.0	682.0	3.0	4.0	2.0	3.0
http://www.astrakhan.vybory.izbirkom.ru/region...	1127	Астраханская область	Красноярская	УИК №1127	344.0	330.0	...	61.0	14.0	132.0	3.0	2.0	2.0	0.0
http://www.crimea.vybory.izbirkom.ru/region/cr...	1231	Республика Крым	Советская	УИК №1231	475.0	450.0	...	6.0	2.0	353.0	10.0	0.0	1.0	2.0
http://www.rostov.vybory.izbirkom.ru/region/ro...	1268	Ростовская область	Мясниковская	УИК №1268	2528.0	2200.0	...	309.0	62.0	1093.0	32.0	11.0	13.0	14.0
http://www.altai_terr.vybory.izbirkom.ru/regio...	108	Алтайский край	Барнаул, Индустриальная	УИК №108	2061.0	1550.0	...	331.0	59.0	884.0	24.0	4.0	12.0	22.0
http://www.altai_terr.vybory.izbirkom.ru/regio...	1470	Алтайский край	Советская	УИК №1470	164.0	130.0	...	22.0	23.0	53.0	1.0	0.0	0.0	0.0
http://www.jewish_aut.vybory.izbirkom.ru/regio...	35	Еврейская автономная область	Биробиджанская городская	УИК №35	24.0	25.0	...	4.0	2.0	16.0	0.0	0.0	0.0	0.0
http://www.bryansk.vybory.izbirkom.ru/region/b...	649	Брянская область	Мглинская	УИК №649	278.0	309.0	...	23.0	8.0	207.0	1.0	0.0	0.0	0.0
http://www.bashkortostan.vybory.izbirkom.ru/re...	1712	Республика Башкортостан	Давлекановская	УИК №1712	1171.0	1190.0	...	111.0	38.0	445.0	5.0	8.0	5.0	4.0
http://www.kabardin-balkar.vybory.izbirkom.ru/...	163	Кабардино-Балкарская Республика	Нальчикская городская	УИК №163	1766.0	1750.0	...	81.0	19.0	1513.0	2.0	2.0	5.0	2.0
http://www.krasnoyarsk.vybory.izbirkom.ru/regi...	324	Красноярский край	Красноярск, Октябрьская	УИК №324	2458.0	1800.0	...	202.0	81.0	1206.0	48.0	5.0	7.0	33.0
http://www.altai_terr.vybory.izbirkom.ru/regio...	1833	Алтайский край	Чарышская	УИК №1833	108.0	110.0	...	18.0	8.0	46.0	0.0	0.0	0.0	0.0
http://www.tatarstan.vybory.izbirkom.ru/region...	2165	Республика Татарстан (Татарстан)	Набережные Челны, Автозаводская	УИК №2165	2322.0	2300.0	...	125.0	68.0	1306.0	25.0	171.0	8.0	10.0
http://www.voronezh.vybory.izbirkom.ru/region/...	3704	Воронежская область	Терновская	УИК №3704	152.0	160.0	...	15.0	5.0	104.0	0.0	1.0	0.0	0.0
http://www.krasnodar.vybory.izbirkom.ru/region...	2951	Краснодарский край	Лабинская	УИК №2951	381.0	379.0	...	23.0	18.0	315.0	2.0	1.0	2.0	0.0
http://www.bryansk.vybory.izbirkom.ru/region/b...	537	Брянская область	Климовская	УИК №537	372.0	370.0	...	24.0	31.0	295.0	0.0	1.0	0.0	0.0
http://www.khakas.vybory.izbirkom.ru/region/kh...	205	Республика Хакасия	Аскизская	УИК №205	1417.0	1450.0	...	143.0	61.0	513.0	5.0	3.0	2.0	1.0
http://www.tula.vybory.izbirkom.ru/region/tula...	1628	Тульская область	Новомосковская	УИК №1628	1849.0	1500.0	...	148.0	87.0	966.0	36.0	13.0	6.0	8.0
http://www.astrakhan.vybory.izbirkom.ru/region...	318	Астраханская область	Астрахань, Советская	УИК №318	1688.0	1700.0	...	131.0	43.0	687.0	32.0	5.0	10.0	14.0
http://www.pskov.vybory.izbirkom.ru/region/psk...	621	Псковская область	Невельская	УИК №621	330.0	350.0	...	26.0	15.0	213.0	2.0	5.0	0.0	0.0

Computation of missing values

The acquired dataset does not contain the voter turnout and votes share between candidates, but both are easily computed using the following formulas:

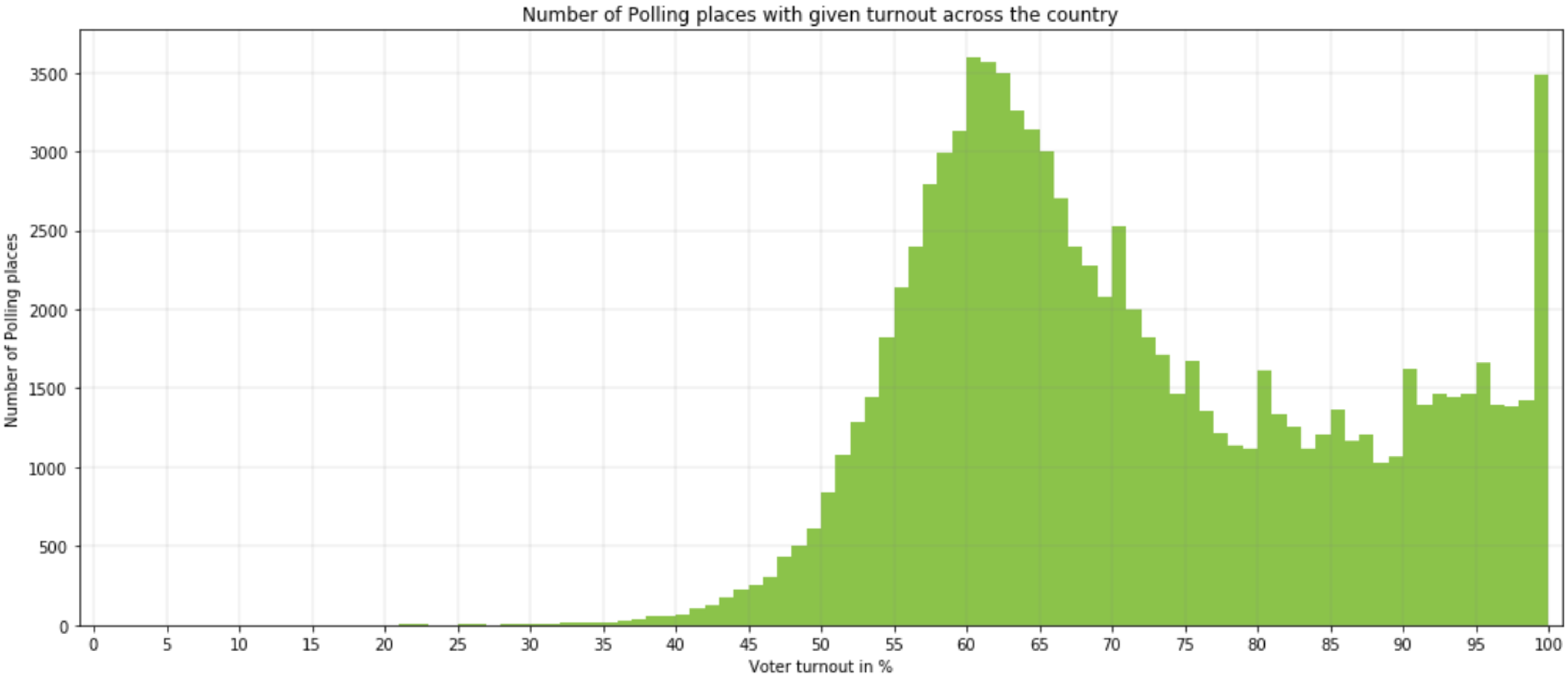
$$turnout = \frac{number\ of\ valid\ bulletins + number\ of\ invalid\ bulletins}{number\ of\ legit\ voters}$$

$$votes\ share\ for\ party^{i} = \frac{votes\ for\ the\ party^{i}}{\sum_j votes\ for\ the\ party^{j}}$$

The formulas can be computed for specific polling station or aggregated for the whole region or country

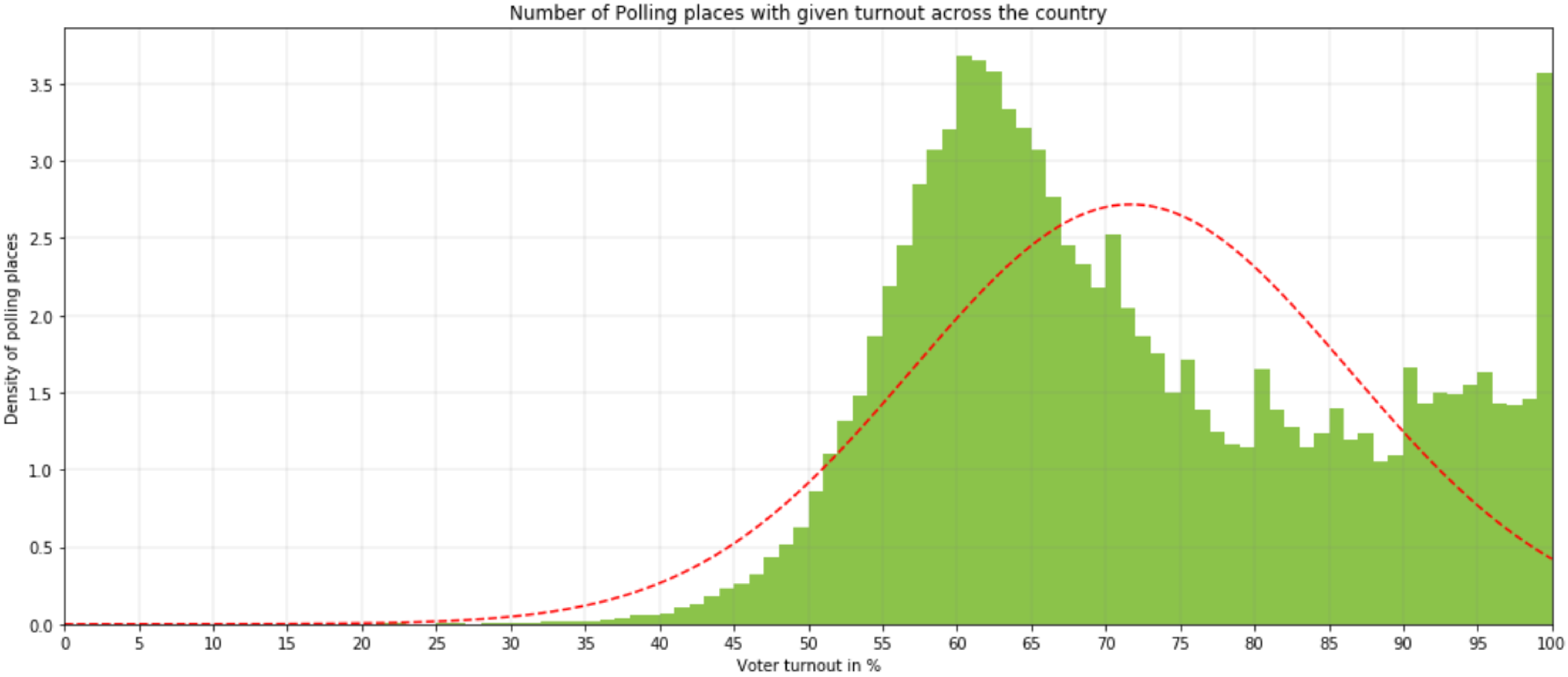
Voter turnout

The following histogram with 1000 bins reflects the distribution of voter turnout across the polling stations of the country



