Competition for water: annuals & perennials

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1 Annual model with seed survival dependent on dry-season length

In the original annual plant model, the population dynamics of a species i are given by:

$$\frac{N_{i,T+1}}{N_{i,T}} = F_i G_i^B t_i^B \tag{1}$$

where $t_i = \left[\sum_{j=1}^{i} \frac{W_{j-1}-W_j}{\sum_{k=j}^{Q} N_k G_k^L}\right]$. Now, assume that the quantity of seeds produced by species i declines linearly in the time between when species i finishes its growing season and the start of the subsequent growing season. As a result, equation 1, becomes:

$$\frac{N_{i,T+1}}{N_{i,T}} = F_i G_i^B t_i^B - \mu_{s,i}\left[T_0 - t_i\right] \tag{2}$$

where $\mu_{s,i}$ is the rate at which the density of seeds of species i declines after being produced and T_0 is the length of the full season from rain to rain. Under this formulation, there is no simple expression for a species' break-even time, $\tau_i^B = t_i^B$. Instead, species i's break-even time is determined by solving the following expression for t_i^B:

$$1 + \mu_{s,i} T_0 = t_i \left[F_i G_i^B t_i^{B-1} + \mu_{s,i}\right]$$

This expression can be solved explicitly for integer values of B, or numerically in general. Regardless, we see that intuitively, increases in T_0 lead to increases in break-even time. This reflects the fact that a longer season translates to more time spent in the soil, during which seeds perish. Increases in $\mu_{s,i}$ likewise result in increased break-even time (because we constrain $T_0 > t_i$). Finally, as in the annual model, increased fecundity and growth rates lead to reductions in break-even time.

2 Annual model with seed survival dependent on dry-season length and persistent seed bank