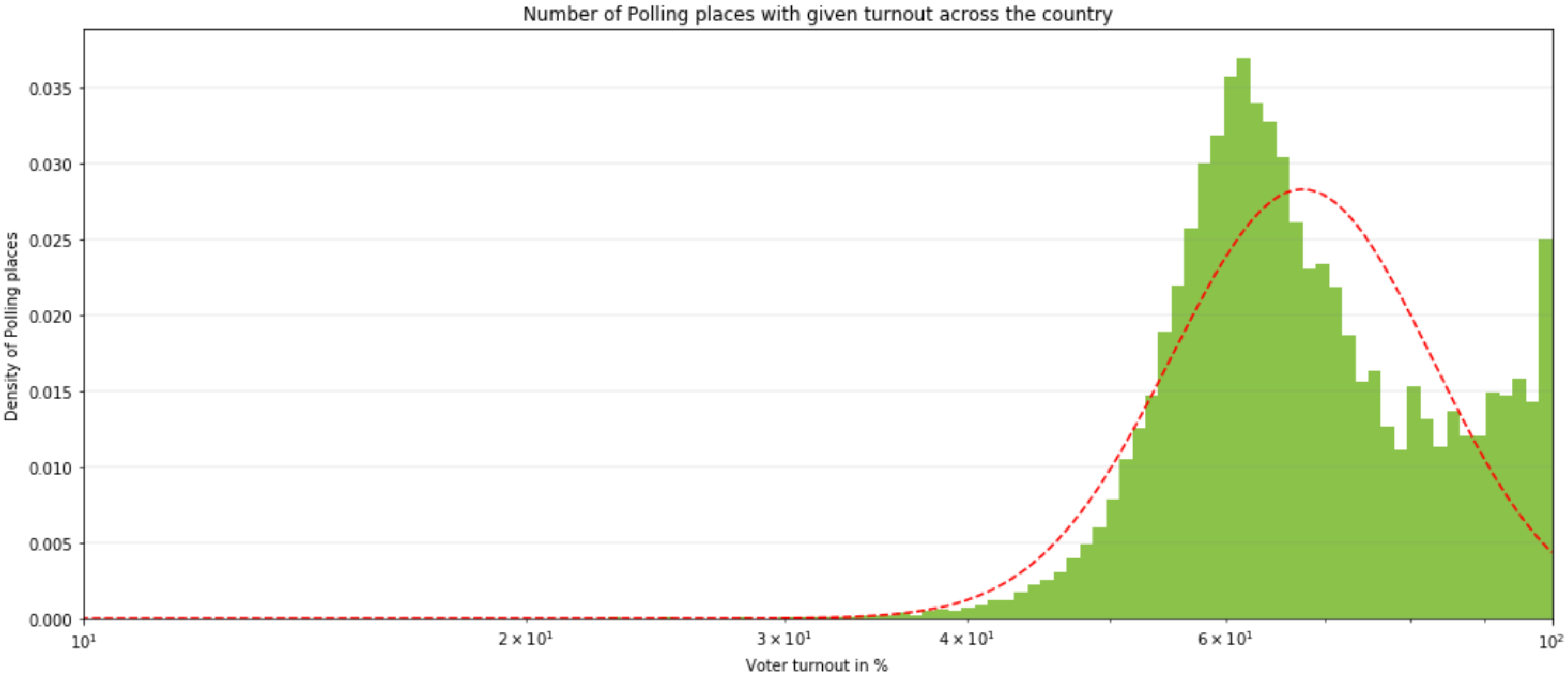
Voter turnout – normality test

According to the normality test with alpha = 0,01 we reject the null-hypothesis that the data is sampled from Gaussian distribution



Voter turnout – log-normality test

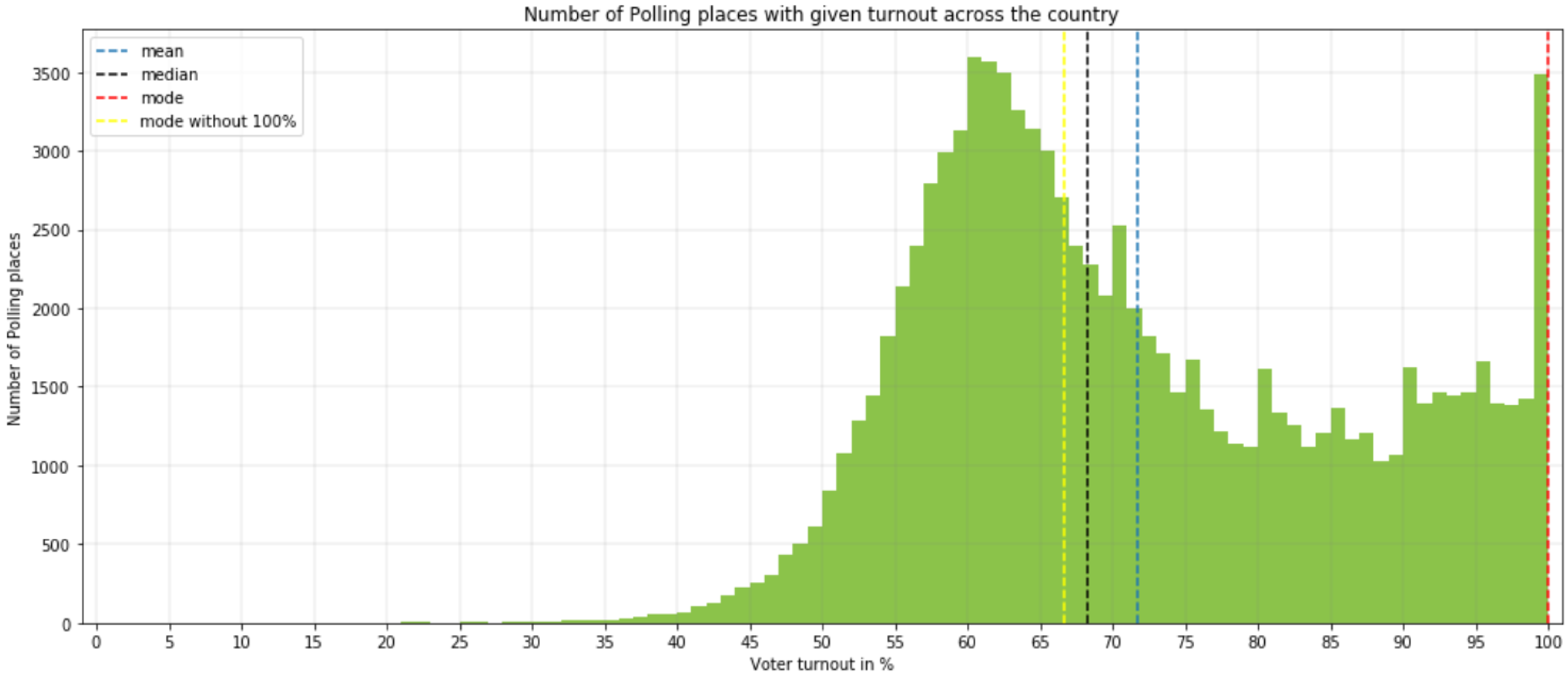
The log-normality test with alpha = 0,01 also rejects the null-hypothesis that the data is sampled from log-normal distribution



Voter turnout

Observation of histogram reveals two anomalies:

- data is positively skewed and tailed to the side of large turnout percentages
- the most frequent turnout percentage is 100%



Voter turnout – findings on histogram

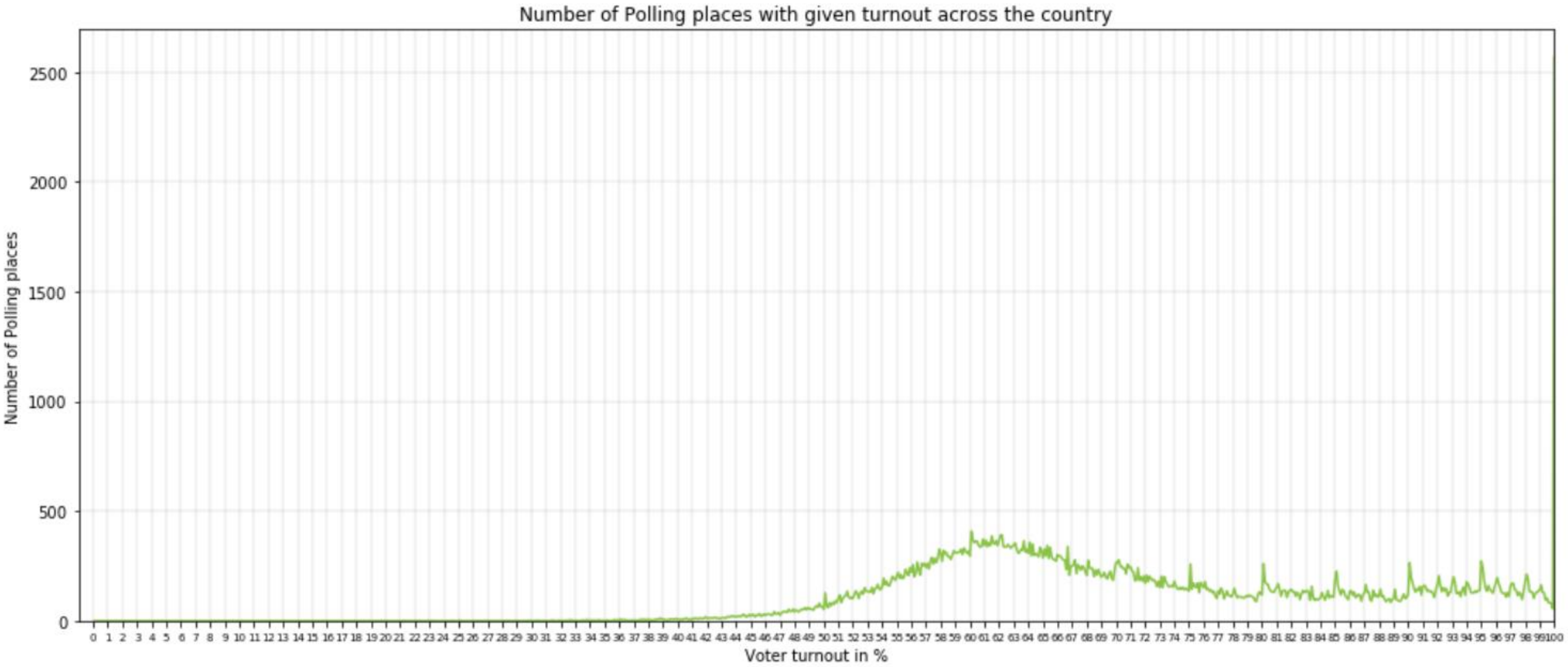
Klimek et al. (2012)[2] states that the positive skewness of turnout data is an evidence of ballot stuffing, that they primarily detect on the results of 2012 Russian presidential election and which can not be described by successful voters mobilization.

They also explain the large number of polling stations with 100 % voter turnout results by urn stuffing. In fact, in Russia many voting stations are attached to state-owned organizations and plants. Such organizations oblige the workers to visit the election and occasionally grant dedicated pauses for voting during the working hours, despite that it is forbidden by the federal law[3].

[2] Klimek, Peter, et al. "Statistical detection of systematic election irregularities." *Proceedings of the National Academy of Sciences* 109.41 (2012): 16469-16473.
[3] Federal law №67-ФЗ http://cikrf.ru/law/federal_law/zakon_02_67fz_n/zakon_02_67_full.html

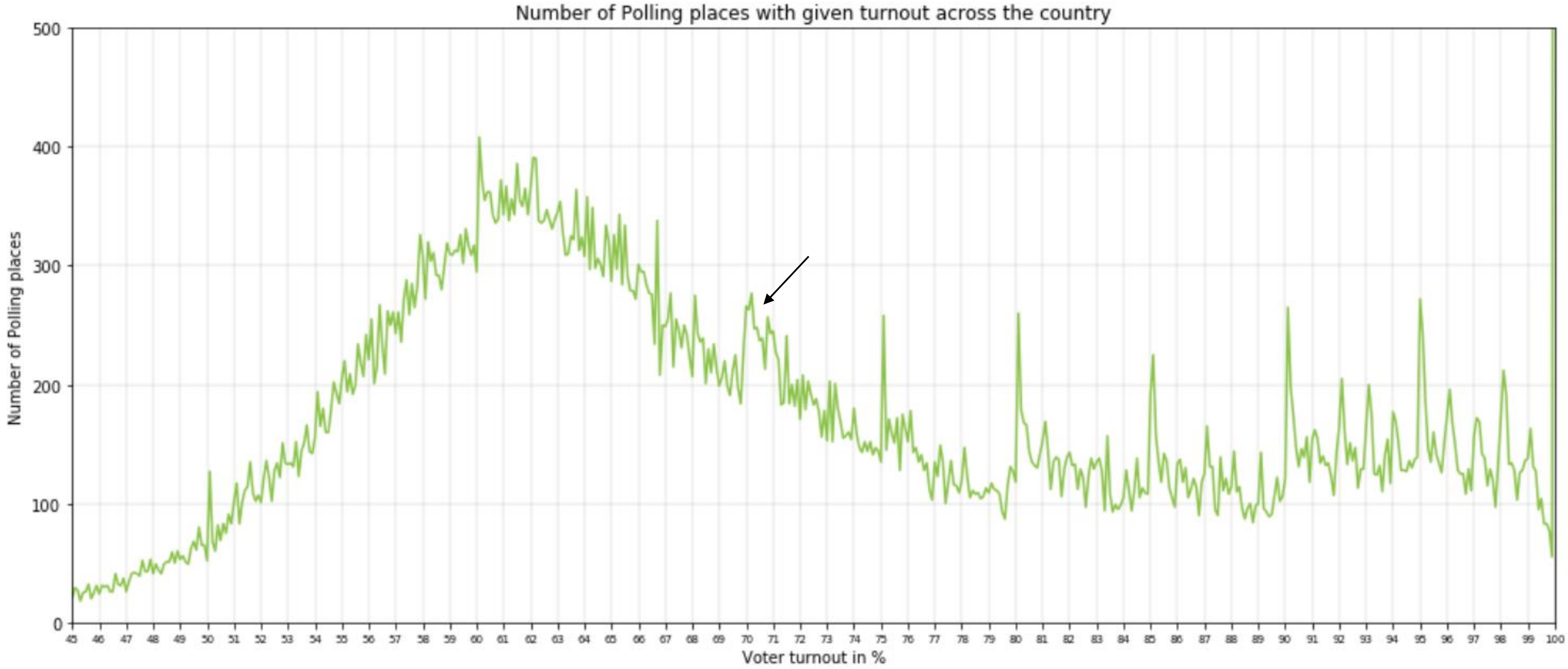
Voter turnout – plot with 0.1% step

Another anomalies can be detected after translation of the histogram into a plot with 0.1% unit step. The outlier point 100 % vertically stretches the plot. For better interpretability we further zoom inside of the right part of the plot.



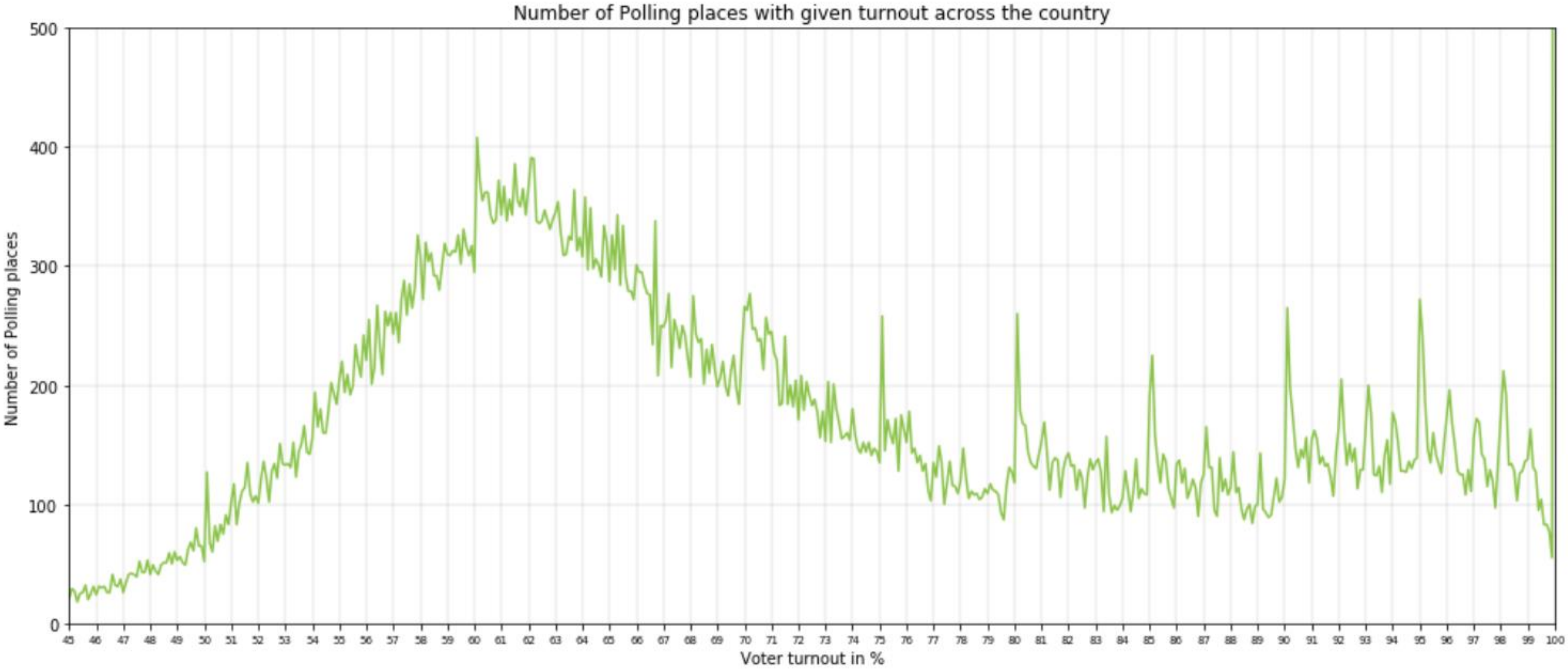
Voter turnout – plot with 0.1% step

Voter turnout forms a plateau on 70 % point, meaning that polling places frequently registered a turnout in a short interval of [70, 71] %



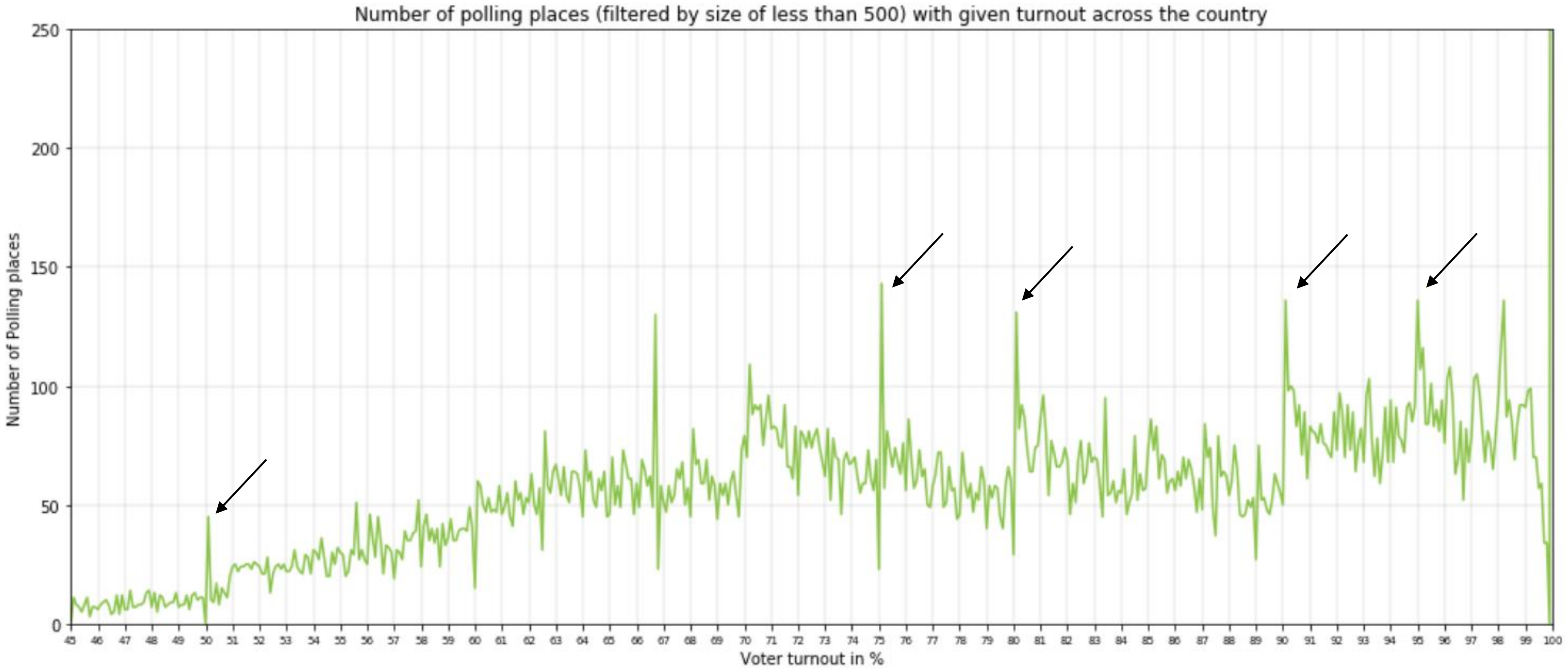
Voter turnout – plot with 0.1% step

Starting from 50 % point on turnout axes every round number corresponds to a local peak. Particularly tall peaks coincide with percentages which are being multiples of 5 or 10.



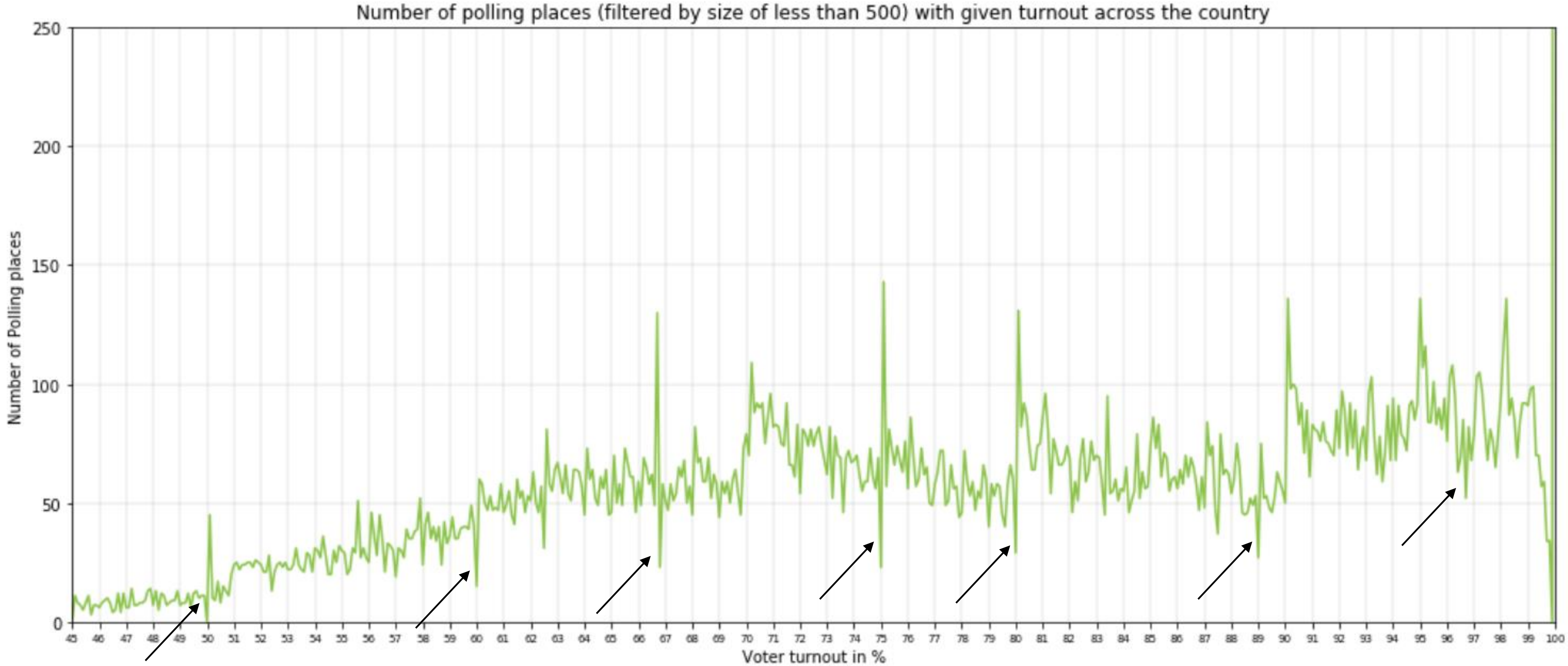
Voter turnout – round number peaks

On the one hand, such peaks can be caused by a property of rational fractions. If both nominator R_n and denominator R_d are integer random values and $R_n < R_d$, then random value $R_f = \frac{R_n}{R_d}$ tends to take some of values corresponding to round numbers with higher probability. Examples of such higher probability values are 1/5, 2/10, 3/15, … , 20/100, etc. By contrast some of the values like 17/69 are difficult to obtain.



Voter turnout – round number peaks

Also such peaks are partially explained by the computational roundoff. In such cases peaks are often followed by proportional in size pits.



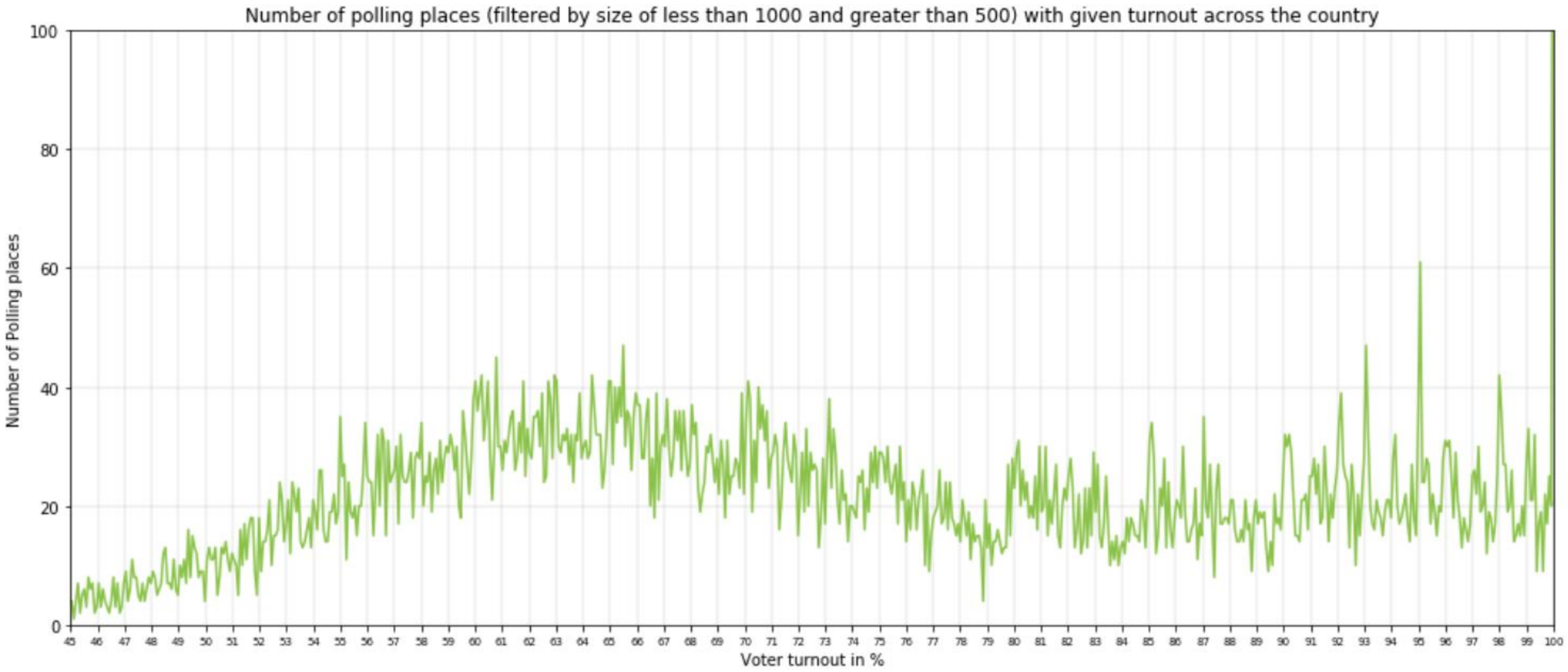
Round numbers

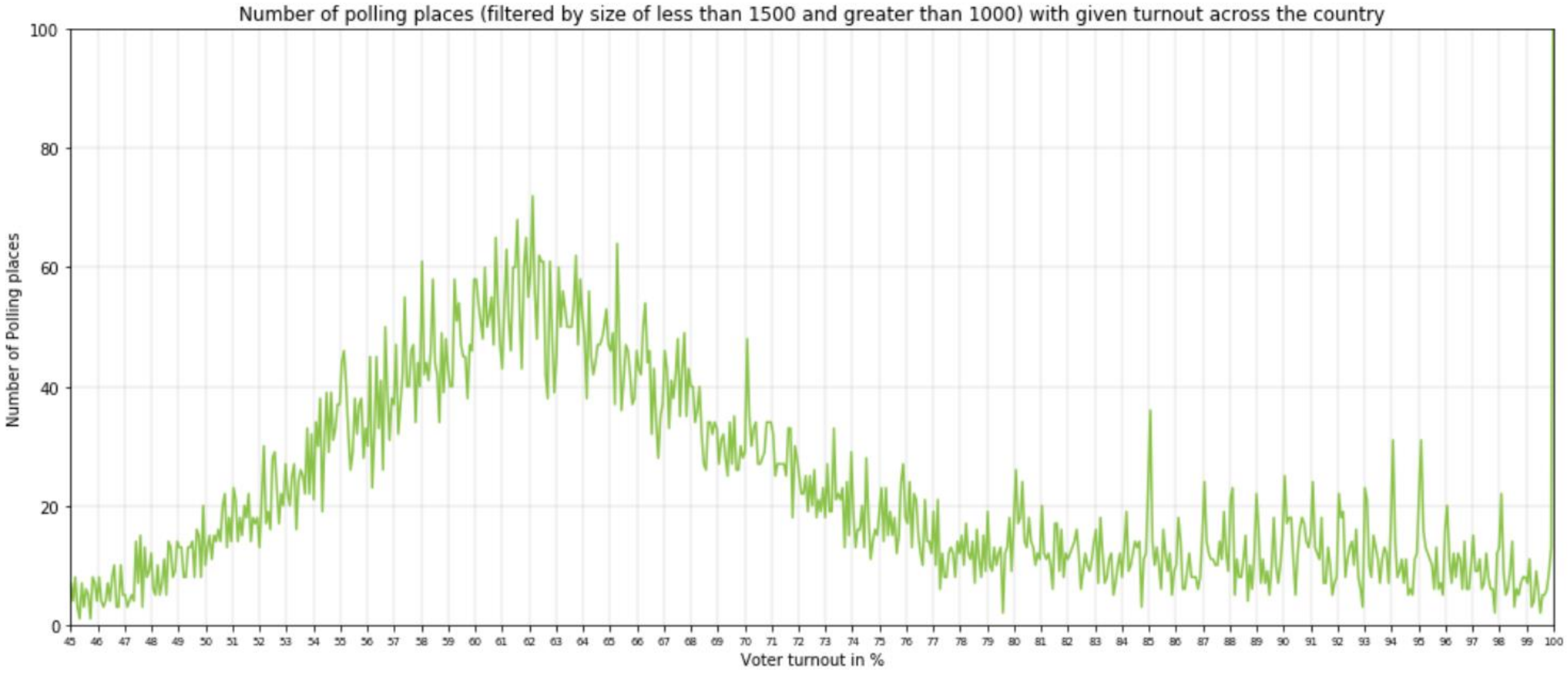
On the other hand, the problems tied to small numbers in rational fractions are insignificant in terms of bigger polling stations.

The plots demonstrate it on two subsets of the dataset specified by polling stations with interval size of attached voters s:

- $500 < s \leq 1000$
- $1000 < s \leq 1500$

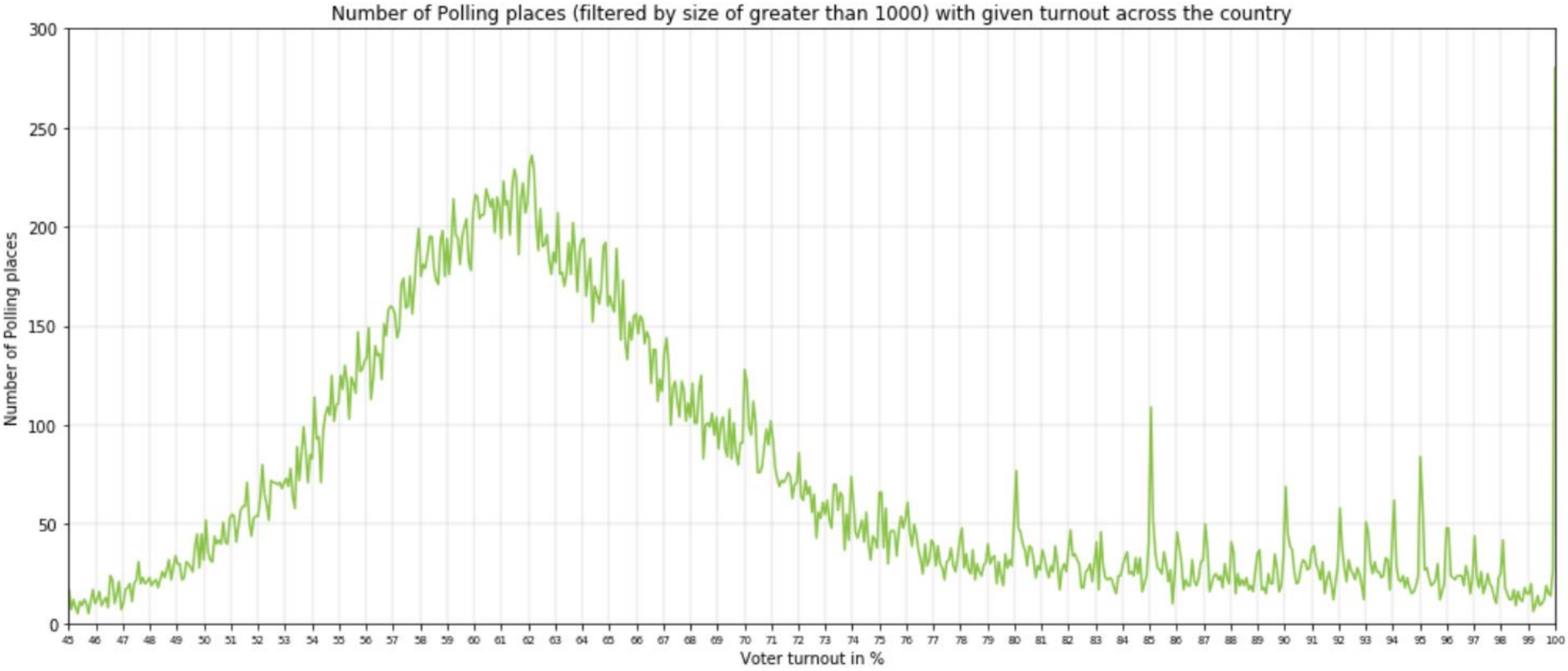
These subsets does not suffer from any of two rational fractions problems mentioned before, but still demonstrate the anomalous behavior around round numbers.





Voter turnout – round number peaks

Taking a subset of only the polling places with the voters size greater than 1000 completely eliminates the small numbers problem. The subset of large polling places still demonstrates the anomalous behavior with peaks on round numbers.



Round numbers – testing the hypothesis

In order to prove the assumption about existing of anomalous peaks we apply the hypothesis testing.

We create two independent and disjoint sets S_f and S_r, where S_f contains all quantitative values corresponding to non-round turnout percentages, and S_r contains the values corresponding only to round percentages except 100 % outlier point.

As long as we proved that the data comes from non-Gaussian distribution we can apply only non-parametric test. The most appropriate test would be the **permutation test** for independent sample-sets because of its specificity and precision. Its hypotheses formulation is following:

$$H_0: F_{X_1}(x) = F_{X_2}(x)$$
$$H_1: F_{X_1}(x) = F_{X_2}(x + \Delta), \Delta \neq 0$$

With α=0.01 we reject H_0 with p=0.0043, meaning that sets S_f and S_r do not follow the same distribution, in fact S_r is taken from a different distribution, what proves our assumption

Voter turnout – findings on 0.1 % step plot

The plateau on an interval of [70, 71] % voter turnout is explained by the 70-70 principle, where one of 70's is exactly a value of voter turnout that the government strived to achieve during elections. Because of this, there are three possible ways of things:

1. Polling stations took the number literally and intentionally attracted exactly 70 % of attached voters
2. The third mode of the dataset is inside of [70, 71] interval
3. Polling stations which observed turnout less than 70 % illegally utilized unused bulletins to increase the turnout and reach the desired bound

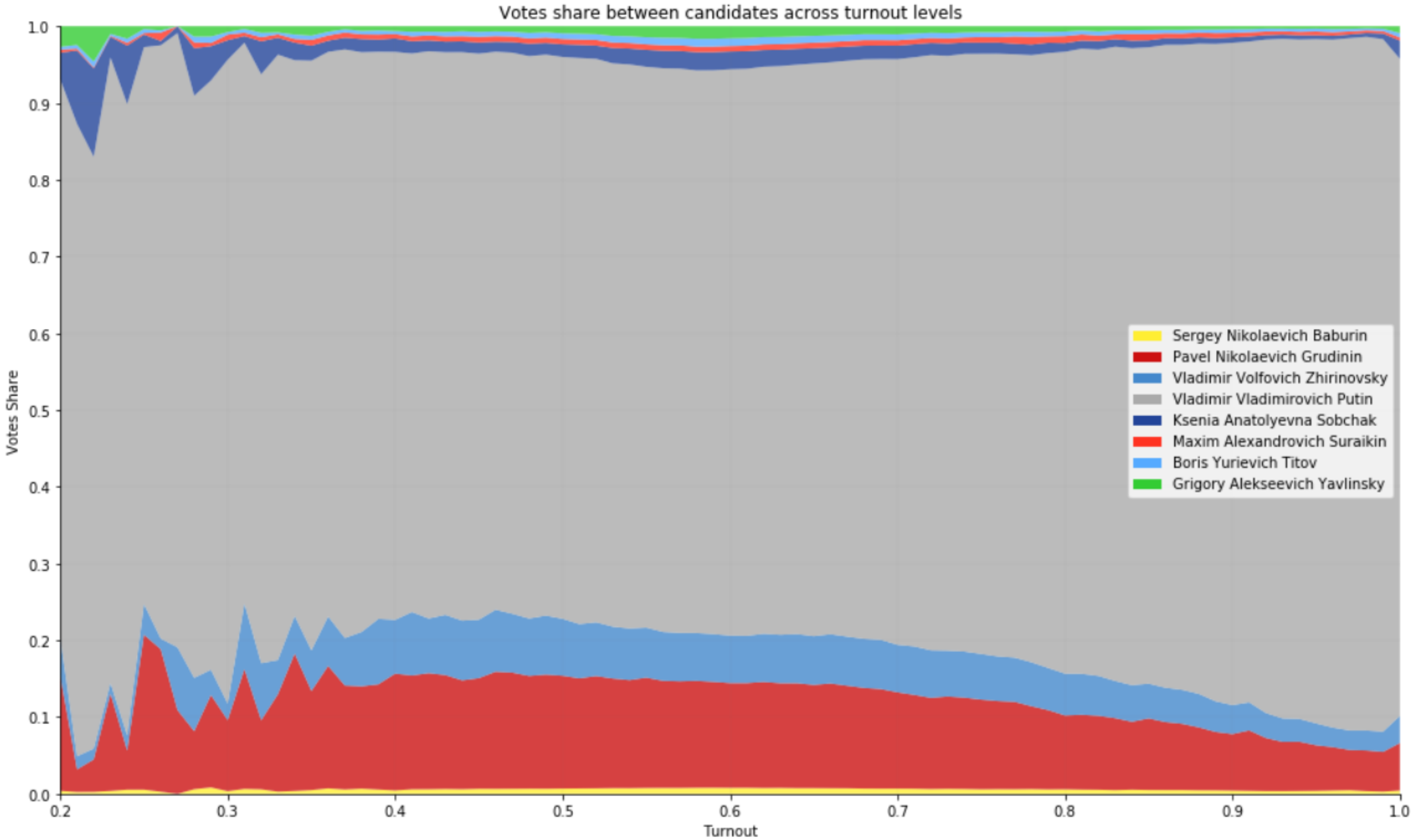
Also, the data gains anomalous behavior on round numbers after 50 % voter turnout point. Round values cause peaks connected to number of polling stations corresponding to specific turnout percentage. The peaks are not caused by rational fractions and are statistically significant. Kobak et al. (2016)[4] connects such activity to fraud but not necessarily malicious. Polling stations tend to count some part (which usually round up to and integer value) of unused during election bulletins in order to illegally add votes for specific candidate and to achieve a higher voter turnout.

[4] Kobak, Dmitry, Sergey Shpilkin, and Maxim S. Pshenichnikov. "Integer percentages as electoral falsification fingerprints." *The Annals of Applied Statistics* 10.1 (2016): 54-73.

Votes share

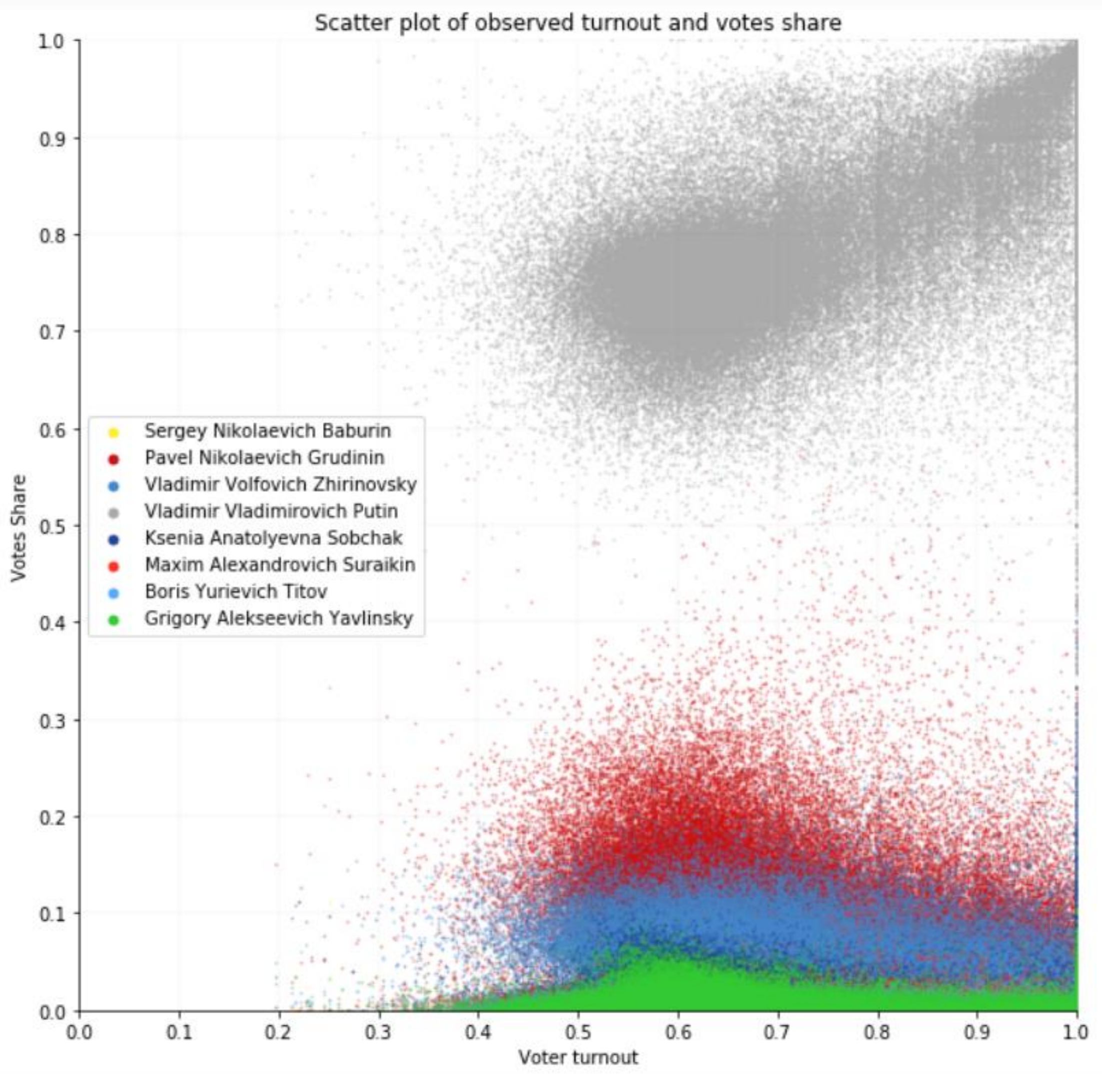
The following stack plot reflects the relation between voter turnout and votes share for candidates.

- The waves in the beginning of axes are caused by an insufficient number of polling places with low turnout
- Vladimir Putin gains the majority of votes across all turnout levels
- The area corresponding to Vladimir Putin starts to substantially increase after a point of 0.6 turnout



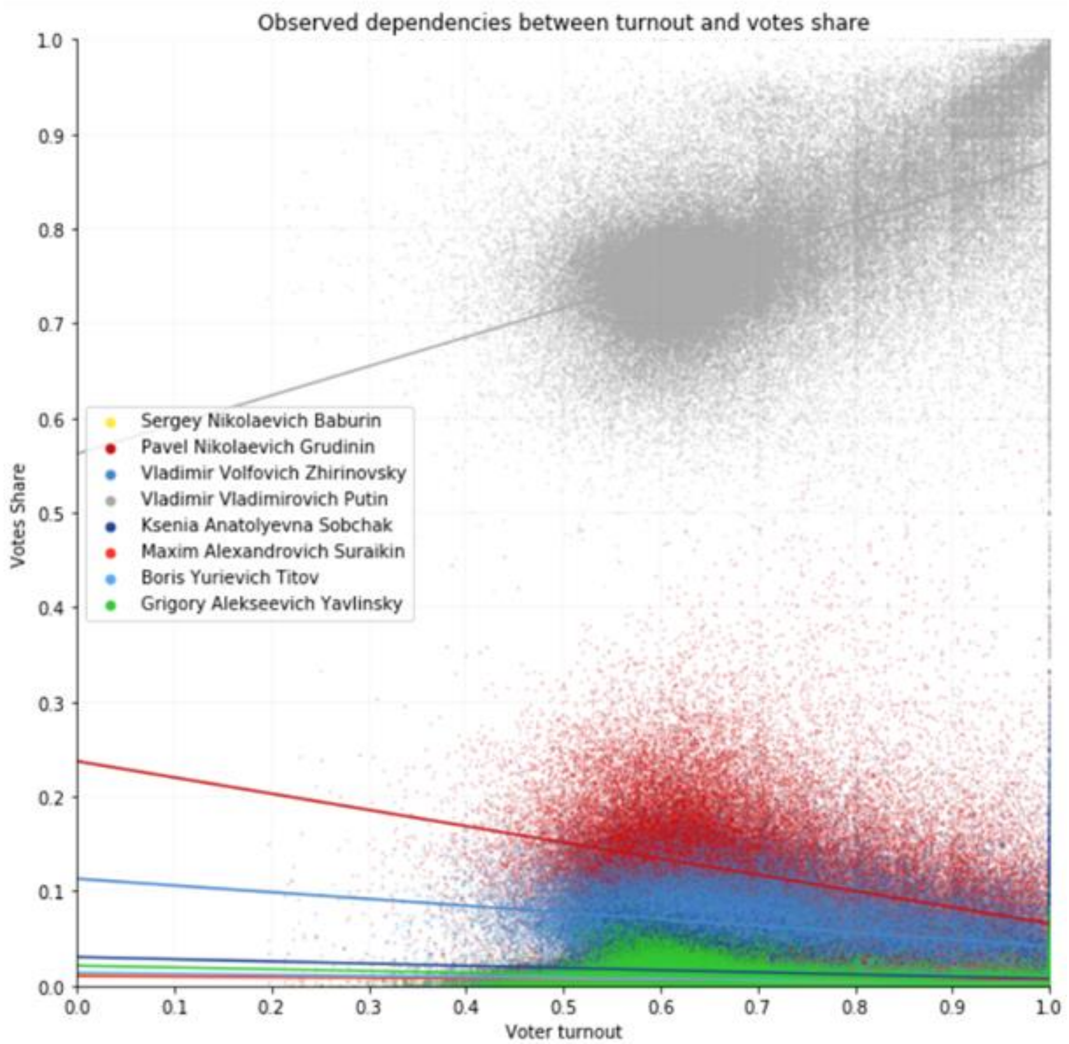
Votes share

The scatter plot of votes share is less clear than a stack plot but reveals an anomaly of a visible correlation between turnout and votes share of candidates



Votes share - correlation

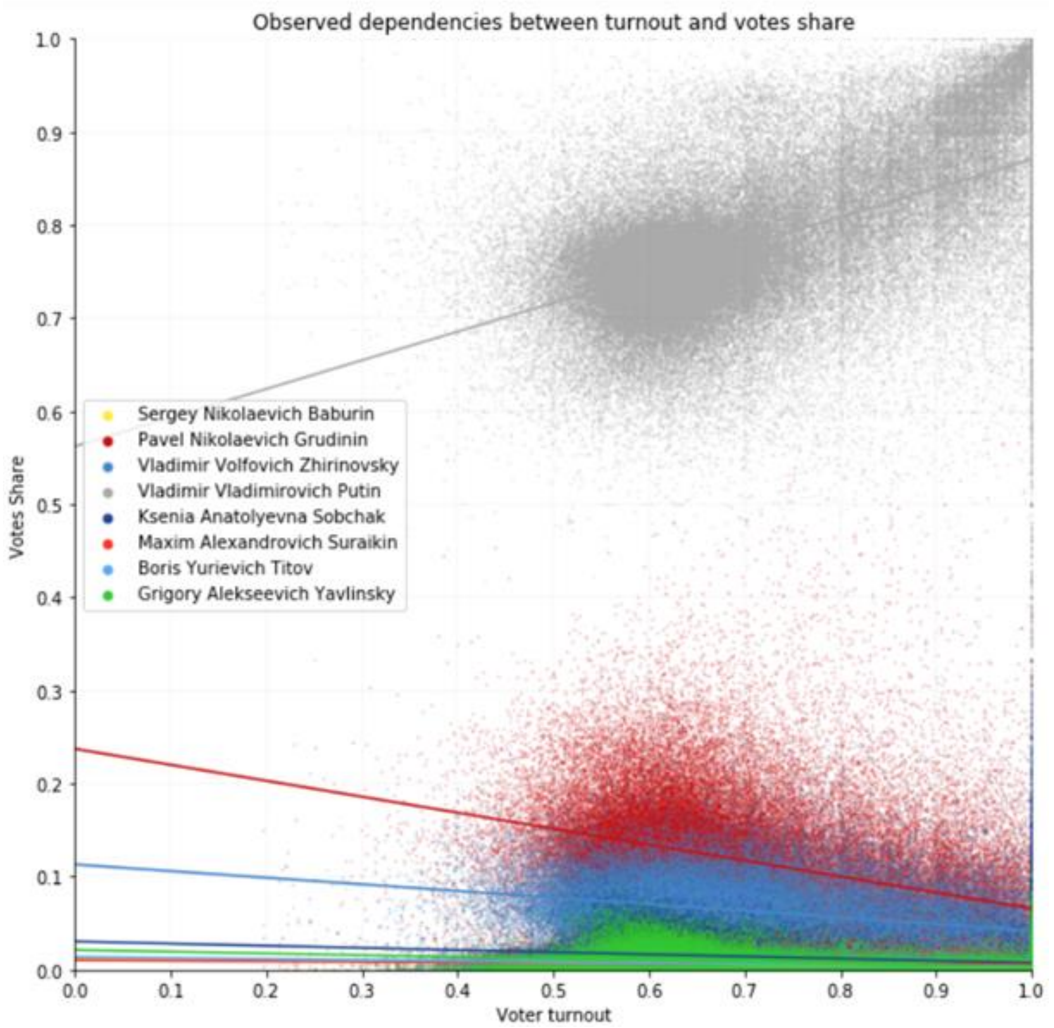
Fitting a linear regression to votes share of every candidate reveals existence of a positive correlation for the leader candidate, whereas every other candidate suffers a negative correlation between the turnout and votes share.



Votes share - correlation

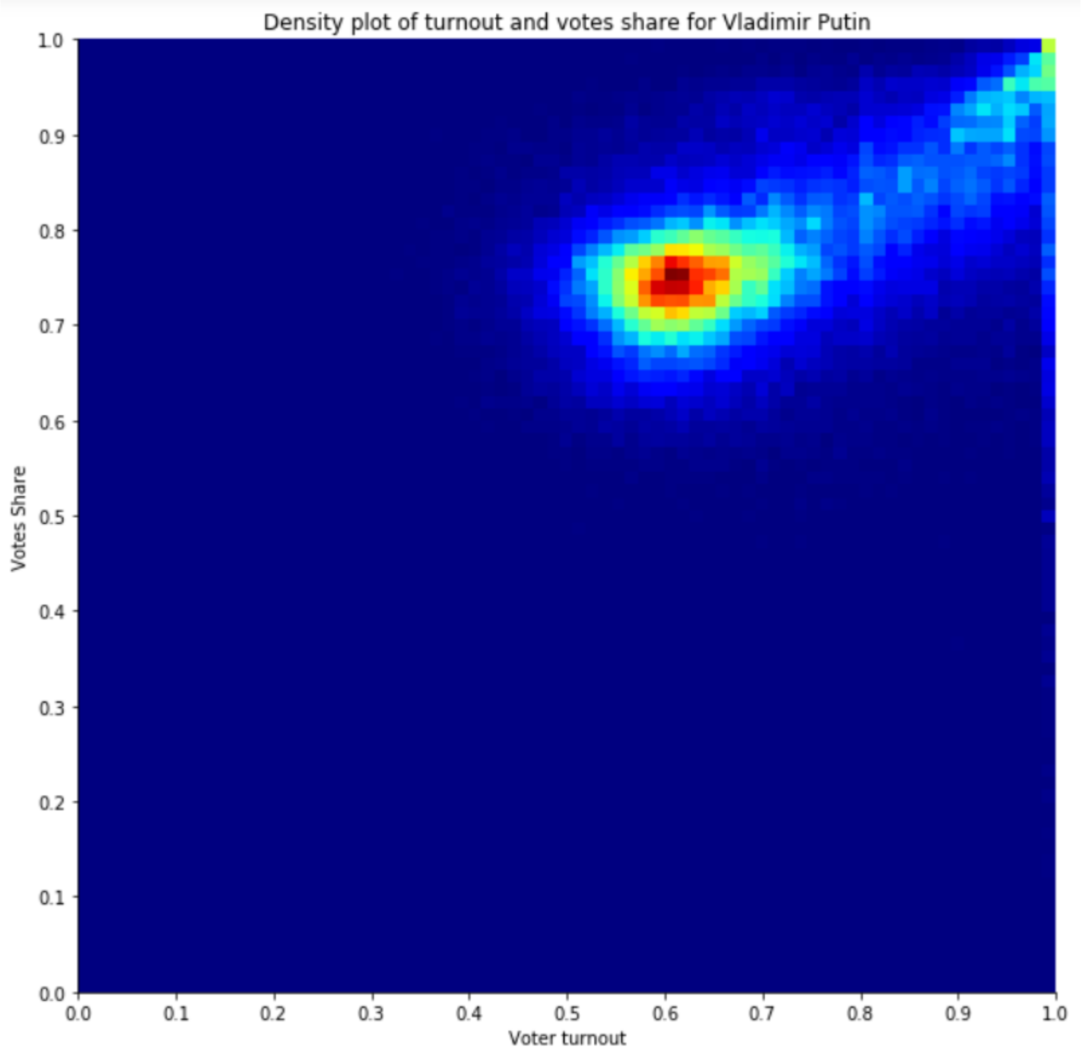
Building the confidence intervals for parameters of built linear regression functions confirms that only the leader candidate has a positive correlation between the turnout and votes share. Furthermore, other candidates except the second leader have non-existent slope coefficients, which mean that they do not significantly lose the votes share with increase of turnout. But the second leader suffers from a significant negative correlation.

```
Sergey Nikolaevich Baburin
Intercept      0.011367  0.011757
Turnout_slope -0.007809 -0.007276

Pavel Nikolaevich Grudinin
Intercept      0.235443  0.239052
Turnout_slope -0.173881 -0.168949

Vladimir Volfovich Zhirinovsky
Intercept      0.111985  0.114221
Turnout_slope -0.072585 -0.069529

Vladimir Vladimirovich Putin
Intercept      0.559681  0.564483
Turnout_slope  0.304791  0.311355

Ksenia Anatolyevna Sobchak
Intercept      0.029960  0.030866
Turnout_slope -0.023698 -0.022460

Maxim Alexandrovich Suraikin
Intercept      0.010027  0.010463
Turnout_slope -0.005542 -0.004946

Boris Yurievich Titov
Intercept      0.013613  0.014027
Turnout_slope -0.010983 -0.010418

Grigory Alekseevich Yavlinsky
Intercept      0.020956  0.021589
Turnout_slope -0.019125 -0.018261
```



Votes share - fingerprints

2-dimensional density plot of voter turnout and votes share of Vladimir Putin reveals another anomaly. The density plot forms 2 distinct fingerprints with centroids at (0.62, 0.73) and (100, 100)



Votes share - findings

Observation of votes share of candidates with relation to voter turnout of polling stations reveals two malicious fraud anomalies

First is the double fingerprint on a scatter plot for a leader candidate where the second peak correspond to extreme (100, 100) point. Such anomaly is always an evidence of reckless fraud according to Klimek et al. (2012)[2] who observes the same anomaly on other Russian elections in 2011 and 2012 and on elections held in Uganda.

The second is the positive correlation between voter turnout in polling stations and votes share of the leader candidate. This is the most severe anomaly which confirms the fraud of both the turnout and votes share. Furthermore, observed linear regression coefficients built on these values point to the fact that malicious polling stations transferred the votes from the second leader candidate to the first one, effectively lowering the concurrency between them.

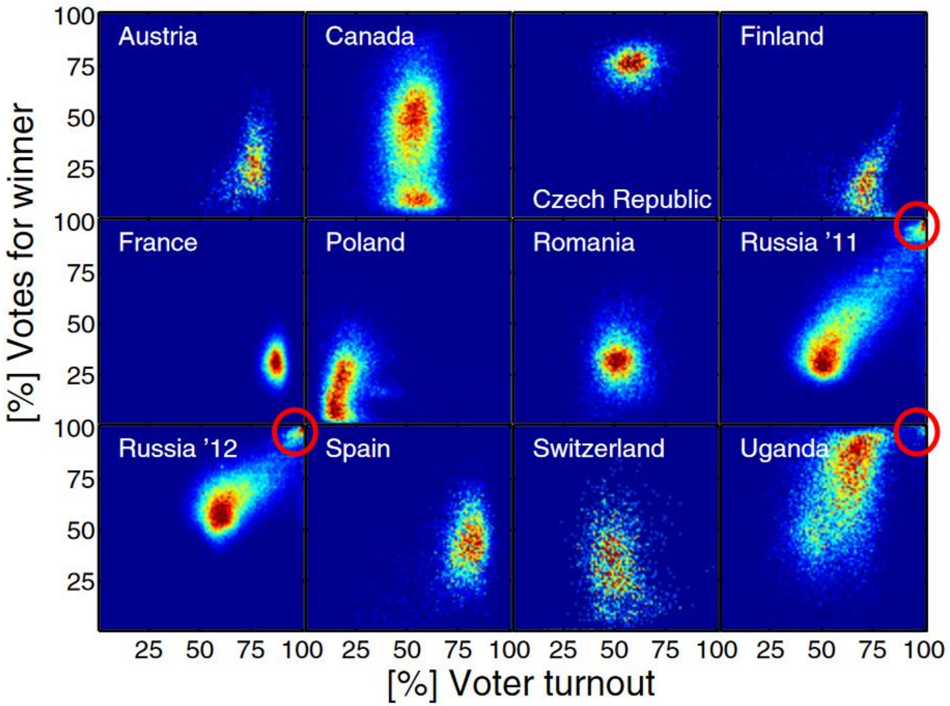
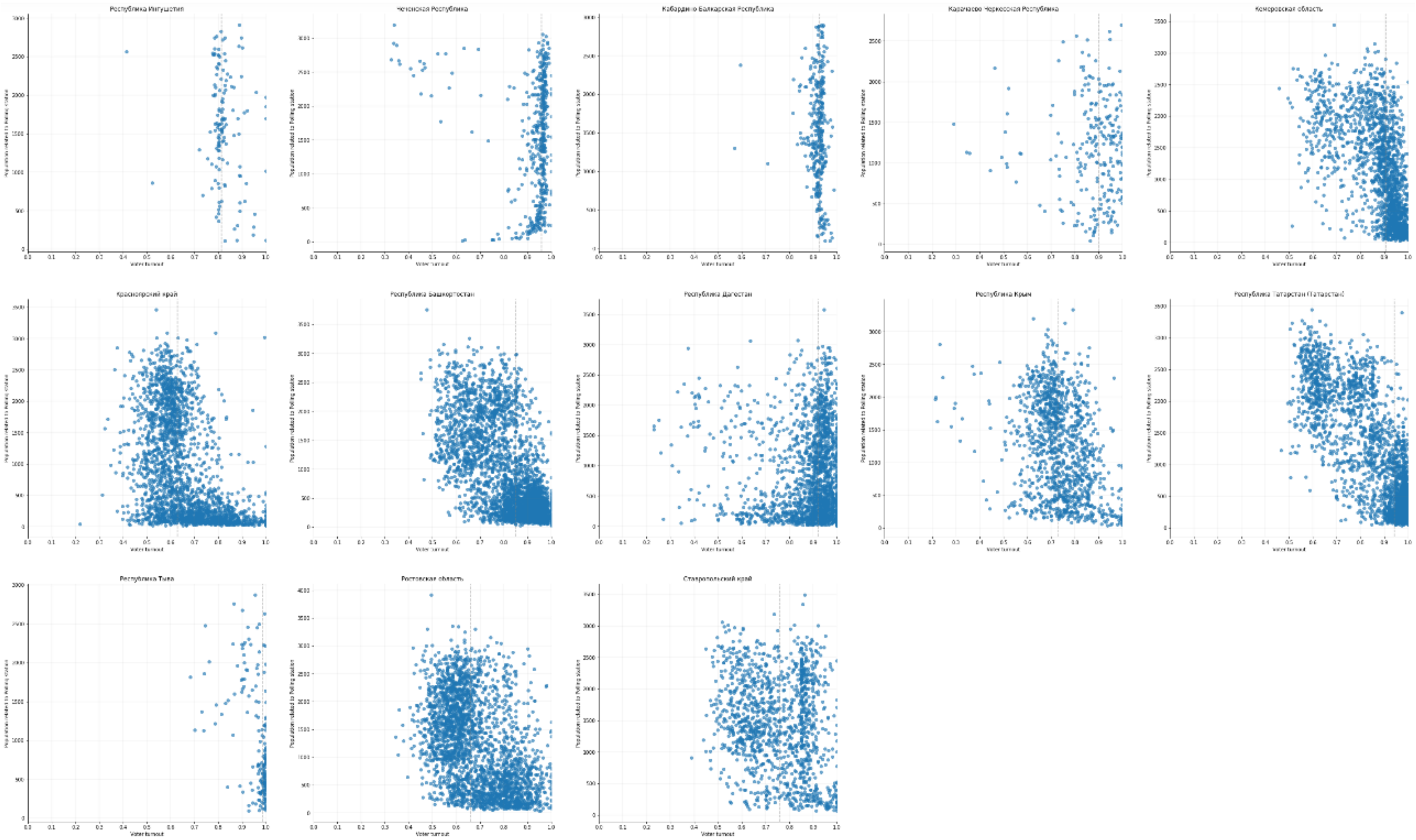


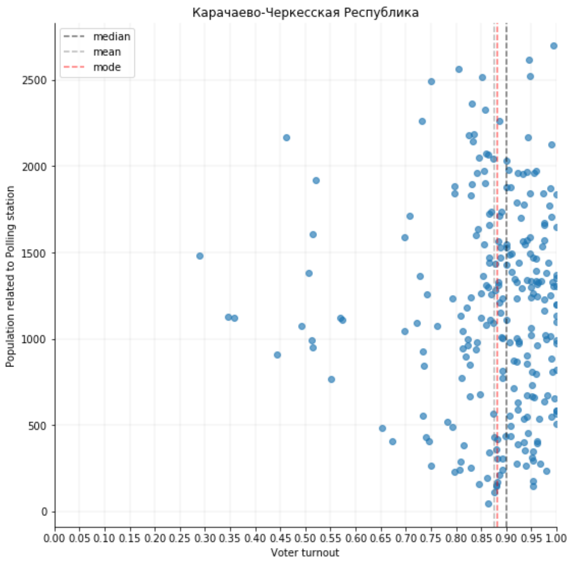
Figure from paper of Klimek et al. (2012)[2] demonstrating double fingerprints

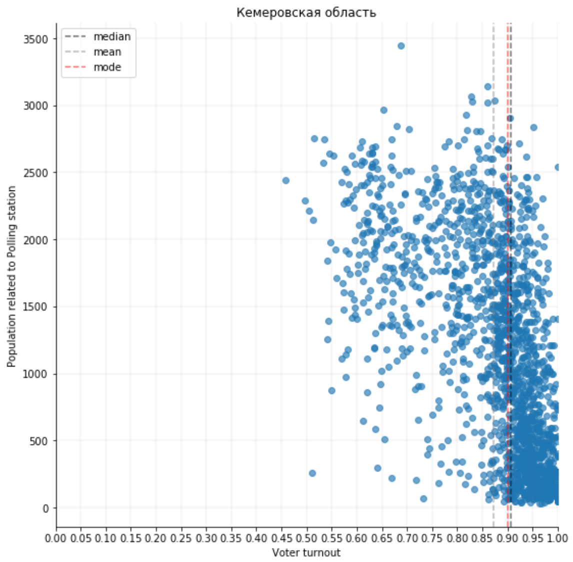
Fraud regions

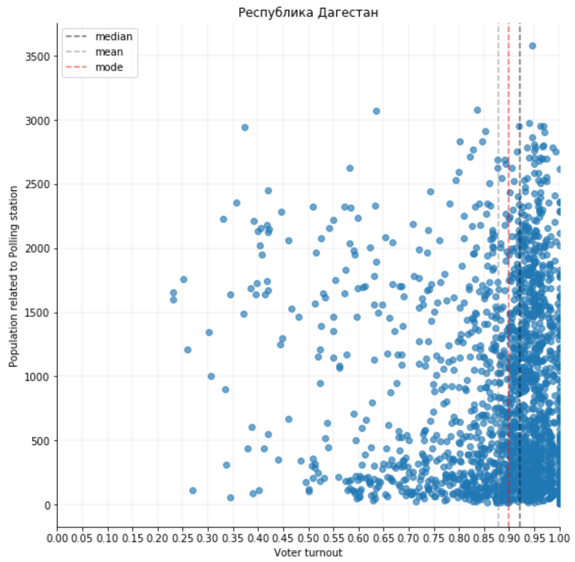
Using extracted information we can observe the most notable in terms of fraud regions in the country. Following plots are constructed for the regions filtered by scripts according to the biased turnout median, observed earlier round number anomaly and correlation anomaly.

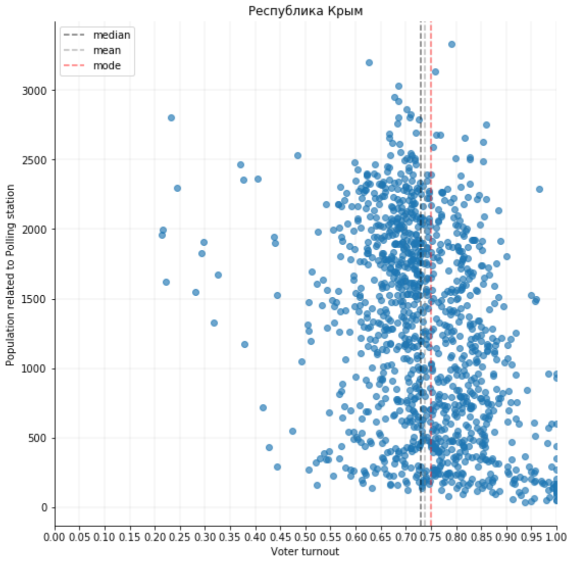


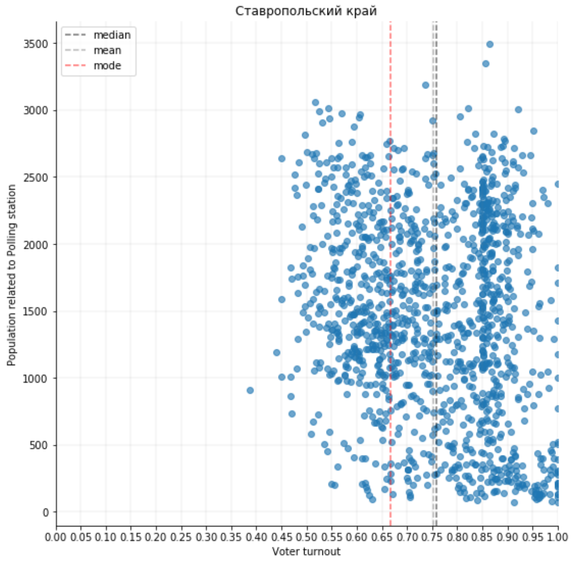
Fraud regions – group 1







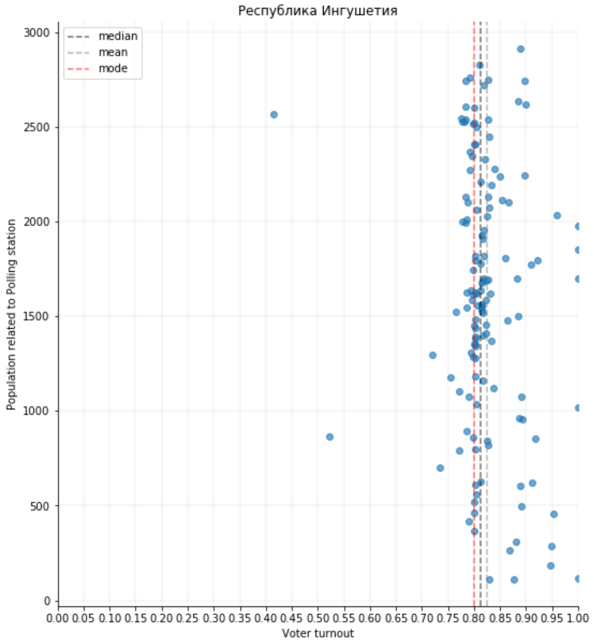


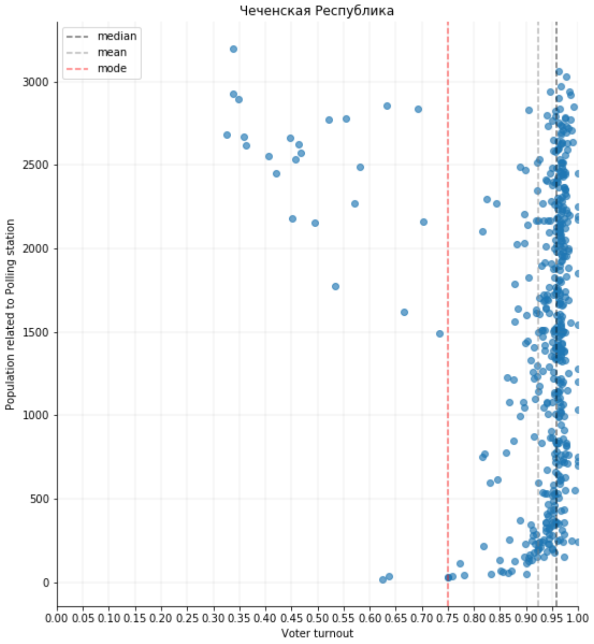


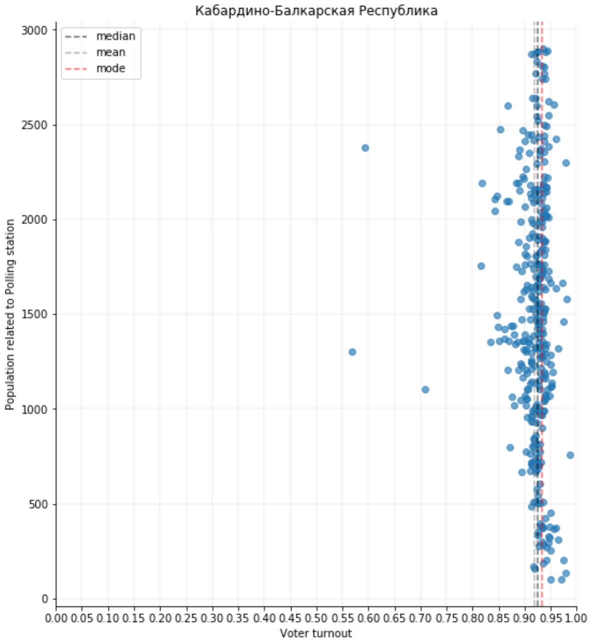
First group of regions demonstrates a strong turnout percentage tendency across the polling stations of all sizes. Median, mean and mode values are tightly placed together demonstrating the value which polling stations tried to fraud. The only exception in the group is the last plot of Stavropol Region, where median, mean and mode values are displaced from the fraud value equal to 0.85 by a strong noise.

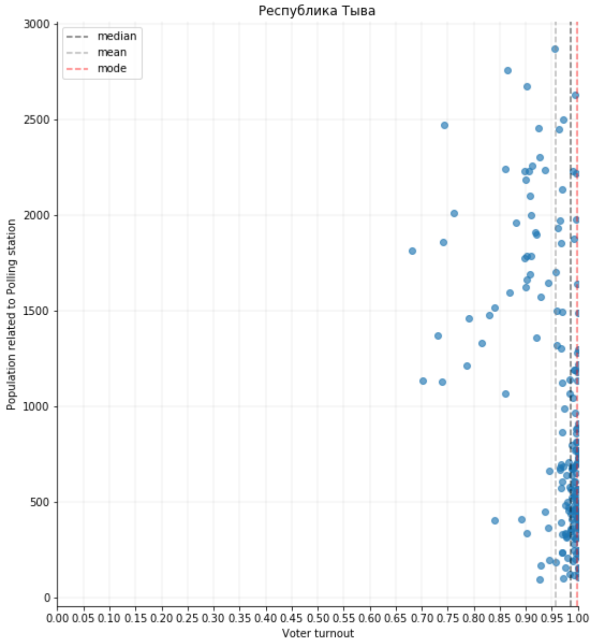
Fraud regions – group 2

The second group demonstrates a more extreme case of regions which completely falsified the election results. Almost every polling station in these regions have the same turnout value. The outlier polling stations are probably the stations kept under surveillance of independent election observers.







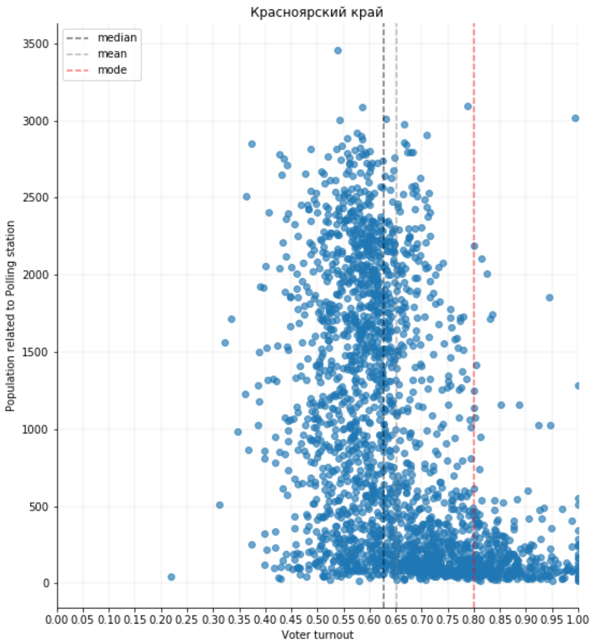


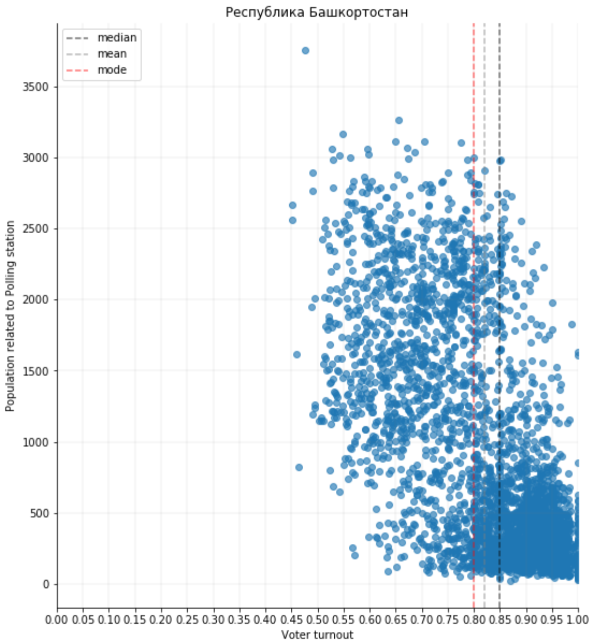
Fraud regions – group 3

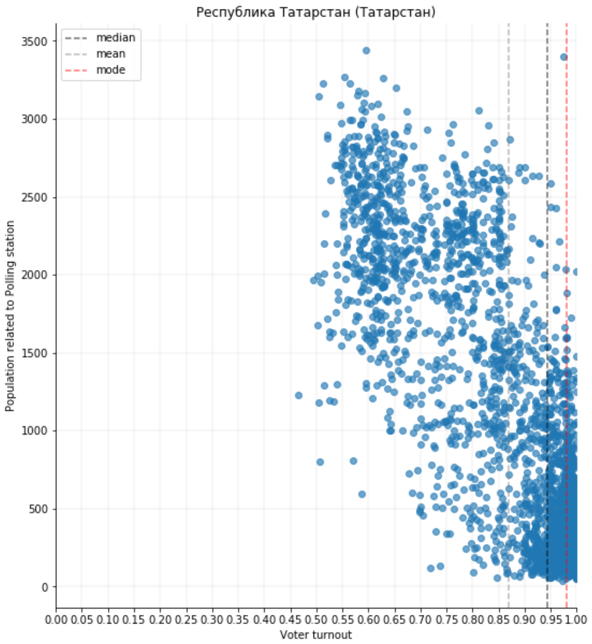
The third group represents large and highly-populated regions that affected the general turnout ratio across the region by falsification of the values corresponding to small polling stations.

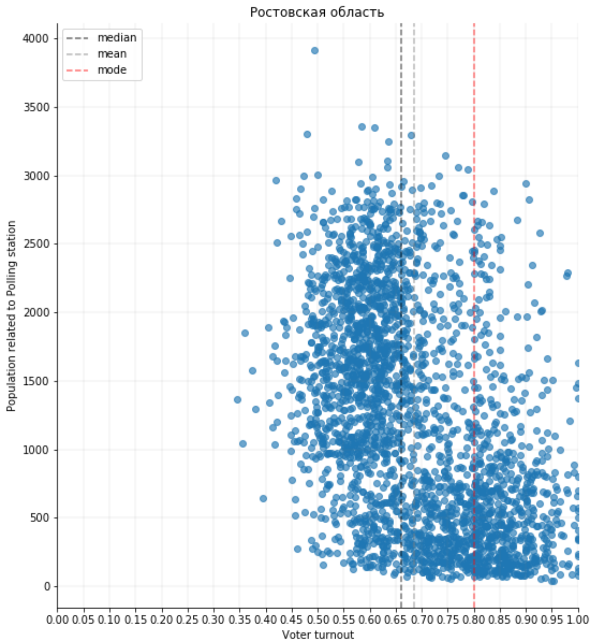
The polling stations of these regions could be clusterized into 2 groups:

1. polling stations with high size of attached population controlled by independent election observers. This group is placed in the middle of the plot and is dispersed across the directions
2. small polling stations in sparsely populated areas affected by falsification. The group is tightly placed in a lower right corner of a plot.









Fraud regions – findings

Every detected region follows an anomalous behavior. The first and the second groups follow the same principle of mass falsification by pulling up the turnout and hereby the leader candidate’s votes share to some predefined on region level threshold. The third group tends to increase the general region value of turnout by falsifications carried on small polling stations. We can agree that the fraud in these regions has been detected correctly because several works[5][6] already pointed to their region-level falsifications.

[5] Alexander Shen. "Elections and statistics: the case of "United Russia"", 2009-2018. arXiv:1204.0307
[6] Kobak, Dmitry, Sergey Shpilkin, and Maxim S. Pshenichnikov. "Statistical anomalies in 2011-2012 Russian elections revealed by 2D correlation analysis." *arXiv preprint arXiv:1205.0741* (2012).

Summary

This work demonstrated an application of election fraud detection methods on the 2018 Russian presidential election data. The study was conducted using only the data collected from official governmental Web-pages of polling stations. The work revealed statistically significant anomalies in voters turnout data distribution demonstrating its positive skewness, abundance of round numbers and enormous peak at 100 % voter turnout. The analysis detected the fraud anomalies in votes share distribution revealing a positive correlation with turnout for the leader candidate and detected a double fingerprint aggregation in its density. Using this information, several most fraud-efficient regions were extracted from the full dataset and grouped together according to some common criteria.

For reproducibility of the results the Jupyter Notebook containing the code used for the study is published on Github:
https://github.com/LeviiBereg/elect2018

Used links and materials

[1] GitLab program repository: https://gitlab.com/modos189/cikinfo
[2] Klimek, Peter, et al. "Statistical detection of systematic election irregularities." *Proceedings of the National Academy of Sciences* 109.41 (2012): 16469-16473.
[3] Federal law №67-ФЗ http://cikrf.ru/law/federal_law/zakon_02_67fz_n/zakon_02_67_full.html
[4] Kobak, Dmitry, Sergey Shpilkin, and Maxim S. Pshenichnikov. "Integer percentages as electoral falsification fingerprints." *The Annals of Applied Statistics* 10.1 (2016): 54-73.
[5] Alexander Shen. "Elections and statistics: the case of "United Russia"", 2009-2018. arXiv:1204.0307
[6] Kobak, Dmitry, Sergey Shpilkin, and Maxim S. Pshenichnikov. "Statistical anomalies in 2011-2012 Russian elections revealed by 2D correlation analysis." *arXiv preprint arXiv:1205.0741* (2012).